S C H O O L W I D E

The Write Path I: Mathematics

Teacher Guide

AVID PRESS

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Written by
Jim Donohue
Tim Gill

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Jim Donohue and Tim Gill—Co-Writers/Revisers

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How to Use This Book

The Write Path I: Mathematics Teacher Guide was designed by and for AVID-trained Math teachers to augment the math texts and syllabi utilized in mathematics classrooms in grades 6 through 12. The lessons offered here are exemplars of lessons that are rich in content and pedagogy. The authors and contributors are hopeful that you will "personalize" these lessons and use them as a springboard to create extensions and improvements that support your grade-level content standards and lessons. A Learning Resource Log is provided on the following pages as a place to record your thinking during the Write Path I Educator Training.

WICR

The AVID-influenced classroom is distinctive with clear evidence of writing as a tool for learning, inquiry methodologies, collaboration, and reading as a tool for learning (WICR). WICR-infused lessons provide students at all levels with an opportunity to practice the literacy skills they must master in preparation for accessing rigorous course work and post-secondary education access and success.

Writing is basic to thinking, learning, and growth. It allows students to think in complex ways by clarifying and ordering experiences. It contributes to self-knowledge and helps students to become better readers. The *Write Path I: Mathematics'* lessons in note-taking, learning logs, and writing about mathematics will provide students with the opportunity to write and think like a mathematician.

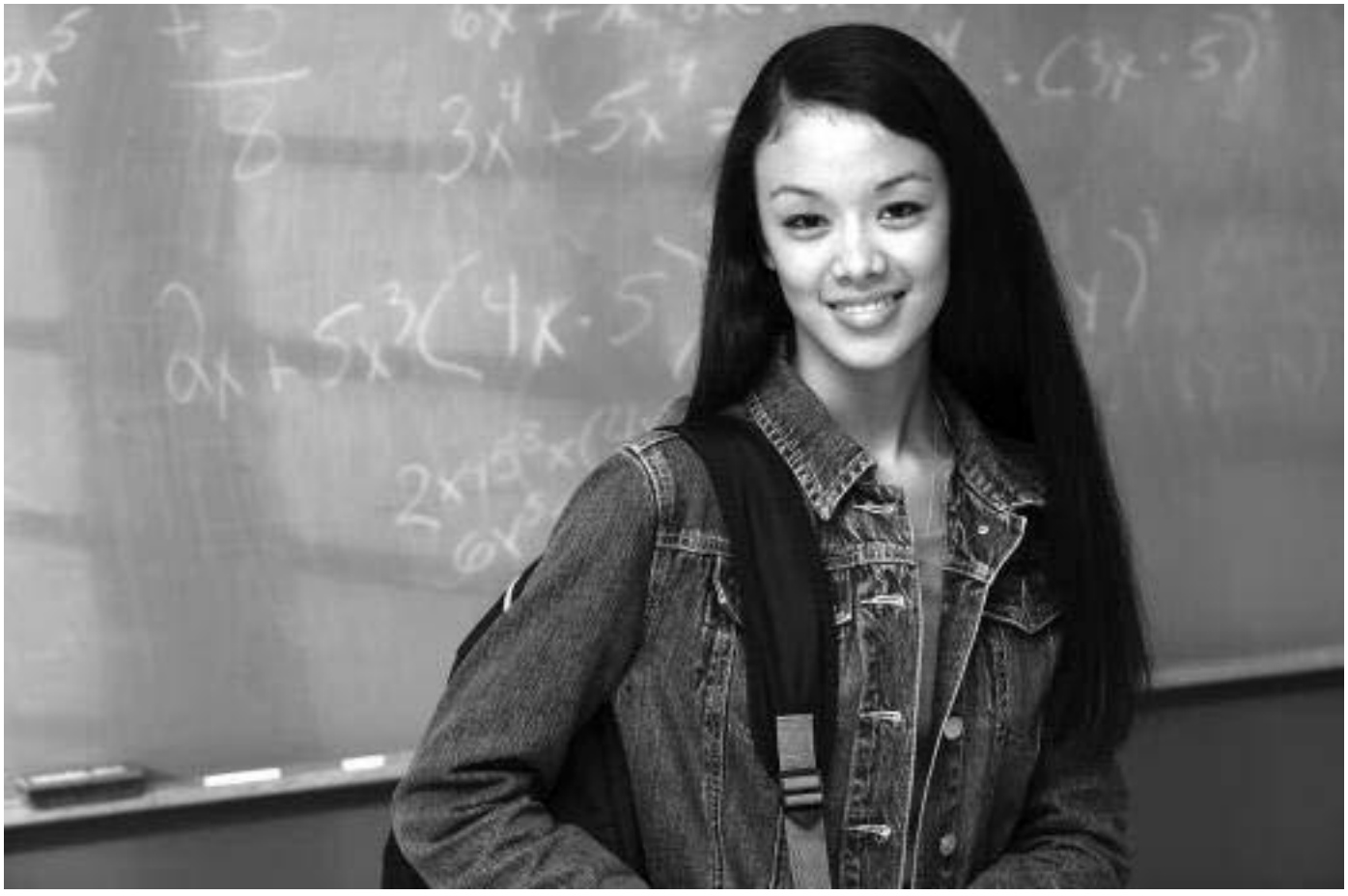
Inquiry immediately engages students with their own thinking processes. It teaches them to think for themselves instead of chasing the "Right Answer." Student ownership for expanded understanding of concepts is the result. *The Write Path I: Mathematics'* lessons introduce students to strategies for crafting higher-level questions using Arthur Costa's model of intellectual functioning. Strategies such as Philosophical Chairs, inquiry-based tutorials, and Socratic Seminars along with skillful questioning will empower students to gain mastery of their own learning.

The AVID philosophy and belief is that students learn best when they are actively manipulating materials by making inferences and then generalizing from those inferences. Collaborative learning groups encourage this kind of thinking. *The Write Path I: Mathematics'* lessons emphasize collaborative group work whenever possible. In small groups, students will ask, explore, and answer questions as they become better listeners, thinkers, speakers, and writers. They will discover ideas and remember them because they will actively engage them. Their teachers will become coaches, carefully guiding them in their learning.

AVID incorporates strategies that help students become more effective readers when used strategically with rich and varied curricula. The reading strategies introduced in *The Write Path I: Mathematics* are designed to make sense of the mathematics text and help students synthesize their understanding. The lessons that are included in *The Write Path I: Mathematics* will help students understand what effective readers do, how to activate prior knowledge, investigate text structure, and utilize text-processing strategies to improve comprehension of their mathematics texts.

The lessons in *The Write Path I: Mathematics* are divided into four themes/units: Writing, Inquiry, Collaboration, and Reading (WICR). Most of the lessons involve more than one theme. Because writing is one of the most important skills, nearly every lesson includes a writing to learn component. In all cases, higher-level thinking and discourse are emphasized.

In addition to WICR strategies, the AVID-influenced classroom is noted for its incorporation of active learning methodologies. In the following section, you will find a summary of some of the more common methodologies that are integrated into the lessons in the *The Write Path Mathematics* books. The authors and contributors have sought to incorporate an active learning methodology that seemed best suited to the particular lesson or concept. Please use these as opportunities to personalize the lesson by substituting the active learning methodology and content of your choice.



Common Active Learning Methodologies

Note: The names of the methodologies may be different in other texts.

Keys to Success when implementing Active Learning Methodologies

- Start small!
- Plan an active learning methodology, try it out, collect feedback, then modify and try it again.
- Start on the first day of class.
- Explain to students why you are doing this and how it will aid them in the learning process.
- Develop classroom routines for transitions.
- Work collaboratively with a colleague while you're implementing active learning methodologies.

Think-Pair-Share

1. Instruct students to think carefully about a specific topic or a question. This may be facilitated by a quick write.
2. Instruct students to find a partner near to them.
3. When you give a signal, one partner shares his/her answer to the question and the reasons that support it, while the other partner listens.
4. The partners exchange roles.
5. The partners prepare to share their answers/responses with the large group.

Think-Pair-Share—Squared

1. Participants listen to a question, concern or scenario.
2. Individuals think and make notes about the questions, concern or scenario.
3. Individuals pair and discuss their responses.
4. Pairs join into groups of four and discuss responses.
5. Foursomes prepare to share their answers/responses with the large group.

KWL

1. Draw three columns on chart paper. Label the columns of the KWL chart; What we Know, What we Want/Need to Know, and What we Learned.
2. Identify a text selection or topic for students to consider during the activity.
3. Ask students to brainstorm and enter information in the columns indicating what they know and want/need to know.
4. Provide students with frequent practice.

Carousel Brainstorming

1. Prepare the same number of wall charts as groups.
2. Assign each group to begin at a specific chart. It may be helpful to assign a different color marker for each group.
3. On the first signal—groups move to assigned charts and generate and record as many ideas as possible for that item.
4. On the second signal—groups rotate clockwise to the next chart, review what previous group wrote,

generate and add additional ideas.

5. On the third signal—groups rotate clockwise to the next chart, review what the previous group wrote, generate and add additional ideas … continue until all groups have written on all charts … then,
6. Ask the students to take a "Gallery Walk" of all charts and be seated.

Jigsaw—Home Group/Expert Group

When discussion of new information is desired, but time is limited, use Jigsaw reading/study groupings.

1. Divide students into small groups. The number of sections of the reading or the number of concepts being reviewed or introduced will determine the number and size of the groups.
2. Assign each member of the group a number that corresponds to the section of the text to be read or the concept to be mastered. Each member is responsible for completing one part of the reading or mastering one of the assigned concepts. Encourage students to take notes.
3. Students then leave their "home" groups and form "expert" groups with other students with the same number. Each "expert" group works on its part of the assignment; members assist each other with questions, clarifications, and summaries. In preparation for going back to his or her "home" group as an "expert," each student rehearses and teaches the lesson to the other members.
4. Students return to their "home" groups and share, discuss information, and teach their part of the assignment.
5. Students reassemble as a whole class and share their thoughts and responses.

Jigsaw Sequencing Groups

1. Cut sections of a solution process, reading or proof into individual parts. Each part should have a complete meaning and show a type of transition at the beginning or the end of that section.
2. Form groups of students that correspond to the number of "jigsaw" pieces.
3. Each group member receives a different piece of the text, problem or proof.
4. Each member of the group must then decide where their piece fits in the text, problem or proof.
5. If a student thinks he/she has the first section of the text, problem or proof, the student must give the reasons why without letting the group read the section. He/she tells the group, "I think I have the first piece because…"
6. If the group agrees that it is the first section, the student reads the text, problem or proof aloud to the group and then places it on the table.
7. The group then proceeds to look for the next section following the same rules as above.

Numbered Heads Together

1. Place students in groups of four.
2. Have students in each group number off from one to four.
3. Ask students a question for discussion or review.
4. Have students discuss the question in their groups, making sure that each member of the group can answer the question if called upon.
5. Select a random number corresponding to a number of a group member.
6. Select one or two students to respond to the question. Additional students with the same number can respond to the question by adding new information to the previous response(s).

Fishbowl

1. Set up a small inner circle of students to demonstrate an activity for the class. Have all other students form a larger outer circle around the inner circle (Fishbowl group) of students.
2. The inner circle (Fishbowl) listens carefully to teacher directions and then demonstrates the activity to the rest of the class.
3. As necessary, clarify and correct the activity steps with the Fishbowl group.
4. Debrief with the entire class.

   Note: The Fishbowl can also be used as a type of Socratic Seminar, where the inner circle of students participate in a discussion and the outer circle students listen and take notes. Later, the outer circle students can comment on the discussion, using their notes and then exchange places with the fishbowl students.

Novel Ideas Only

1. Place students in groups and assign groups to list ideas about a given topic. Set a time limit for the task.
2. Have a spokesperson from each group stand and share one "novel" idea from the group's list.
3. Students in each group must listen attentively to ensure that no group repeats information already provided by another group. (Each group spokesperson can only give information not previously mentioned.)
4. As students hear an item shared by another group, they check it off their own group's list
5. Each spokesperson sits down after they have either read or checked off all the items on their list.
6. The activity continues until all "novel" ideas about the topic have been shared and all students are again sitting down.

Novel Ideas—Four Corners

1. Allow students to divide themselves into four groups based on their perceived level of understanding or mastery of a question or concept.
2. Ask the groups to brainstorm all that they know about the question or concept.
3. Ask a representative from the level one group to share all that was on their group's brainstorm list.
4. Proceed in turn with each sequential group allowing them to share new information not previously mentioned.
5. Finish with the group that perceived themselves as having mastered the material.
6. Clarify misconceptions and misstatements.

Inside/Outside Circles or Parallel Lineups

1. Divide students into two equal groups.
2. Place half the group in the inner circle directly facing a member of the second half of the group in an outer circle. (Alternatively, form parallel lines.)
3. Provide a limited amount of time for the partners to quiz each other on vocabulary, review questions or to discuss another teacher-designated topic.
4. Have the outer circle move to the left (or right) two or three partners down. With parallel lineups, have one or two persons at one end of the line walk quickly to the other end of the line, and all other move one or two spaces to face a new partner.
5. Repeat step 3.

Give One/Get One

1. Ask each student to make a list of ideas related to a teacher-generated topic or question on a sheet of paper. Give students two to three minutes to create as long a list as possible.
2. Tell students to draw a line after their final idea.
3. Have students stand with their list in hand and talk, one on one, with as many other students as they can in a period of three to five minutes.
4. Students must give each other student they meet an idea from their list; they must also write down one new idea from each partner's list.
5. At the end of the activity, create a class list of information completed from the individual lists of students.

Talking Chips

1. Have students each create three name cards ("Talking Chips").
2. During discussion groups, have student take out their name cards ("Talking Chips"). Tell them that when they are ready to contribute to the discussion they must place one of their chips in the center of the table. When they do this, all other students at the table must stop talking and listen attentively.
3. When students have used up all of their talking chips, they must wait for others to use theirs up, too, before they can contribute to the discussion again.
4. Once all chips are in the center of the table, they can be redistributed and all participants invited to join in the discussion again.

Take Five

This process is used to gain consensus decision-making. It is an effective way to assess group needs and gather information for problem-solving.

1. Divide the group into smaller groups of four or five students each.
2. Provide quiet time for each student to complete a 5 to 10 minute quickwrite.
3. Provide time for groups to collaborate and brainstorm.
    a. Each student should share his or her writing one at a time.
    b. Groups should look for common themes and record consensus.
    c. Each group should then share their top agreements/priorities with the larger group.
4. The larger group records common themes/priorities.

Parking Lot

1. Provide students with sticky notes on which they can record questions and concerns. Designate a location in the room for students to "post" their questions and concerns.
2. Encourage students to add to the Parking Lot at any time.
3. Check the Parking Lot frequently and address any notes that have been posted.

Consultation Groups

1. Divide the group into small issue groups based on needs or interests.
2. Utilize a Jigsaw structure or other reporting out or sharing strategy for the "experts" to share what they know or have learned.

Whip Around

1. Divide students into small groups of four to five students each.
2. Present a question or discussion prompt.
3. Give a time limit, usually two to three minutes.
4. Going around the group sequentially, each student is provided an opportunity to comment on the question or discussion prompt.
5. A student may pass one time, but must comment the next time it is his or her turn.

Popcorn

1. Give students an opportunity to share ideas and comments with the whole group. Students do not have to raise their hand.
2. Standing to share improves thinking, keeps comments short and provides an opportunity to include movement.
3. Record the ideas/comments on chart paper.

Learning Logs—Minute Papers

1. Provide students with the opportunity to synthesize their knowledge and to ask unanswered questions during a few minutes at the end of the class. Writing prompts could include:
   - What was the most important thing you learned today?
   - What important question remains unanswered?
   - Variations of these questions, and the student questions and answers they generate, enhance your students' learning process and provide you with feedback on students' understanding of the subject material.

Concept Mapping

1. Ask students to create visual representations of models, ideas, and the relationships between concepts.
2. Ask them to draw circles containing concepts and lines, with connecting phrases on the lines, between concepts. These can be done individually or in groups,
3. Provide an opportunity for students to share, discuss, and critique the work of their peers.

Note-checking Pairs

At the end of a class segment (after 10 to 15 minutes) ask students find a "Shoulder Partner" to review their notes. The note review activities could include:

- Summarize the three important points.
- Choose the most important idea that will appear on the exam.
- Check the completeness and accuracy of your partners notes.
- Use the notes to solve an example problem.
- Write questions in the left column of their Cornell notes.
- Use the notes to work on a teacher-generated question.

Collect the student work as a formative assessment.

Games

Games such as Buzz counting games, Jeopardy, matching, mysteries, group competitions, puzzles, charades, Scrabble, Pictionary, etc. can be designed to introduce or review specific vocabulary and concepts.

Why Vertical Teams?

Part I: Case Study

California Mathematics Standards

California Mathematics Content Standards, which expect enrollment in Algebra for all students at the eighth-grade level, are now more closely aligned with the competitive realities of admission to the state's better four-year universities. In California and nationally, barriers exist that inhibit schools in their attempts to successfully meet this new expectation. The gap between the mathematics that has historically been taught to most eighth graders and the newly adopted state standards is tremendous. The shortage of well-prepared mathematics teachers is another limiting factor.

The truth is that students who do not take a minimum of five years of college preparatory mathematics culminating in Advanced Placement Calculus or Advanced Placement Statistics are far less likely to be admitted to the nation's more competitive public and private institutions. The most likely scenario for such a sequence requires that students successfully complete college-preparatory Algebra as eighth graders.

Nevertheless, the legislation of standards by itself cannot change the attitudes and practices of those educators, parents, and students who for years have accepted college-preparatory Algebra at eighth grade as an honors course, with limited access, reserved only for the "best and brightest." It will take the ongoing work of educators committed to equity and access to profoundly impact the systems that have effectively denied access and impeded success for some groups of students far more than others.

The mandate of "Algebra for all" and the shift of that curriculum from the high school level to the middle school will require additional assistance for middle-level mathematics teachers, parents, AVID coordinators and tutors, all of whom will need to support young people from Algebra through Calculus. One means of creating viable support systems is through the formation of Mathematics/AVID Vertical Teams.

The Concept of Mathematics/AVID Vertical Teams

To help counteract the barriers that have prohibited equal access to honors and Advanced Placement (AP) mathematics courses, it is recommended that middle schools and high schools form Mathematics/AVID Vertical Teams. Their challenge should be to address the question:

> *"What would it take to build a mathematics program so strong and inviting that a large percentage of students—perhaps every student—could be prepared to successfully complete challenging mathematics courses, such as calculus, before leaving high school or upon entering college?"*

The Advanced Placement Program Mathematics Vertical Team Toolkit 1998, funded by The College Board and Dana Center, suggests the vision, evidences the need, and recommends the purposeful and necessary work of a mathematics-focused vertical team.

AVID: Advancement Via Individual Determination

AVID is a nationally recognized program designed to give students who ordinarily would not be programmed into rigorous, academic, college-preparatory classes the opportunity to take such classes and the support necessary to succeed in them.

The core of this support structure is the AVID coordinator, whose elective AVID course aims to help students develop the skills they will need to succeed in rigorous classes. By coupling high academic expectations with strong, persistent support, ordinary students are enabled to do extraordinary things. Writing to Learn, Inquiry,

Collaboration, and Reading to Learn (the WICR methodologies) are emphasized, and students experience college entry tests and writing activities. They learn and practice study skills, they participate in collaborative study groups facilitated by college tutors, and they enrich their secondary years by learning about college and careers. Fears of a system that seems unapproachable are assuaged as their confidence and academic achievement increase. AVID parents become partners in the preparation for college, and teams of educators at the school follow an action plan to fulfill the mission of AVID.

AVID is designed to increase schoolwide learning and performance. The mission of AVID is to ensure that all students, and most especially students in the middle, are capable of completing a college-preparatory path and:

- Will succeed in the most rigorous curriculum;
- Will enter mainstream activities of the school;
- Will increase their enrollment in four-year colleges; and
- Will become educated and responsible participants and leaders in a democratic society.

"The students of the 21st century desperately need educational opportunities. The long-term economic success of the nation and the perpetuation of democracy are going to hinge on building an education system to accommodate and prepare all of our students. A society that defines itself as a democracy is obligated to create and sustain public education for the full, broad sweep of its citizenry."

—Mary Catherine Swanson, AVID Founder

Tomorrow's Jobs and the Need for a Better Educated Populace

"In recent years, the level of educational attainment of the labor force has risen dramatically. The trend toward higher educational attainment is expected to continue. Projected rates of employment growth are faster for occupations requiring higher levels of education or training than for those requiring less. Workers in occupations requiring higher levels of education have higher incomes. Many of the occupations projected to grow most rapidly between 1992 and 2005 are among those with higher earnings."

—*Occupational Outlook Handbook,* US Dept. of Labor's Bureau of Labor Statistics

"Increasingly, over the past 30 years, new jobs have been filled by people with a college degree, including more than 90% of the new jobs created since 1980."

—*Source:* McCarthy & Verney, *Immigration in a Changing Economy: California Expansion,* RAND 1997

The US Department of Labor predicts that, "For 12 of the 20 fastest growing occupations, an associate degree or higher is the most significant level of postsecondary education or training."

—Retrieved February 24, 2008, from <http://www.bls.gov/oco/oco2003.htm>

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Part II: Advanced Placement Calculus

Brief History

The Advanced Placement Program began over four decades ago to enable students to complete college-level studies while they are still in high school and to obtain college placement or credit on the basis of their

performance on rigorous AP® examinations. The AP program is administered by the College Board which contracts with Educational Testing Service (ETS) for technical and operational education services.

There are numerous AP courses and examinations offered in a variety of disciplines including Calculus AB, Calculus BC, and Statistics.

Most university applications specifically ask for a list of all AP courses taken in high school, and admissions officers use AP coursework as a significant part of their selection process. Students who take AP courses and exams have a distinct advantage in being accepted to a university. Because AP coursework is more rigorous, students with the AP advantage are better prepared for college coursework.

AP Mathematics Prerequisites

Prior to enrolling in an AP Calculus or AP Statistics course, students are expected to take the following sequence of courses: Algebra I, Geometry, Algebra II/Trigonometry, Precalculus or equivalent integrated courses that cover the same content and have prior college approval. Since these courses are sequential, students must begin the sequence in the eighth grade or attend summer school.

Students enrolled in Advanced Placement courses experience:

- Significant increase in mathematical knowledge
- Increased chances of acceptance by the best universities
- Increased number of possible university majors
- Increased confidence in their ability to handle university-level work
- Cost-saving advantages through placement in advanced standing
- Time-saving advantages through advanced placement
- Priority registration at some institutions

Access for AVID Students to Rigorous Mathematics in High School

The AVID mission and practices have always expected students to enroll in rigorous coursework, but most students at most schools are not yet being accepted into AP Calculus or AP Statistics as high school seniors. Traditionally, a majority of students do not take or successfully complete the prerequisite Algebra course in the eighth grade and so they are not eligible for the highest track of mathematics in high school.

In *Class Struggle: What's Wrong (and Right) with America's Best Public High Schools* (1998), Jay Mathews concludes that students will strive for the best if they get the chance. He estimates that at least 25,000 students are told each year that they cannot take the AP courses they want, and another 75,000 students and probably far more have the ability to do well in such courses but do not ask to enroll because no one encourages them to do so.

CREATE (Center for Research, Evaluation and Training in Education) published the following recommendation in an executive summary based on findings from a longitudinal study of the AVID program.

> "Increase the emphasis on algebra in middle-level AVID. Early indicators in this study show that, in the first semester of high school, students who took algebra in middle grades out-performed students who did not. **In fact, algebra was the single most critical predictor of GPA and college preparation credit accumulation in the one term studied thus far."**
>
> —*Longitudinal Research on Middle-Level AVID: Year 2 Report – Executive Summary,* 1998

As part of their ongoing work, vertical team members will want to examine the attitudes, practices, and policies that exist within their communities and effectively limit access to rigorous mathematics. Discussions related to these issues are more powerful when connected to appropriate data. As stated previously, teams may want to collect data to describe the local situation and make comparisons to state and national realities.

Part III: Common Concerns about Placement of AVID Students in Honors-Track Math Courses

Frequently when teachers, counselors, and parents are asked why they hesitate to enroll AVID middle school students in pre-Algebra and Algebra courses, the following concerns surface in the conversation. For real change to occur, vertical team members need to anticipate the typical fears of others, evaluate their own beliefs and practices, and become proactive in changing the attitudes and climate at their schools. If the vertical team is going to increase the number of students succeeding in Algebra through Calculus and Statistics, it must carefully and critically review current local practices which determine student placement.

Some of the most commonly asked questions and concerns from parents, teachers, and students are presented here with suggested responses.

1. **Concern:** AVID students' self-esteem will be damaged if they don't do well in Algebra.

   Response: At the middle school level, both educators and parents are dedicated to educating the whole child. There is great concern for the student's self-esteem. Yet, real self-esteem does not come from getting good grades in mediocre programs. Rather, self-esteem develops as a result of meeting with success in new and challenging work. Students know the difference. Students know when the work they are doing matters. They also know when the system is praising them through inflated grades for work that is substandard or remedial. A grade of C or D in Algebra might translate to the achievement of higher-level standards more than an A or B in a subject that simply revisits material already taught in previous grades.

   AVID builds self-esteem by believing in its students and in their ability to meet high academic standards. AVID stands by its members when the learning becomes difficult and reminds them that their self-determination is the key to their success.

2. **Concern:** Teachers will need to water down their curriculum.

   Response: It is essential that Algebra teachers continue to maintain high standards in their math classes. It is imperative that the essential content as identified through articulation and vertical team consensus be taught.

   Teachers need staff development to hone their own math skills and to develop strategies that work well with a diverse group of students. They need to learn and practice methodologies designed to increase their effectiveness with a diverse student population.

   Teachers need to trust that through the support of tutors in the AVID elective, students will receive opportunities to strengthen their arithmetic skills as well as additional help with their rigorous Algebra curriculum.

3. **Concern:** Unless students have adequate English language skills they cannot be enrolled in advanced math courses.

   Response: Students who have English language difficulties will indeed find Algebra challenging, especially if the mathematics curriculum at their school is based on acquisition of skills through the process

of solving word problems. However, when collaborative group work is promoted in the classroom, as it is in the AVID elective, students can overcome their language barrier. Much of the symbolism used in mathematics is universal, and placement in honors level mathematics courses is often the arena where students whose first language is not English can excel.

4. **Concern:** Grades will suffer.

   Response: Research on the AVID program has found that as much as two years may be required for students to develop the skills for academic success as demonstrated by C or better grades in college-preparatory classes. This further substantiates the need for enrollment in honors courses, specifically Algebra, at the middle school level. The research also suggests that when students persist and teachers hold them to high standards, their grades improve in time to ready them for the university.

   Even a poor grade in Algebra at the eighth-grade level assures that the student will be placed in Algebra in the ninth grade, thus enabling the child's continued tracking in a college prep curriculum. AVID provides the infrastructure, the immediate intervention strategies, and the support students need to succeed.

5. **Concern:** Parents will worry about their children's placement.

   Response: It is true that parents will be concerned should their child bring home poor grades and may want to have the counselor move the student to a less rigorous mathematics class. But parents need to remember that what matters most at the middle school level is the track to which their children are gaining access. Parents need to be more concerned about access to rigorous curriculum that will open the doors of higher education to their children than they are to grades. A student who "fails" Algebra will know more than a student who is never given the opportunity to be in the Algebra classroom.

6. **Concern:** Heterogeneous classes hurt GATE students.

   Response: "We have mountains of research evidence indicating that homogeneous grouping doesn't consistently help anyone learn better ... no group of students has been found to benefit consistently from being in a homogeneous group." (Oakes, J. (1985). *Keeping track: How schools structure inequality*. New Haven, CT: Yale University Press.)

Rigor in Mathematics

Mathematics teachers are essential members of the AVID site team, dedicated to equity in the classroom and to providing students with access to academic rigor *(AVID Essential 4)*. The importance of mathematics teachers to the future success of students has been delineated in numerous contemporary research findings. Clifford Adelman states in *Answers in the Toolbox* (1999) that "the academic intensity and quality" of a student's course of study is a far more powerful predictor of bachelor-degree attainment than class rank, grade point average, or test scores. He finds that this impact is "far more pronounced for African-American and Latino students than for any other group."[1]

In 2006, Adelman revisited his *Answers in the Toolbox* and the critical role of mathematics teachers was more clearly defined. In *Answers in a Toolbox Revisited*, Adelman states that the "highest level of mathematics reached in high school continues to be a key marker in precollegiate momentum, with the tipping point of momentum toward a bachelor's degree now firmly above Algebra 2."[2] In addition to college access and success, a rigorous curriculum "predicts greater skill in the workforce and greater wage-earning potential. An extensive study conducted by ETS found that 84 percent of highly paid professionals and 61 percent of "well-paid, white-collar" professionals had taken Algebra 2 or higher-level mathematics courses while only 30 percent of low-to-moderately skilled and low-paid workers had done so."[3] These findings make a strong case for all schools to provide all students, not just those enrolled in "college prep," with a rigorous academic program including preparation for and access to Algebra II and beyond. Sadly, not all schools provide equal access to rigor.

Research findings from *Answers in the Toolbox Revisited* (2006) indicate "Latino students … are far less likely to attend high schools offering Trigonometry (let alone Calculus) than white or Asian students, [and that] students from the lowest socioeconomic status (SES) quintile attend high schools that are much less likely to offer any math above Algebra 2 than students in the upper SES quintiles."[2] These finding, among others, make it clear that mathematics teachers are critically important on the AVID site team in ensuring not only equity and access, but that academic support, encouragement, and rigor are a part of every students' school experience. The extent to which opportunities that require high-level cognitive mathematical processing are offered to students at all levels is the charge of the math leaders in our schools and districts. However, these opportunities will not be available to students if a clearly defined scope and sequence of skills and concepts are not identified and vertically aligned.

What is Rigor in Mathematics?

The vital role of rigor in school curricula is unmistakably supported by research. However, an agreed upon definition of what constitutes rigor in mathematics is more elusive. Many professionals in the educational community have embraced the definition put forward in *Teaching What Matters Most: Standards and Strategies for Raising Student Achievement*[4] in which the authors argue that, "rigor is the goal of helping students develop the capacity to understand content that is complex, ambiguous, provocative, and personally or emotionally challenging."

A statement issued by the Institute for Learning captures much of what many educators in mathematics support:

> *Academic rigor in a thinking curriculum holds that students must be exposed to a rich knowledge core that is organized around the mastery of major concepts. This curriculum should provide students with regular opportunities to pose and solve problems, formulate hypotheses, justify their reasoning, construct explanations, and test their own understanding.*[5]

Several versions of standards released by The National Council of Teachers of Mathematics (NCTM) support these definitions of rigor, which promote mathematical thinking, reasoning, and understanding (NCTM, 2000, 1991, 1989). The NCTM has repeatedly extended a philosophy of students as "active constructors of mathematical knowledge, and teachers are to serve as facilitators of students' learning by providing classroom experiences in which students can engage with rich mathematical tasks, develop connections between mathematical ideas and between different representations of mathematical ideas, and collaboratively construct and communicate their mathematical thinking."[6] Briefly stated, students must be given an opportunity to investigate important and worthwhile mathematics with understanding that goes beyond procedural knowledge. This being said, it becomes the imperative of the AVID site team, and more specifically, that of the mathematics leaders on the site team, to help their schools and colleagues define and integrate rigor throughout their curricula.

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Learning Resource Log

Use the Learning Resource Log to record your ideas for modifications to the lessons, keep a summary of the training, and have a quick reference when you return to the classroom.

Tip: Recording your ideas now will make it easier to integrate them into your learning plan later on.

Lesson Name	Location	Notes/Modification

Learning Resource Log (Continued)

AVID Proven Achievement. Lifelong Advantage.

Lesson Name	Location	Notes/Modification

Lesson Name	Location	Notes/Modification

Learning Resource Log (Continued)

AVID Proven Achievement. Lifelong Advantage.

Lesson Name	Location	Notes/Modification

Unit One: Writing in Mathematics

Introduction to Writing in Mathematics

Writing to Learn in Mathematics

It is critical that the mathematics curricula of the 21st century incorporate opportunities for students to become effective communicators. The National Council of Teachers of Mathematics (NCTM) highlights the importance of communication in their standards.

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- analyze and evaluate the mathematical thinking and strategies of others; and
- use the language of mathematics to express mathematical ideas precisely.

Writing to learn in mathematics will benefit students in a number of ways. The practice promotes clear thinking, helps students make connections between math and other disciplines, and it raises the students' awareness of what is known and not known when problem-solving. Writing to learn also helps students raise questions about new ideas, organize their thinking, and perhaps most importantly, it helps students construct meaning out of complex material.

The practice of writing to learn is also beneficial to teachers because it provides invaluable insight into the students' thinking and level of understanding. Those teachers who add writing to their pedagogy will often find it easier to recognize and diagnose their students' conceptual problems. The time teachers invest in helping students clearly explain their thinking will be recouped in instructional time later on as lessons become more prescriptive in nature. Students too often remark, "I can do it but I can't explain it." What is the value to the student to "do" math if they cannot communicate about it? The number one issue separating successful college students and potential employees is the ability to communicate effectively. It will not be enough for 21st century students to master the concepts of mathematics. They will need to be equipped with the skills to successfully communicate highly technical information.

The syntax, vocabulary, and structure of writing in mathematics are unique and the mastery of them requires direct explicit instruction and ongoing opportunities to practice. While some students will resist writing in math class, they need to understand that writing is the single most powerful tool for thinking and learning.

The contemporary mathematics curricula offers a wide variety of entry points for Writing to Learn activities. Daily note-taking, journals, quickwrites, extended response questions, reports, portfolios, and learning logs are only a sampling of the formal and informal writing opportunities in the mathematics classroom.

Getting Started

Start small. Incorporate Cornell note-taking into the daily routine. Try quickwrites or a journal with a particular unit and utilize a rubric to help students understand what it takes to "write like a mathematician." Spend time modeling technical writing and formal oral communication. Students will experience the use of the language of mathematics, begin to understand the kind of writing that is expected of them, and know that it will support their learning. Encourage students to talk about mathematics and to practice their academic language. Use prompts to guide writing and critical reflection. Scaffold activities that will lead to formalized technical writing.

Becoming Better Communicators

Students take a great risk putting their thinking on paper and potentially exposing their lack of deep understanding of what they are doing and why they are doing it. They must simultaneously wrestle with style and content while making public their ideas. For students who do not understand a math concept or a problem solution, the process may seem overwhelming. They are trying to explain new material that they do not fully understand to someone whose understanding is greater than their own. For students who think they do understand the math, the process seems wasteful. "I know what I am doing; I can show my work symbolically; why do I have to talk about it in words?" It is up to the teacher to help such students realize the benefit that writing provides even when it exposes their areas of limited understanding or confused thinking. The mathematics teacher can emphasize that writing helps students remember what they are learning as they find connections with prior knowledge, and that writing about mathematics will be necessary to communicate with others whether inside or outside the mathematics community.

Rubrics

Many writing activities include holistic scoring guidelines that students can use to shape their writing and to evaluate their own work and the work of other students. Prewriting for many assignments might be prefaced with discussion of the rubric. Once students are familiar with rubrics, asking them to develop rubrics of their own can be a valuable exercise. The more students learn to use a well-developed rubric to guide their writing, the easier it will be for a teacher to assess student work, thus encouraging both teacher and students to spend more time writing. A general scoring rubric is provided here as a guide in developing an agreed upon rubric in the classroom.

General Scoring Rubric	
4 Points	• Contains a complete response with a clear, coherent, and unambiguous explanation. • Includes a clear and simple diagram, if appropriate. • Communicates effectively to an identified audience. • Shows an understanding of the question and the mathematical ideas/processes. • Identifies all the important elements of the question. • Includes examples and counterexamples. • Gives strong supportive arguments.
3 Points	• Contains a good response with some, but not all of the characteristics outlined above. • May include an incomplete diagram. • The identified audience may be unclear and the ideas are communicated less effectively. • The understanding of the question and the mathematical ideas/processes is not clear. • Includes examples but counterexamples may not be included or may be unclear. • May include minor errors of execution but not of understanding.
2 Points	• Contains a complete response, but the explanation is muddled. • Presents an incomplete argument. • Includes diagrams that are inappropriate or unclear, or the response fails to provide diagrams when it would be appropriate. • Indicates some understanding of the mathematical ideas/process, but in an unclear way. • Shows clear evidence of understanding some important ideas/processes, while also making one or more fundamental errors.
1 Point	• Omits parts of the question in the response. • The response includes major errors or incorporates inappropriate strategies.

Models

Models, anchor papers, and templates can be powerful teaching tools. Ask students to read the models for ideas. Engage them in critiquing the work. This will help them develop a map for their own writing. The *Problem-Solving and Guide for Solution Write-up* is provided as a model for developing an agreed upon template for the classroom.

Problem-Solving and Guide for a Solution Write-up

SOLVING THE PROBLEM

I. Read the problem carefully again and again.

II. Restate the Problem

- Use your own words.
- Include all parts of the problem including specific information such as lengths of line segments, size of angles, number sets mentioned, etc.
- Include any given figures and draw them to scale.
- Include any given formulas.

III. Search for a Solution

- Identify words and symbols you don't know. Look up their meanings and write them down.
- Think about the problem
- Use a graph, a data table, algebraic reasoning, technology or a combination of these to investigate the problem.

WRITING THE SOLUTION

IV. Describe Your Solution Process

- Describe how you got started.
- What strategies have you used to solve the problem?
- Include any charts, graphs, lists, geometric figures, drawings, manipulatives that you used or created.
- If you only found a partial solution, state what it is.
- If you know your solution is incomplete or wrong, explain how you know.
- If you have a general solution, state what it is and support it with specific examples.
- State whether you think there could be other correct solutions and support your position.

V. Reflect about What You Have Learned

- What mathematics was required to be able to solve the problem?
- What advanced mathematics helped you solve the problem in a more sophisticated way?
- How, if at all, does this problem relate to previous work you have done?
- How and when did collaboration with peers assist you in reaching a solution?
- What "real-world" applications might there be for problems like this?

Individual and Collaborative Writing

Quickwriting

Through quickwriting activities, students can explore ideas without fear of criticism and without the premature editing that can inhibit expression. Writing encourages writing. By reducing anxiety about writing and producing material that can become a foundation for further writing, quickwriting is an excellent tool for prompting the thought and focus central to the entire writing process.

Clustering

Based on the premise that working with the natural rhythms of the brain to create writing produces work that is rich in meaning, clustering is a nonlinear brainstorming process that helps writers discover the ideas and patterns of organization that characterize strong writing. The practice not only promotes creative writing, but it also produces material that is abundant in memories, metaphor, and wholeness, and as such, its application to the academic terrain of analytical writing (or even test review) is equally impressive.

Listing and Grouping

Listing helps students recall what they already know about a topic and discover what they may need to find out about it. Lists alone sometimes suffice as a prewriting activity. There may be other occasions when students can use their lists to generate groupings of items that may be helpful as they consider how to organize the information they will write about.

Guided Reflection

Guided reflection can take the form of class discussion or individual student conferences with the teacher and/or tutors about the writing in which students are engaged.

Choosing a Topic

Some assignments are designed to provide students with exposure to a variety of discourse modes and topics. Students may need some coaching to identify and narrow topics, but experience with defining topics will be good practice for the many times in college when students will be expected to do so.

Choosing an Audience, Purpose, and Format

Activities that require students to write for different audiences and purposes greatly enhance the legitimacy of the writing process. By providing students with flexibility and choice in audience, purpose, and format, writing takes on the dimension of a real world venture, and the decision-making process required by such variety produces energetic revision and refreshing writing. Discussions that address audience, purpose, and form remind students of the importance of the choices they make as writers.

Planning

Students often benefit from help planning the organization of their writing. Strategies like outlining and grouping, as well as individual conferences with other students, tutors, and teachers are helpful. Making available copies of various forms of graphic organizers reinforces the importance of planning writing before drafting.

Individual and Collaborative Drafting

Done individually, drafting produces writing that is ready for response and revision. Done collaboratively, drafting incorporates a variety of points of view forged into writing that is also ready for response and revision. *Note:* Students often need reminders that during the drafting stage of the writing process they should focus on content and logical organization rather than mechanical correctness.

Individual and Response Group Editing

Response to drafts and revision comprise a stage of the writing process frequently abandoned by students working too close to deadlines and teachers unfamiliar with classroom groupings and/or guidance of students as they learn to respond to each other's work. The college preparatory class provides a perfect opportunity to help students integrate this portion of the process into their work with writing.

As frightening as it may be for students, responding to each other's work and soliciting critique of their own work helps them to develop a sense of audience that reaches beyond the teacher alone. By giving and receiving responses, students begin to think of themselves as writers whose work must communicate with others. If students are taught the characteristics of good writing—instruction that often demands the direct involvement of teachers—and are convinced that their commentaries are met with appreciation and often with tangible results in the evaluation stage, they can gain the courage to contribute to the successful writing of others. As students read each other's work, they become better at identifying good and bad writing. They become better writers.

Oral Response Groups

Groups that mix students by strengths and areas needing improvement in writing can provide consistency of commentary as students work through revisions of various writing assignments and prompt unusual unity around the task of writing. Changing group composition by assignment and/or to blend experienced writers with novices can afford students a variety of responses to their writing and increase the camaraderie of an entire class. Students are often self-conscious reading their writing aloud; they are sometimes inclined to dismiss their work or apologize for it with comments like, "This is just a first draft," or "My baby brother kept me up until four this morning, so this isn't very good." Since response groups should be safe places for even the most unpolished writing, students may need reminders that "no apologies" should precede reading their work.

Another tendency of some writers when presented with critique is to become defensive or to try to explain what they intended the essay to say. Students need guidance to relax into the safety of accepting critique as helpful without feeling the need to defend or explain their work. Finally, students often need a great deal of coaching to phrase specific and meaningful commentary. Lacking self-confidence or fearful of hurting feelings, students sometimes slip into excessively general commentary such as, "It's good," or "It just needs a little work," which doesn't give writers much information to shape revision. Have listeners write down comments as they listen. Then they can provide specific oral comments that provide useful information to the writer. Helping students with

phrasing, directly addressing some of the causes of unnecessarily general commentary, and modeling effective commentary by joining groups during response sessions can contribute greatly to the honesty and thoroughness of student critique.

Written Response

When oral response groups are not feasible, or as a supplement to oral response group commentary, written critique can provide ideas for revision. Students should be encouraged to frame comments similar to those expressed in response groups directly on each other's papers. Written commentary can be reviewed by individual writers and discussed with other students, tutors, or the teacher as students prepare for revision.

Revision allows students to make use of the comments they receive from response groups and written critique and to redefine what it is that they want to communicate in a piece of writing. For many students, revision is a painful process. Nevertheless, revision is evidence that a writer really cares about a piece of writing. As students revise, they need to revisit the critical areas of audience, purpose, and form. In addition, they need to review the components of writing pertinent to the type of writing they are producing. Revision is a teachable skill. Students benefit from reminders of their power as writers: While critique should be considered seriously, revision need not automatically respond to all suggestions for revision mentioned during critique. Part of the integrity of the writing process is assuming ownership for the decisions made that acknowledge critique but preserve the writer's own vision. Final editing for cosmetics familiarizes students with standard mechanics and guides them toward submitting writing that is free from mechanical distractions.

AVID writing lessons adhere to the following "writing process" sequence:

- Study and discussion of samples
- Pre-writing
- Drafting
- Peer evaluation
- Revision
- Peer evaluation
- Final revision
- Teacher evaluation
- Discussion and revision
- Reflection...

Technical Writing Tips for Mathematics:

- Create an outline
- Use a word processor and equation editor
- Use complete sentences (begin with a capital letter; use a subject and verb)
- Double space
- Start with a short summary when writing a report
- Use your textbook as a guide for style and syntax
- Don't start a sentence with a symbol
- Use standard and accepted symbols
- Do not use symbols in a sentence unless they are part of an equation
- Do not insert words in place of symbols in equations
- Leave a double space between a symbol and text
- Write variables in *italics* or **bold** face
- Avoid starting a sentence with a number
- Spell out numbers that begin a sentence
- Use care, pronouns can confuse the reader
- Use active verbs
- Generally expressions used as nouns are singular and the verb should agree
- State a theorem and provide an example before proving it
- Avoid being judgmental, i.e., "It is easy to see that..."
- Place math expressions centered on their own line
- Equations are dependent clauses
- Use headings to divide your paper into different sections
- Number all tables and write a caption for each table
- Number all figures in a separate numbering system from the tables
- There is no substitution for formal mathematical terminology
- Check calculations with care

1.1: Cornell Notes

Topic

- Introduction to Cornell Notes

Objectives

Students will:

- Become familiar with the Cornell note-taking system
- Develop specific note-taking skills
- Develop facility in writing questions that synthesize the notes' content
- Develop skills in writing a summary of daily notes
- Understand how to use Cornell Notes to review and study for exams

Timeline

- One 50-minute class period to introduce students to the purpose and format of Cornell Notes

WICR Strategies

- Writing to Learn
  - — Write notes in a two-column format
  - — Write questions that synthesize the content of the notes
  - — Write daily summaries of notes

NCTM Standards

Communication

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- analyze and evaluate the mathematical thinking and strategies of others; and
- use the language of mathematics to express mathematical ideas precisely.

Connections

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- recognize and use connections among mathematical ideas;
- understand how mathematical ideas interconnect and build on one another to produce a coherent whole; and
- recognize and apply mathematics in contexts outside of mathematics.

Representation

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- create and use representations to organize, record, and communicate mathematical ideas;
- select, apply, and translate among mathematical representations to solve problems; and
- use representations to model and interpret physical, social, and mathematical phenomena.

Rationale

One of the cornerstone AVID strategies is Cornell note-taking. The Cornell note-taking system originated at Cornell University based on research done in the area of memory and learning theory. It is a very valuable system because it takes students through the cycle of learning. It is much more than just a way to record information. Teaching Cornell note-taking will take time, however it will be one of the most valued skills that students will take with them when they enter college. In AVID-inspired classes, note-taking is considered an essential skill and therefore will improve with time and practice. The Cornell note-taking system incorporates what students do with their notes once they have taken them. By using Cornell Notes consistently, students learn to see writing as a tool for learning in mathematics as well as other subject areas.

The Cornell note-taking system is a system that students need to be taught and reviewed regularly. It is not intended to change how teachers deliver information, but rather how students record and interact with that information.

Vertical Alignment

- Formal note-taking in mathematics is a critical skill that can be introduced and refined at all levels.

Materials/Preparation

- *Student Handout/Overhead Transparency 1.1a:* "Class Notes/Learning Logs/Textbook Notes"
- *Student Handout/Overhead Transparency 1.1b:* "Taking Cornell Notes—Some Tips"
- *Student Handout 1.1c:* "STAR Note-taking Strategy"
- *Student Handout 1.1d:* "Tips for Studying with Notes"
- *Student Handout 1.1e:* "The Cornell Note-taking System"
- *Student Handout 1.1f:* "Common Math Abbreviations"
- *Student Handout 1.1g:* "DLIQ Summary"
- *Student Handout 1.1h:* "Cornell Note-taking Checklist"
- *Student Handout/Overhead Transparency 1.1i:* "Student Samples"
- Initially students should be provided with *Student Handout/Overhead Transparency 1.1a:* "Class Notes/Learning Logs/Textbook Notes" and *Student Handout/Overhead Transparency 1.1b:* "Taking Cornell Notes—Some Tips" to help them with the structure of Cornell Notes. Once the routine has been established, no special materials are required.

Instructions

- Ask students to complete a quickwrite explaining how they learned to take notes.
- Use "Popcorn" or another group sharing protocol for students to share their training in note-taking.
- Ask students if anyone has taught them what to do with their notes once they have taken them.
- Brainstorm with students why taking notes might be a good skill to learn.
- In pairs or groups, have students list some effective ways to use notes.
- Explain to students that there are several skills needed to become an effective note-taker, for example:
  - — Know what to write down.
  - — Be able to listen to what the teacher says and write it down at the same time.
  - — Learn how to use abbreviations.
  - — Use symbols and/or indentations on the note page to organize notes while writing.
  - — Know what to do with notes after taking them. (See *Student Handout 1.1d:* "Tips for Studying with Notes.")
- Distribute and review *Student Handout/Overhead Transparency 1.1a:* "Class Notes/Learning Logs/Textbook Notes," *Student Handout/Overhead Transparency 1.1b:* "Taking Cornell Notes—Some Tips," and *Student Handout 1.1c:* STAR Note-taking Strategy" to introduce the structure, advantages, and uses of Cornell Notes. Show students how to set up their paper for Cornell Notes and identify the five parts of the format (heading, topic, notes, questions, and summary).
- Using the information in *Student Handout 1.1e:* "The Cornell Note-taking System," give a 7–10 minute lecture.
- Ask students to take notes on the right-hand side of their paper.
- After the lecture, ask students to pair-share their notes with a partner and encourage them to add to their notes if they missed any information. Train students to use a different color of ink when they add to their notes so that they can see what they missed.
- Survey the class to see if anyone used abbreviations and share those with the whole class.
- Distribute *Student Handout 1.1f:* "Common Math Abbreviations" for a list of commonly used abbreviations. Encourage students to begin to create their own list of abbreviations. *Tip:* Asking students to think about how they use abbreviations in "instant messaging" may help them to understand the concept.
- Review the importance of highlighting the main idea(s) and key words.
- Teach students how to use the questioning column. Students generate questions that can be answered with their notes on the right. Explain to students that these questions may appear later on a test or quiz.
- Ask students to write one to three questions.
- Have some students share their questions with the whole class. (There should be duplication and/or overlap).
- As students become familiar with Bloom's Taxonomy or Costa's Higher Levels of Questions, you may require that they include the higher-level questions in their notes. Initially students will most likely write Level One or Level Two questions (see section on "Inquiry" for more information).

- Share a well-written summary to model the characteristics of a "good" summary. Describe how the summary provides a "big picture" and ties the main ideas together to reflect learning.
- Distribute and review *Student Handout 1.1g:* "DLIQ Summary."
- Provide time for students to work in pairs or individually to write their summaries.
- Ask students to share their summaries with the whole class. As they do this, take time to point out which parts are the most effective. Teaching students effective summarizing will take time, but ultimately this skill will improve their writing overall. Continue to reinforce the difference between re-telling information versus connecting the main ideas to show new learning.
- Distribute *Student Handout 1.1h:* "Cornell Note-taking Checklist" and review each of the descriptors.
- Ask students to complete a self-assessment of their notes using the rubric and then compare it with an assessment by a partner using the rubric.
- Survey class to see how students scored. Discuss how taking notes is a skill and the expectation is that students will improve with time.
- Provide students with opportunities to practice using the Cornell Notes rubric (see *Student Handout 1.1h:* "Cornell Note-taking Checklist").
  - — Post copies of the sample math notes or other exemplars.
  - — Divide students into small groups.
  - — Ask groups to use the rubric and reach a consensus score for the each of the sample/exemplar notes.
  - — Lead a class discussion about what characteristics contributed to "good" notes.
- Collect student notes regularly and grade them using the rubric previously introduced.
- Review and problem-solve difficult note-taking situations.
- Emphasize note-taking skills consistently and hold students accountable for all of the steps they need to do on their own outside of class to get the most use of their notes.

Higher-Level Questions

Level Two

- Compare the Cornell note-taking system with traditional notes.

Level Three

- Describe how Cornell note-taking skills might be useful in a practical or real-world setting.

Formative Assessment

- Use a Cornell Notes rubric occasionally to give students feedback on their notes.
- You can also use these rubrics and have students do peer checks of other student's Cornell Notes.

Class Notes/Learning Logs/Textbook Notes

Level 2: sort, infer, analyze, sequence, organize, solve, explain, compare, contrast, classify, isolate, characterize, make analogies.

Level 3: conclude, criticize, reorganize, justify, judge, estimate, predict, speculate, make a model, extrapolate, apply a principle, interpret, hypothesize, if/then

Name: _____

Class: _____

Period: _____ Date: _____

Topic

Study/Review Questions	
Connections, Summary, Reflection, Analysis:	

Class Notes/Learning Logs/Textbook Notes

Level 2: sort, infer, analyze, sequence, organize, solve, explain, compare, contrast, classify, isolate, characterize, make analogies.

Level 3: conclude, criticize, reorganize, justify, judge, estimate, predict, speculate, make a model, extrapolate, apply a principle, interpret, hypothesize, if/then

Name: _____

Class: _____

Period: _____ Date: _____

Topic

Study/Review Questions	
Connections, Summary, Reflection, Analysis:	

Class Notes/Learning Logs/Textbook Notes

Level 2: sort, infer, analyze, sequence, organize, solve, explain, compare, contrast, classify, isolate, characterize, make analogies.

Level 3: conclude, criticize, reorganize, justify, judge, estimate, predict, speculate, make a model, extrapolate, apply a principle, interpret, hypothesize, if/then

Name: _____

Class: _____

Period: _____ Date: _____

Topic

Study/Review Questions	
Connections, Summary, Reflection, Analysis:	

Taking Cornell Notes—Some Tips

Level 2: sort, infer, analyze, sequence, organize, solve, explain, compare, contrast, classify, isolate, characterize, make analogies.

Level 3: conclude, criticize, reorganize, justify, judge, evaluate, estimate, predict, speculate, make a model, extrapolate, apply a principle, interpret, hypothesize, if/then

Name: _____

Class: _____

Period: _____ Date: _____

Topic *Note-taking Strategies*

Study/Review Questions	
How can you use the speaker's style to identify important points?	*Become familiar with the speaker's style.* *Listen for important points that might be emphasized when the speaker:* · *Pauses or slows down* · *Repeats a point* · *Modulates the volume of her/his voice* · *Uses introductory phrases (e.g., "The four main points are" or "Note the relationship")* · *Writes on the board* · *Gestures or uses visual aids* *Write only the important ideas such as name, examples, terms, definitions, effects, evaluations, cross references: make it brief, but clear. Use abbreviations for familiar words.*
How can you keep-up with the speaker?	***Speaker says:*** *"An altitude of a triangle is the perpendicular segment from a vertex to the opposite side or the line that contains the opposite side. An altitude can lie inside, on, or outside the triangle."* ***Notes say:*** *Altitude of Δ is ⊥ from vertex to opp side or line containg opp side.* · *Can be inside, on or outside Δ*
How should you use your notes to review?	*Use notes to review:* · *Develop study questions and identify the main ideas.* · *Fill in details for clarity.* · *Look up and add to the definitions of new words/terminology.* · *Identify information that is unclear and/or questions that need to be answered; write and mark them so they can be easily found; get answers to the questions from other students and/or the speaker.* · *Add symbols to highlight important ideas and key words.* · *Delete irrelevant information.* · *Review the overall organization of the material: add symbols to make the organization clear or rewrite for clarity as needed.* · *Write a summary of the significant ideas.*

Connections, Summary, Reflection, Analysis:

Three important note-taking strategies are reviewed in the notes. Identifying important points and main ideas, using abbreviations to paraphrase information provided during the class and the use of notes for review are outlined. The important cues in identifying main points and the use of questions to help with review are particularly helpful strategies as is the writing of summaries.

Name: _____ Quarter: _____

Begin Date: _____ Period: _____

STAR Note-taking Strategy

1. Put your name, period, class, and date in upper right-hand corner.
2. Give your notes a title.
3. Draw a vertical line down the paper to divide it into sections of about one-third and two-thirds.

T = Take Notes

1. PARAPHRASE the text or lecturer in the right-hand column.
2. Listen to decide which parts of the information are most important. Notice if the lecturer seems to stray from the topic.
3. Use whatever it takes to cue your own memory system. For example, use capital printing, underlining, arrows, or even pictures.
4. Don't get hung up on spelling. If you know what you mean, that is what counts. If you use this information later for another assignment or an essay, check for proper spelling then.
5. Use abbreviations that work for you. Develop your own shorthand.

A = After Class

1. Within five minutes of class, or as soon as humanly possible, edit your notes. Reread them looking for places to make additions, deletions, or clarifications.
2. Work with a partner to review your notes whenever possible.
3. Use a highlighter or underlining to emphasize important points.
4. Note any points that need to be clarified with the lecturer in the next session.
5. Finally, fill in the left-hand column with questions, icons, symbols, pictures, and memory keys.

R = Review Notes

1. Review notes regularly, after class, at least once a week.
2. Cover the right-hand column with blank paper. Read aloud or rewrite the right-hand column by using the cues in the left-hand column.
3. Paraphrase the answers.
4. Reflect by summarizing the notes, relating the subject to yourself, or relating the subject to personal experiences.

Tips for Studying with Notes

Study/Review Questions	**Topic** *Tips for Studying with Notes*
How can the format of Cornell notes help with studying for a test?	· *Spread out or hold notes so that right side of page is covered; review ideas and answer study questions from the left-hand column; use right-hand section as an answer key.* · *Engage in an oral quiz with others using study questions from the left-hand column.* · *Cover the right-hand column with blank paper; write out answers to the left-hand study questions and explanations of main ideas.*
What should you write in the summary section?	· *Write summaries of the most important material in the summary/reflection section.* · *Write a quiz for others using the notes; exchange and correct.* · *Write anticipated test questions beyond those already in the left-hand column and write answers to the questions.*
How should you use notes to review?	· *Look over notes frequently to keep information and questions still unanswered fresh in mind.* · *Recite information from notes.*
How can you use notes when studying in a group?	· *Exchange notes with others to flesh out information and understanding.* · *Use notes in study groups to provide a common ground of material for reference and review. Rewrite notes if necessary.*

Connections, Summary, Reflection, Analysis:

The use of Cornell note-taking will aid in the recording and recall of classroom activities. The student will have opportunities to systematically record and review main ideas and details. In addition, the Cornell note-taking system will provide a method of enduring mastery through the notes review and reflection process.

Name: _____ Quarter: _____

Begin Date: _____ Period: _____

The Cornell Note-taking System

What are the advantages?

Three Advantages:

1. It is a method for mastering information, not just recording facts.
2. It is efficient.
3. Each step prepares the way for the next part of the learning process.

What materials are needed?

Materials:

1. Loose-leaf paper or graph paper to be kept in the binder.
2. 2½ inch column drawn at left-hand edge of each paper to be used for questions.
3. 3–4 lines left at the bottom of page for summary section.

How should notes be recorded?

During class, record notes on the right-hand side of the paper:

1. Record notes in paragraphs, skipping lines to separate information logically.
2. Don't force an outlining system, but do use any obvious numbering.
3. Strive to get main ideas down. Facts, details, and examples are important, but they're meaningful only with concepts.
4. Use abbreviations for extra writing and listening time.
5. Use graphic organizers or pictures when they are helpful.

How should notes be refined?

After class, refine notes:

1. Write questions in the left column about the information on the right.
2. Check or correct incomplete items:
   - Dates, terms, names.
   - Notes that are too brief for recall months later.
3. Read the notes and underline key words and phrases.
4. Read underlined words and write in recall questions in the left-hand column (use keywords and very brief phrases that will trigger ideas/facts on the right). These are in addition to the questions.
5. Write a reflective paragraph about the notes at the bottom of the page.
6. If possible, compare notes with a study buddy.

What are the ways to recite notes?

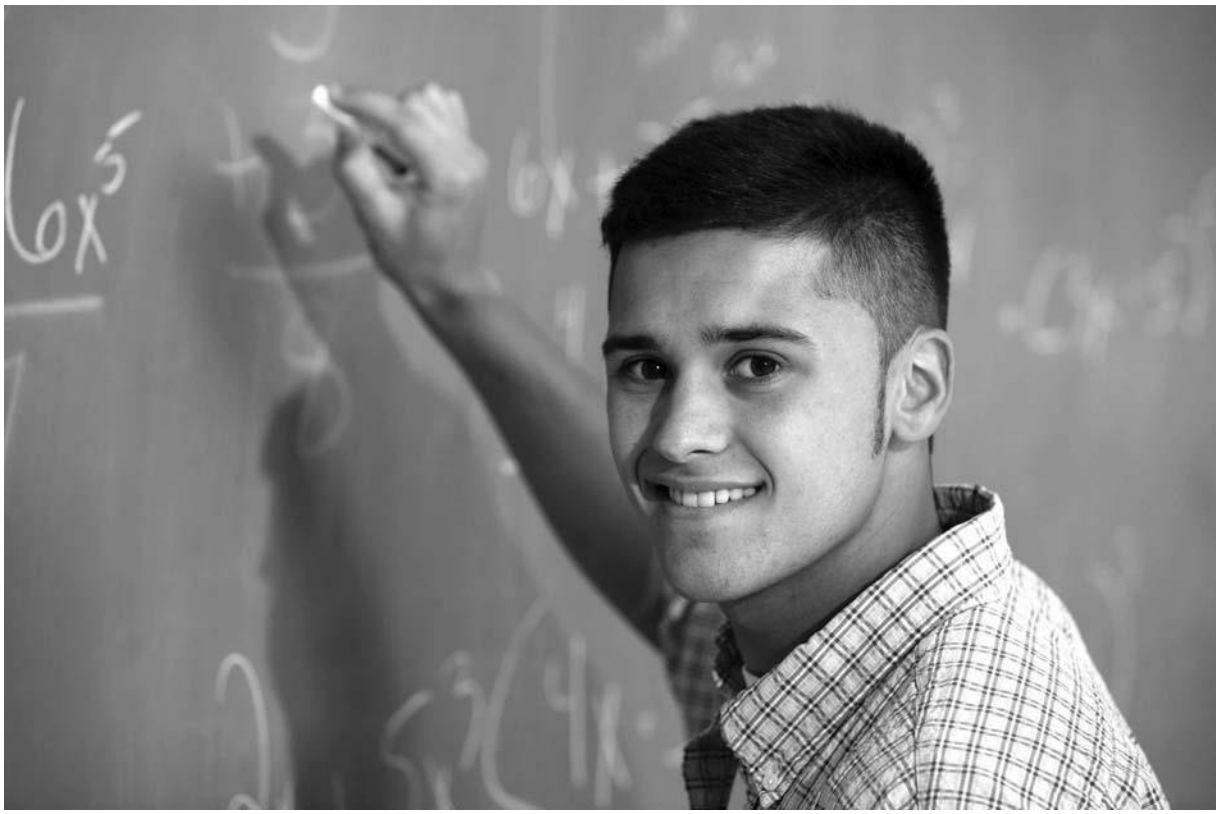
Recite notes three ways:

1. Cover up right side of page. Read the questions. Recite information as fully as possible. Uncover the sheet and verify information frequently. (This is the single most powerful learning tool!)
2. Reflect on the organization of all the lectures. Overlap notes and read recall cues from the left side. Study the progression of the information. This will stimulate categories, relationships, inferences, personal opinions/experiences. Record all of these insights! REFLECTION = KEY TO MEMORY!!
3. Review by reciting, reflecting, and reading insights.

What are the six steps of this system?

This system in brief:

1. Record lectures in the main column.
2. Within 8 hours, read over notes to fill in gaps and to make notes more legible.
3. Identify main ideas and write questions in left-hand column based on main ideas.
4. Recite by covering main column and expanding on recall questions. Then verify.
5. Write a summary at the bottom of page.
6. Review your notes regularly. Short, fast, frequent reviews will produce better understanding and recall.



Common Math Abbreviations

Name _____ Date _____ Period _____

Common Short Cuts For Note-taking—Abbreviations/Acronyms

For	4	Factorial	!
To	2	Difference/change	Δ
With	w	Therefore	$\therefore$
Without	w/o	Perpendicular	$\perp$
Within	w/i	Mean	μ
And	& or +	Pi	π
Minus	–	Theta – used for angles	θ
Equal/same	=	Sigma – standard deviation	σ
Not equal	$\neq$	Infinity	∞
School	sch	Union	$\cup$
No/not ever	$\varnothing$	Intersection	$\cap$
Part	prt	Then – implies	$\rightarrow$
Point	pt	Empty set	$\varnothing$
Be	b	Sum/summation	Σ
Between	b/w	Similar	~
Reference	ref	Approximately equal	$\approx$
Symbols	$> < \geq \leq$	Congruent	$\cong$
If and only if	IFF, $\longleftrightarrow$	Parallel	$\parallel$

Additional Suggestions

- Make names and titles into acronyms after writing them the first time.
- Write first few syllables of long words and complete the word when reviewing notes.
  - -- coll Collect
  - -- comm Communicate
- Write words deleting vowels until notes are reviewed.
  - -- spk Speak
  - -- commnct Communicate
  - -- commnty Community

Think of some of your own short cuts.

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____
7. _____
8. _____
9. _____
10. _____

DLIQ Summary

Include the following in a DLIQ summary:

A. What did we ***do*** in class?

B. What did we ***learn*** in class?

C. What was ***interesting*** in class?

D. What ***questions*** do I have?

E. Check your summary to be sure the details support the topic and the concept in your notes.

Topic: _____

_____

Concept: _____

_____

DLIQ

D _____

L _____

I _____

Q _____

Write three to five sentences using DLIQ:

Cornell Note-taking Checklist

Name_____ Period _____

Do your notes have the following characteristics?

1. Consistent Cornell physical format, notes dated and titled, readable — 3 pts
2. Use of abbreviations, key words/phrases, underlining, starring — 1 pt
3. Main ideas are easily seen; correct sequencing of information — 1 pt
4. Questions are completed on left hand side; Level 2 and 3 questions — 3 pts
5. An accurate, complete summary follows the notes — 2 pts

Characteristics **Date**					
1. Consistent Cornell physical format, notes dated and titled, readable					
2. Use of abbreviations, key words/phrases, underlining, starring					
3. Main ideas are easily seen; correct sequencing of information					
4. Questions are completed on left hand side; Level 2 and 3 questions					
5. An accurate, complete summary follows the notes					
Total Points					

Rubric

Consistent Cornell physical format, notes dated and titled, readable

3. Vertical line drawn 2.5 inches from the left margin. Heading is complete with name, date, subject. The notes are titled. Notes are adequate in length.
2. Minor problem with format
1. No date or no title; short
0. Fails to use Cornell note-taking format or date and title are missing or notes are inadequate in length

Use of abbreviations, key words/phrases, underlining, starring

1. Techniques used throughout
0. Too much verbiage

Main ideas are easily seen; correct sequencing of information

1. Information is complete and in correct order
0. Notes confusing

Questions are completed on left hand side; Level 2 and 3 questions

3. A substantive number of higher order thinking questions are noted in the left margin which are answered in the notes to the right
2. Level 1 questions are many; level 2 and 3 questions minimal
1. Level 1 questions only
0. No questions in the left hand margin

An accurate, complete summary follows the notes

2. Detailed summary covers the main topics of the notes
1. Summary is generic or incomplete
0. Summary missing

Student Sample 1

Class Notes
If there was no class lecture this week, write a paragraph about what you learned and/or questions about what you didn't understand.

Topic: Distance Formula

Name: Student A

Class: Algebra

Period: 6

Date: 1/11

Questions/Main Ideas:	Notes:
What is the distance formula?	The distance formula calculates distance based on rate and time. Distance = rate · time $D = rt$
Example 1:	How far will a train travel at 85mph for 4 hours? $D = ?$ $r = 85$ mph $t = 4$ hours $D = 85(4)$ $D = 340$ miles
Example 2:	How far will a truck travel at 65mph for 3.5 hrs? $D = ?$ $r = 65$ mph $t = 3.5$ hrs. $D = 65(3.5)$ $D = 227.5$ miles

Summary:

The distance formula measures distance based on rate and time. Distance = rate times time. This is often used with word problems.

Student Sample 2

NAME
DATE
REMOVED

TITLE: INTRO TO THE VOLUME OF CYLINDERS

OUTCOME	UNDERSTAND THE CHARACTERISTICS OF VOLUME OF CYLINDERS
PLAN	WORK IN SMALL GROUPS TO LEARN ABOUT CYLINDERS
HOMEWORK	INVESTIGATE & GRAPH vol vs RADIUS, vol vs hgt w/ FIXED RADIAL SURFACE AREA
CYLINDER PROPERTIES	(90 SEC. QUICK WRITE) WRITE ENTIRE TIME, CHILD WRITE SAME WORD OVER & OVER 3-D FIGURE, CIRCULAR IN SHAPE, FLAT AT BOTH ENDS ROLL WHEN LAID FLAT, STATIONARY WHEN UP RIGHT TWO CIRCLES CONNECTED BY CURVED SIDES, CIRCLE, CIRCLE, CIRCLE, CIRCLE, CIRCLE, CIRCLE, CIRCLE, CIRCLE BRAIN STORM - PARENTAL NOTES FIXED FIGURE - 2 CIRCLES + 1 RECTANGLE POP CORN CLASS DATA • NET • $Ch + A_B = S_A$ • 3-D SHAPE • LOOKS LIKE A BEAR CAN • DIAGONAL THROUGH IT = ELIPSE • GOOD TO STORE ROUND STUFF • spin → RECTANGLE & HANG FROM CENTER & SPIN — YOU GET A CYLINDER • $V = \pi r^2 h$ r=radius h=height

WHAT IF?
IS THE $LS_A \cong$ FOR
C=8½ C=5½
DOES THE OVERLAP Δ THE OUTCOME? IF SO HOW?

PHYSICAL CHAT
QUICK WRITE
WHICH CYLINDER HOLDS THE MOST M&M'S?
SAME RECTANGLE - ROLL HOT DOG — SHORT WIDE
HAMBURGER — TALL NARROW
SHORT & WIDE WILL HOLD MORE, ...
FORMULA $V = \pi r^2 h$ RADIUS IS SQUARED & RADIUS OF SHORT & WIDE HAS A LARGER RADIUS

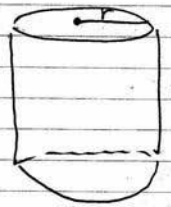
(OUTPUT SIDE)
STUDY QUESTIONS

WHAT ARE THE PARTS OF A CYLINDER?

HOW DO YOU FIND THE VOLUME OF A CYLINDER?
R=3 H=5

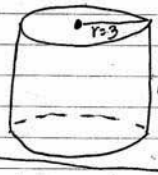
SUMMARY
DLIQ
DO
LEARN
INTERESTING
QUESTIONS?

(INPUT SIDE)
GUIDED NOTES
DRAW A LARGE CYLINDER AND LABEL ITS PARTS
(RECORD INSTRUCTIONS & SEE IF OFF W/ BLOCKING)



r=radius
h=height

DRAW A LARGE CYLINDER
r=3 h=5
← THE BLOCKING INDICATES DIRECTIONS THAT WERE GIVEN



COME UP TO THE SIDE & CHECK AGAIN

FIND THE VOLUME OF A CYLINDER
r = 3cm h = 5cm IN TERMS OF Pi

$V = \pi r^2 h$
$= \pi 3^2 \cdot 5$
$= 3.14 \cdot 9 \cdot 5$
$= 3.14 \cdot 45$
$= 45\pi \text{ cm}^3$

WE MEASURED THE VOLUME OF CYLINDERS
WE LEARNED THE VOLUME OR "V" IS EQUAL TO
$V = \pi r^2 h$
AND THAT THE RADIUS OF THE CYLINDER (CIRCULAR SURFACE AREA) LSA DOES NOT MEAN THE VOLUME IS THE SAME, EVEN IF LSA IS CONGRUENT.
WHAT AFFECTS WOULD THE OVERLAP HAVE ON A CYLINDER?

Student Sample 3

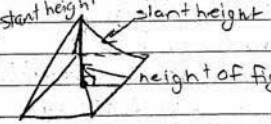
Lesson 12.3
SA of Pyramids and Cones

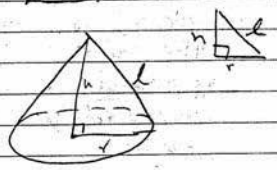
NAME
CLASS
DATE

REMOVED

What is the slant height?

★ slant height: this is the height of the slanted lateral faces of a pyramid or cone.



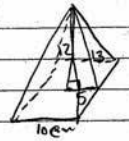


What is the formula for pyramids and cones?

→ Pyramid: $SA = B + \frac{1}{2}Pl$

→ Cone: $SA = \pi r^2 + \pi r l$

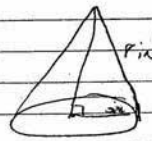
examples →



$\frac{1}{2}bh$ $(.5)(10)(13)$ 65cm.

Total SA = $13 + \frac{1}{2}Pl$
$= 5^2 + \frac{1}{2}(40)l$
$= 10^2 + \frac{1}{2}(40)(13)$
$= 100 +$
$= 340 cm^2$

examples →



$SA = B + \pi r l$

$SA = \pi r^2 + \pi r l$
$SA = 3.14(5^2) + (3.14)(5)(8)$
$SA =$ 204.3 in^2

lesson 12.3:
14: 3 out E
Quiz Thurs, all SA + 12.1

Summary: Today's lesson was on the surface area of pyramids and cones. To calculate this, you need to find the slant height, which is the height of the slanted lateral faces of a pyramid or cone. To find SA for a pyramid, you use $SA = B + \frac{1}{2}Pl$ and for cones, you should basically use $SA = \pi r^2 + \pi r l$. There is an example of each was given.

Student Sample 4

Lesson 8.3

Similar Figures

NAME
CLASS
DATE

REMOVED

What are similar polygons?

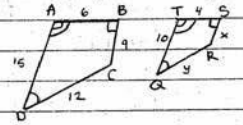
1) Corresponding ≅ Angles

2) Corresponding sides that are proportional

(same shape; different sizes)

same shape; different sizes

Write out proportionality statement



Corresponding ∠'s:

∠A ≅ ∠T ; ∠D ≅ ∠Q

∠C ≅ ∠R ; ∠B ≅ ∠S

(set A) All from larger figure → $\frac{AB}{TS} = \frac{DC}{QR} = \frac{AD}{TQ} = \frac{BC}{SR}$ ← (set b) All from smaller figure

$\frac{6}{4} = \frac{12}{y} = \frac{15}{10} = \frac{9}{x}$

$48 = 6y$ $\quad$ $90 = 15x$

$y = 8$ $\quad$ $x = 6$

⊛ DO NOT USE a found value to find another (if possible)

⊛ To have similar figures, all ratios of corresponding sides must be equal

Scale Factor

If 2 figures are ~ (similar), then the ratio between corresponding sides is the scale factor

VERY IMPORTANT⊛ The perimeters of 2 similar figures have a ratio of the scale factor.

Summary – Similar polygons are figures that have corresponding congruent angles and corresponding sides that are proportional. They have the same shape; but different sizes. An important fact to remember is: DO NOT USE a found value to find another value, if possible. To have similar figures, all ratios of corresponding sides must be equal. If two figures are similar, then the ratio between corresponding sides is the scale factor. The perimeter of two similar figures have a ratio of the scale factor.

Student Sample 5

Class Notes
If there was no class lecture this week, write a paragraph about what you learned and/or questions about what you didn't understand.

Topic: Mark up + Discounts

Name: [illegible]

Class: Math

Period: 6

Date: 1/19

Questions/Main Ideas:

Analyze and solve this equation.

Analyze and solve this problem.

Notes:

(1) Cost $1.50
percent of markup: 70%
$1.50 x .7 = $1.5

(2) cost: $38
% of markup 58%
$38 x .58 = $22.04

(3) cost: $111.00
% of markup 50%
$111.00 x .5 = $555

(5) regular price $24.50
% of discount 20%
$24.50 x .2 = $4.90

(7) cost $6
percent of markup 75%
% of markup 75%
6 x .75 = 4.50
sp = 4.50 + 6 = $10.50

(9) cost $149.99
% of markup 100%
$149.99 x 1 = 14.99
$149.99 + $149.99 = $299.98

Summary: This page is about mark up and discounts. To get the answer you have to take the percent markup and multiply it with the cost. If you have a discount you would take the percent and multiply it with the price. Also if you have a markup you have to add it with the markup you got and the cost.

1.2: Practicing Cornell Notes

Topic

- Solving systems of equations

Objectives

Students will:

- Refine Cornell note-taking skills
- Review alternative solution processes for systems of equations
- Develop higher-level questioning skills
- Write a description of a solution process

Timeline

- One 50-minute class period to review and practice Cornell Notes, to explore alternative solution processes, and complete a "Jigsaw" activity
- One 50-minute class period to allow students to solve the *Student Handout 1.2b:* "Phone Company Problem" using Cornell Notes and alternative solution processes, and to write a summary of their notes

WICR Strategies

- Writing to Learn
  - Write an exemplar set of Cornell Notes
  - Write an explanation of a problem solution
  - Explain how and why the student group selected a particular process
  - Write a rationale for a new pricing structure
- Inquiry
  - Evaluate alternative solution processes
  - Explore alternative pricing structures
- Collaboration
  - Work collaboratively to understand a problem
  - Work to agree upon a solution process
  - Assign roles to group members
  - Assist peers in developing exemplar Cornell Notes
- Reading to Learn
  - Practice a reading strategy while reading a problem

NCTM Standards

Focal Point Grade 8

Algebra: Analyzing and representing linear functions and solving linear equations and systems of linear equations

Algebra

Instructional programs from pre-kindergarten through grade 12 should enable all students to represent and analyze mathematical situations and structures using algebraic symbols.

Communication

Instructional programs from pre-kindergarten through grade 12 should enable all students to use the language of mathematics to express mathematical ideas precisely.

Connections

Instructional programs from prekindergarten through grade 12 should enable all students to recognize and apply mathematics in contexts outside of mathematics.

Rationale

"Practicing Cornell Notes" provides a framework for using and refining individual technical writing skills as well as building high-level questioning skills through the design and analysis of an experiment. Students working collaboratively are given opportunities to practice and provide evidence of mastering technical vocabulary, technical writing, and key communication skills.

Vertical Alignment

- The "systems of equation" content of this activity is designed for students that have been exposed to various strategies for solving systems problems. However, the concept of integrating note-taking strategies can be done at any level.

Materials/Preparation

- Cornell note-paper
- Flip chart
- Color markers, pens, or pencils
- *Student Handout 1.2a:* "Systems of Equations Review" in Cornell Notes format
- *Student Handout 1.2b:* "Phone Company Problem"
- *Student Handout 1.1c:* "STAR Note-taking Strategy"
- *Student Handout 1.1d:* "Tips for Studying with Notes"
- *Student Handout 1.1h:* "Cornell Note-taking Checklist"

Instructions

This lesson should be done after students have had the opportunity to work with systems of linear equations. They should be able to solve a system using several different methods.

Day One

- Divide the class into groups of three or four.
- Review the Cornell note-taking strategy.
- Distribute and discuss the optional *Student Handout 1.1c:* "STAR Note-taking Strategy" and/or *Student Handout 1.1d:* "Tips for Studying with Notes."
- Distribute the partially completed Cornell Notes *Student Handout 1.2a:* "Systems of Equations Review."
- Brainstorm alternative solution processes.
  - On a flip chart (modeling Cornell Notes) review the alternative solution processes (guess and check, graphing, substitution, elimination, and matrices).
  - Discuss the advantages and disadvantages of each. Give examples and discuss real world applications of each process. Discuss when not to use each process.
- Use a "Jigsaw" activity to have each group use different processes to solve the problem presented in the notes handout.
- Provide time for the groups to complete a solution and for each student to complete a set of Cornell Notes.
- Utilize a "Gallery Tour" or other group share-out process for students to share and assess their work and the work of others.

Day Two

- Distribute *Student Handout 1.2b:* "Phone Company Problem."
- Ask students to select a solution process and write their solution on the right side of their Cornell Notes.
- Ask groups to revisit the problem by looking for an alternative pricing structure that would change the consumers' company choice after the "break even point."
  - Ask groups to illustrate their pricing structures using functions, tables, and graphs.
  - Provide time for groups to complete (in their notes) additional graphs and an explanation of their solution.
- Provide time for different groups to discuss alternative solutions to the extended problem.
- Provide time for pairs of students to review and annotate their notes.
- Provide time for students to write "Study Questions" in the left column of their notes.
- Have students work together as a class to review and refine "Study Questions."
- Provide time for students to work together in writing a summary of their notes.
- Use "Popcorn" or other share-out activity to share and refine summaries.
- Collect and copy exemplar notes to use as "anchor papers" for future instruction in Cornell note-taking strategies.

Higher-Level Questions

Level Two

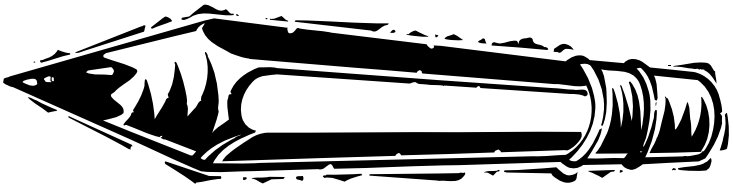
- Which solution process is the best/worst? Justify your answer.
- What are examples of when to use each of the possible processes?
- What is represented by the slope and y-intercept of the graph of the solution?
- What influenced your group in choosing a variable to be represented on the y-axis?

Level Three

- What are some "real world" applications of this type of problem?
- What are the benefits of each of the alternative pricing structures?
- What influences companies when they are designing a pricing structure?
- How would a company change their pricing structure to become more competitive after the break-even point?
- Many people believe that system of equation problems appear more in real world situations than any other type of equation solving process. Do you believe this is true? Why or why not?

Formative Assessment

- Did students understand the Cornell note-taking process? Use *Student Handout 1.1h:* "Cornell Note-taking Checklist" to evaluate their understanding.
- Did students understand and use a variety of solutions?
- Did students use effective collaborative strategies?
- Did students' notes represent their group's thinking?
- What was the quality of the class discourse?
- What was the quality of individual notes?
- Did students' solutions and discussion represent an understanding of the relationship between the mathematics and a "real world" application?



Name: _____

Date:_____ Period: _____

Systems of Equations Review

Name all of the different ways to solve a system of linear equations problem.

Solve this system two different ways:

$x + 2y = 13$

$x - y = 10$

Example problem:

You want to have business cards printed. The executive style card will cost $25 plus 2¢ per card. The economy style card will cost $10 plus 5¢ per card. Write a system of equations to represent the pricing plan for the two different card styles. Solve the system in two different ways. Explain how the solution will inform your decision on which cards to purchase.

Connections, Summary, Reflection, Analysis:

Phone Company Problem

Two competitive cell phone companies, General Telephone and Western Regional, provide special service rates to families who have more than two phones. For their family plan, General charges $40 per month plus $0.15 per minute. Western charges $45 per month plus $0.10 per minute for their family plan.

- Create two equations to describe the two plans.

- Which service would be the best choice?

- Explain how you arrived at your answer.

- How did your notes benefit you as you worked through the problem?

- How would the companies adjust their pricing structure after the break-even point?

- Illustrate your solution with a graph.

1.3: Learning Logs

Topic

- Learning logs

Objectives

Students will:

- Summarize their learning
- Reflect on their learning
- Become aware of the learning process
- Identify gaps in their learning
- Make connections to prior knowledge
- Improve written language skills

Timeline

- 15 minutes at the end of class or an activity

WICR Strategies

- Writing to Learn
  - — Write learning logs to summarize learning

NCTM Standards

Communication

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- analyze and evaluate the mathematical thinking and strategies of others; and
- use the language of mathematics to express mathematical ideas precisely

Connections

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- recognize and use connections among mathematical ideas;
- understand how mathematical ideas interconnect and build on one another to produce a coherent whole; and
- recognize and apply mathematics in contexts outside of mathematics.

Rationale

The *"Learning Log"* is a writing technique designed to help students focus on what they are learning in their classes by having them write their thoughts, reactions, and responses. Every class or learning experience does not involve a lecture or require formal notes. In the 21st century classroom, students may have done an activity, conducted an experiment, or worked on a long term project. Taking notes is more difficult in these situations. In situations like these, the learning log provides an opportunity for the synthesizing and clarifying which normally happens in the reflective summary found in notes. Learning logs are personal, but focused on specific content. By making personal connections with the material/content, the writer constructs their own understandings and more meaningful learning takes place. At the conclusion of an activity or lesson that did not include formal note-taking, a learning log can be assigned as a tool for summarizing and making connections. They may be done either in class or as a homework assignment.

Vertical Alignment

The learning log is a tool used in the AVID-inspired class at all grade levels to have students review learning trajectories. Its value in the mathematics classroom is in its ability to assist students in seeing the connections between prior knowledge and present learning. Completing a learning log will encourage students to slow down and focus on the processes involved in solving a problem or analyzing a concept. It also reinforces the importance of vocabulary and problem solving.

Materials/Preparation

- *Student Handout 1.3a:* "Learning Logs"
- Initially students should be provided with the student handout to help them recognize the information they want to include in a learning log. Once the routine has been established, no special materials are required.

Instructions

- Distribute *Student Handout 1.3a:* "Learning Logs."
- Review the purpose of completing a learning log.
- Model the completion of a learning log with students.
- Share learning log exemplars with students.

Higher-Level Questions

Level Two

- How does this topic relate to what you have done in the past?

Level Three

- Predict how this topic will relate to what you will be doing in the future.

Formative Assessment

- Are all sections of the students' learning logs completed?
- Are complete sentences being used?
- Did students capture the important elements of the lesson?

Learning Logs

Name: _____

Date(s): _____ Subject: _____

Guiding Questions

Write a paragraph of at least 5 sentences about what happened in class and what you learned. Below are some questions you may want to answer:

- What did you learn?
- What did you find interesting?
- What questions do you have about what you learned?
- What were the main ideas?
- What did you understand best?
- How will you find more information?
- How does the current information relate to what you have already learned in class?
- How might it relate to what we will be doing in the future?

1.4: Let Me Think About This—Reflective Journal

Topic

- Summary and reflection

Objectives

Students will:

- Make sense of new material they have learned
- Reflect on their work
- Make connections with prior learning
- Practice technical writing skills

Timeline

- 5–10 minutes at the end of class to reflect on new material and to practice technical writing skills

WICR Strategies

- Writing to Learn
  - — Write a summary and review
- Collaboration
  - — Work with a partner to review and annotate notes

NCTM Standards

Communication

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- communicate their mathematical thinking coherently and clearly to peers, teachers, and others; and
- use the language of mathematics to express mathematical ideas precisely.

Connections

Instructional programs from pre-kindergarten through grade 12 should enable all students to recognize and use connections among mathematical ideas.

Rationale

Writing to learn is a key skill in acquiring deeper understanding in all subject areas. Through daily practice, self-assessment, and guidance, students are given the opportunity to improve facility, fluency, and to develop skills in the precise use of mathematical language. Vocabulary and language usage can only be improved through regular practice. This activity provides a framework for using and refining individual technical writing skills.

Vertical Alignment

- Reflective journal writing can be introduced at any level.

Materials/Preparation

- *Student Handout 1.4a:* "Reflective Journal"

Instructions

- Provide students with five to ten minutes to work in pairs to review and annotate their Cornell Notes.
- Review a technical writing strategy you would like students to practice.
- Distribute *Student Handout 1.4a:* "Reflective Journal."
- Require that the room be quiet.
- Ask students to provide evidence of their understanding while completing their *Reflective Journal.*
- Encourage students to pose questions from the current lesson or previous lessons.

Note: A review of the reflective journals will inform future lesson planning.

Higher-Level Questions

Level Two

- How is the work that we did in class today related to what we have done previously?
- How is the content of this class related to other classes?

Level Three

- Can you relate what we are doing in class to a "real world" application?

Formative Assessment

- Did students respond to the prompt?
- Was the writing clear and concise?
- Did students provide details of the day's work?
- Did students utilize the technical writing strategies they are learning?
- Were the students able to use mathematical vocabulary?

Name: _____ Date: _____

Reflective Journal

Name of the Course: _____

In class today we…

(Describe what topics were covered, what problems were worked on, what presentations were made by students and teachers, or how otherwise you used your time.)

I learned…

(Sum it up in a few sentences using standard English. Be specific, include examples as evidence of your understanding.)

One or two questions or comments I still have are…

(You may start this sentence with, "I don't understand how to…," or "I didn't understand the difference between…," or "I still don't know why…," or "When am I supposed to…," or "At last I understand…,")

Question #1:

Question #2:

My plan for getting help with my homework, should I need it, is to…

1.5: Quickwriting, "What do you think?"

Topic:

- Writing to a prompt

Objectives

Students will:

- Demonstrate pre-instructional knowledge
- Demonstrate post-instructional mastery of a concept
- Develop and demonstrate technical writing skills
- Write clearly about topics in mathematics
- Clarify thinking about topics in mathematics
- Master skills needed on extended response assessments
- Have time to practice/process new concepts
- Practice making connections between concepts
- Identify areas of confusion

Timeline

- 5–10 minutes
- Additional time as needed to redraft, work collaboratively on a redraft, and to complete self and/or peer assessments

WICR Strategies

- Writing to Learn
  - — Write to a prompt
- Collaboration
  - — Participate in collaborative assessment activities
  - — Participate in collaborative writing or rewriting activities
  - — Master assessment rubrics
  - — Compare writing with anchor papers
- Reading to Learn
  - — Read and respond to peer writing

NCTM Standards

Communication

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- analyze and evaluate the mathematical thinking and strategies of others; and
- use the language of mathematics to express mathematical ideas precisely.

Connections

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- recognize and use connections among mathematical ideas;
- understand how mathematical ideas interconnect and build on one another to produce a coherent whole; and
- recognize and apply mathematics in contexts outside of mathematics.

Rationale

The study of math requires that students continually think at higher levels. One skill builds upon another, and as each skill is achieved another is introduced. Part of the responsibility of teachers is to help students integrate new information with prior knowledge. Writing is a tool that enhances student learning. Math teachers and students alike often question how to authentically incorporate writing into the math classroom. The quickwrite is an easy and obvious tool. A quickwrite is a literacy strategy that is designed to give students the opportunity to activate prior knowledge and reflect upon their own thinking and learning, and provides an easy way to incorporate writing in a math classroom.

Vertical Alignment

- The Quickwrite Strategy can be used at all levels.

Materials/Preparation

- Prepare a selection of writing prompts
- Paper and pencil
- A kitchen timer, helpful but not necessary
- Poster paper
- Color markers
- Review the "Technical Writing Tips for Mathematics" (See the *Introduction to Writing in Mathematics*).

Instructions

- Explain the directions for a "quickwrite" to students. *Note:* Tell students that during quickwrites, the room must be quiet with no distractions for the entire allotted period of time.
- Ask students to:
  - — Write as quickly as they can.
  - — Practice good technical writing skills.
  - — Write for the entire time allotted. Their pencil should never leave the page during a "quickwrite."

- — Always restate the prompt.
- — Use Standard English.
- — Express their thoughts clearly and concisely.
- — Use examples to demonstrate ideas.
- — Use the appropriate math symbols along with their meaning when first introduced.
- — If possible, explain their reasoning in more than one way.
- — Monitor their time.
- — Ask a peer to proofread their work, then make the agreed upon revisions before they submit it.
- — If available, compare their work with "anchor papers."

- Read the prompt or write it on the board or overhead.
- Advise students of the time allotment. *(2–10 minutes)*
- Start a timer.
- Distribute a rubric for self or peer assessment.
- A variation could be done on a poster either individually or collaboratively.
- Use a "Gallery Tour" or another protocol for sharing work.
- Ask students to keep all quickwrites for review at the end of a unit.
- This type of assignment can be used anytime in a lesson, at the start, middle, or end.
  - — At the beginning of a lesson it can be used to access prior learning or investigate what students already know about a particular topic.
  - — During instruction, a quickwrite is a way to determine how well students are processing new ideas and to asses their progress.
  - — At the end of a lesson it can be used to summarize, make connections, or assess whether students have met the desired learning objective.
- The prompts used for quickwrites can be either content specific or affective.

Sample Quickwrite Prompts

- What is the purpose of studying math?
- What are your strengths in math? What are your weaknesses?
- What does it mean to "simplify" an expression or equation? How is that similar or different from "evaluating"?
- What is the Order of Operations? Why is it useful?
- Which of the properties of math do you think are most important? Why?
- How are the properties of math useful?
- What are the inverse operations? What role do they play in solving equations?
- How can you subtract a negative number? Why does this work?
- What is an irrational number? How can you do computations with irrational numbers?
- Why is slope defined as "rate of change"?

Higher-Level Questions

Level Two

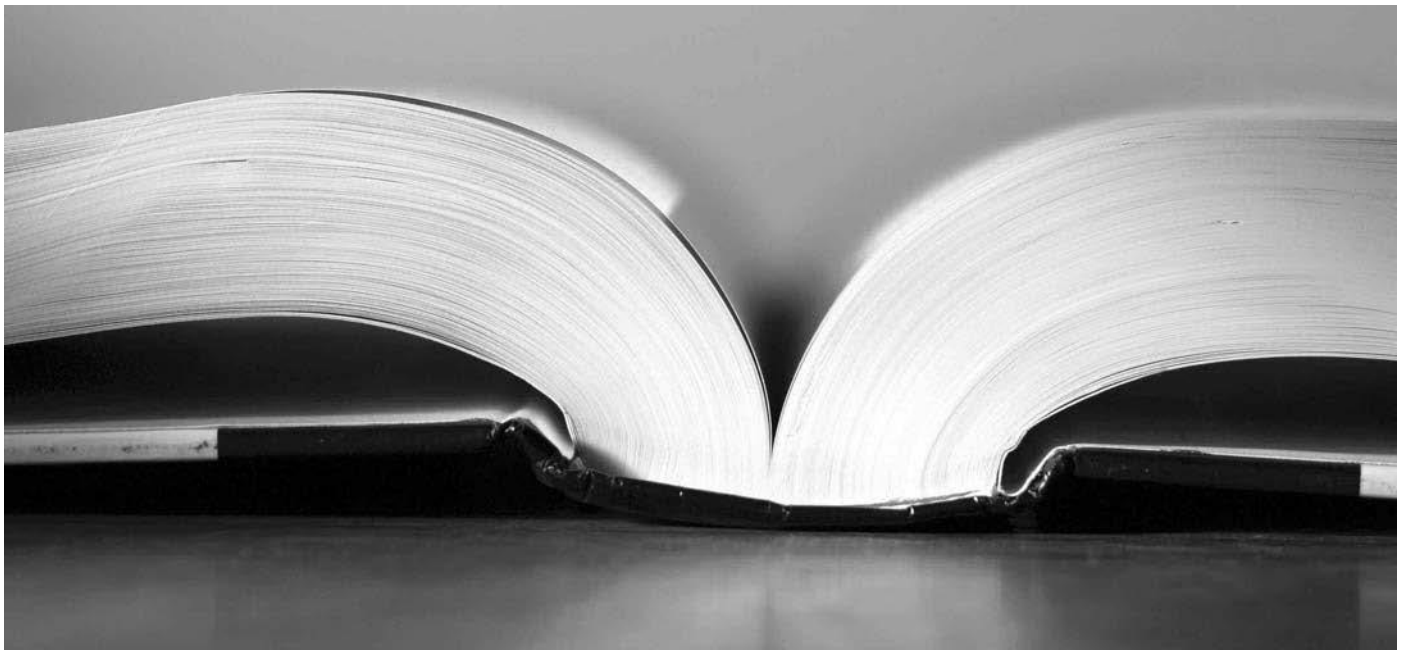
- In what ways has your thinking been clarified through your writing?
- In what ways is the following quote true? *"I don't know what I think until I write it down."*

Level Three

- When writing an extended response for an item on an assessment, how can you ensure that your response is clear, coherent, and unambiguous?

Formative Assessment

- Use "quickwrites" to assess student background knowledge prior to instruction.
- Have students read through their quickwrites for the unit. Ask them to identify sentences that need clarification or revision and to revise the quickwrites either individually or collaboratively. Ask students to write how their thinking has changed or ask them to write about what they have learned that could have supported their original ideas.
- Use quickwrites to help students synthesize their thinking and to assess their level of mastery of concepts.
- "Quickwrites" are often ungraded or graded as "credit' or "no credit."
- Review writing for logical arguments and reasoning.
- Assess the degree to which arguments are supported.
- Extend quickwrite topics into formal essays.



1.6: Math Autobiography

Topic

- Writing to learn

Objectives

Students will:

- Become acquainted with "Writing to Learn" as a topic in mathematics
- Practice writing skills
- Demonstrate their writing skills
- Have an opportunity to introduce themselves to the teacher

Timeline

- One 50-minute class period to finish first draft, plus homework time to edit

WICR Strategies

- Writing to Learn
  - — Write a math autobiography
- Reading to Learn
  - — Read autobiography in a small oral response group

NCTM Standards

Communication

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- organize and consolidate their mathematical thinking through communication; and
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others.

Rationale

The *"Math Autobiography"* provides students with a bridge between the kind of writing that they have done in their English and humanities classes and the writing that they will be doing in mathematics.

Vertical Alignment

- The "Math Autobiography" activity can be completed at any level.

Materials/Preparation

- Pen, highlighters, and paper

Instructions

- Communicate to the students that in order to get to know them better and for them to get to know you, they will be writing about themselves.
- As a prewriting activity, ask students to list specific things that they would like to include in each of the following categories. The only required category is experience(s) from previous math class(es). They may choose to use a graphic organizer to plan their writing.
  - A description of their family
  - Where they have lived
  - Where they have traveled
  - Experience(s) from previous math class(es)
  - Their education and career goals
  - Their outside interests, hobbies, sports or special talents
  - A word picture of the classroom they most remember where learning math was safe and enjoyable
- Provide students time to complete their first draft in class (15–20 minutes).
- Ask students to select one or more sentences that they are most proud of and highlight them.
- Use a "Popcorn" activity or other group share-out activity to provide an opportunity for students to share their favorite parts of their writing.
- Assign homework for students to edit their first draft.

Higher-Level Questions

Level Two

- What are some of the things that make learning math non-threatening and possible?

Level Three

- Which are the best prewriting strategies that you can use to plan your writing activities?

Formative Assessment

- Assess the length and fluency of student writing.
- Evaluate the complexity of student sentences.
- Are most students' math experiences positive or negative?
- Do most students feel comfortable sharing their writing?
- Do most students improve their first draft through the editing process?

1.7: Crossing the River

Topic

- Application of linear equations

Objectives

Students will:

- Work collaboratively on a multi-step mathematical inquiry
- Explore a variety of problem-solving strategies
- Write a detailed explanation
- Present solutions orally

Timeline

- One 50-minute class period for students to develop a rubric and begin exploring problem-solving strategies using manipulatives
- One 50-minute class period for guiding students by inquiry to the equation and setting expectations for the final write-up, plus time outside of class for drafting and editing written explanations
- Time at the beginning of class on the third day for students to present their work orally

WICR Strategies

- Writing to Learn
  - Describe the problem
  - Complete a problem write-up using a template
- Inquiry
  - Utilize manipulatives to discover patterns and potential solutions
- Collaboration
  - Work in small groups to understand a problem and explore solutions
- Reading to Learn
  - Utilize reading comprehension strategies to understand a complex problem, solution processes and problem write-up guide lines

NCTM Standards

Focal Point Grade 7

Number and Operations and Algebra: Developing an understanding of operations on all rational numbers and solving linear equations

Focal Point Grade 8

Algebra: Analyzing and representing linear functions and solving linear equations and systems of linear equations

Algebra

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- understand patterns, relations, and functions;
- represent and analyze mathematical situations and structures using algebraic symbols;
- use mathematical models to represent and understand quantitative relationships; and
- analyze change in various contexts.

Problem Solving

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- build new mathematical knowledge through problem solving;
- solve problems that arise in mathematics and in other contexts;
- apply and adapt a variety of appropriate strategies to solve problems; and
- monitor and reflect on the process of mathematical problem solving.

Reasoning and Proof

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- make and investigate mathematical conjectures; and
- develop and evaluate mathematical arguments and proofs.

Communication

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- analyze and evaluate the mathematical thinking and strategies of others; and
- use the language of mathematics to express mathematical ideas precisely.

Connections

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- recognize and use connections among mathematical ideas;
- understand how mathematical ideas interconnect and build on one another to produce a coherent whole; and
- recognize and apply mathematics in contexts outside of mathematics.

Representation

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- create and use representations to organize, record, and communicate mathematical ideas;
- select, apply, and translate among mathematical representations to solve problems; and
- use representations to model and interpret physical, social, and mathematical phenomena.

Rationale

Mathematical thinking develops over time. Working on a rigorous question over several days can deepen understanding of connections and approaches to problem solving. Students are empowered by utilizing critical thinking, reading, and writing skills and by exploring a variety of strategies to solve problems. By writing detailed and thorough explanations of what they have done, students become better communicators of mathematical concepts and become conscious of their thinking processes while solving rigorous problems. The skills learned and practiced in this activity transfer directly to scientific inquiry. The work that students do on this activity can establish a foundation for completing "Problems of the Week," and other long-term projects.

Vertical Alignment

- Parts of this activity can be introduced at the middle level. The generalizations developed and writing required at the end of the activity will challenge students in higher-level mathematics.

Materials/Preparation

- *Student Handout 1.7a:* "Problem-Solving and Guide for a Solution Write-up"
- *Student Handout 1.7b:* "Crossing the River"
- *Student Handout 1.7c:* "Questions to Guide Students to the Equation for Crossing the River"
- *Teacher Reference Sheet 1.7d:* Answer Key for *Student Handout 1.7c*
- *Overhead Transparency/Student Sample 1.7e:* "Sample Write-up Poster for Crossing the River"
- *Overhead Transparency/Student Sample 1.7f:* "Sample Write-up for Crossing the River"
- Sample rubric (See the *Introduction to Writing in Mathematics*)
- Review the Technical Writing Tips for Mathematics (See the *Introduction to Writing in Mathematics*)
- Manipulatives such as: two sets of different types of beans, two sets of different colored cubes, two sets of different types of candy, etc. ...

Instructions

Day One

- Distribute and review *Student Handout 1.7a:* "Problem-Solving and Guide for a Solution Write-up." This guide will help students begin the problem-solving process and assist their writing of the solution.
- Distribute *Student Handout 1.7b:* "Crossing the River."
- Using the sample rubric as a guide and the "Writing the Solution" portion of *Student Handout 1.7a,* ask students to develop a rubric to assess their solution write-up. Emphasize that they will be describing, as clearly as possible, what the problem is asking them to do, as well as writing about one or more solutions and several ways to arrive at them.
- Once the rubric is in place, encourage students to find a solution using manipulatives during class.
- Ask students to look for more that one way to solve the problem.

Day Two

- When students return to class, distribute *Student Handout 1.7c:* "Questions to Guide Students to the Equation for Crossing the River." Ask students to complete the handout in small groups.
- Using *Student Handout 1.7a:* "Problem-Solving and Guide for a Solution Write-up," ask students to prepare a 2–3 page typed report or a report written in ink that follows Standard English rules for the following day.
- Encourage students to be as specific as possible. Another student should be able to follow their reasoning process and be convinced that the solution is correct.
- Remind students that although it may be necessary to make assumptions in order to solve a problem, they must clearly state what those assumptions are.
- Review the "Technical Writing Tips for Mathematics."
- Tell students that they will be asked to present their solution write-up orally to the class the following day. Encourage students to utilize the class-generated rubric as a guide for self-assessment.
- If needed, share the student sample write-up *1.7e* and *1.7f.*

Higher-Level Questions

Level Two

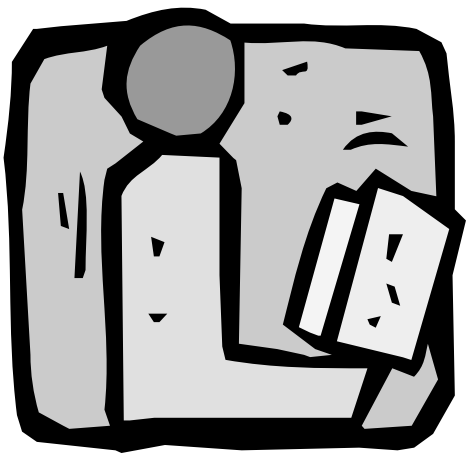
- When is the use of manipulatives effective in exploring potential solutions to a problem?

Level Three

- Can you think of "real world" situations similar to "Crossing the River" where mathematics can be used to find the outcome? (This question can be used to begin a Socratic Seminar following this activity).

Formative Assessment

- Assess student writing to ensure understanding of the problem.
- Monitor collaborative group skills.
- Assess students' use of manipulatives in seeking a solution.
- Ask students to explain the patterns that they have noticed.



Problem-Solving and Guide for a Solution Write-up

SOLVING THE PROBLEM

I. Read the problem carefully again and again.

II. Restate the Problem

- Use your own words.
- Include all parts of the problem including specific information such as lengths of line segments, size of angles, number sets mentioned, etc.
- Include any given figures and draw them to scale.
- Include any given formulas.

III. Search for a Solution

- Identify words and symbols you don't know. Look up their meanings and write them down.
- Think about the problem
- Use a graph, a data table, algebraic reasoning, technology or a combination of these to investigate the problem.

WRITING THE SOLUTION

IV. Describe Your Solution Process

- Describe how you got started.
- What strategies have you used to solve the problem?
- Include any charts, graphs, lists, geometric figures, drawings, manipulatives that you used or created.
- If you only found a partial solution, state what it is.
- If you know your solution is incomplete or wrong, explain how you know.
- If you have a general solution, state what it is and support it with specific examples.
- State whether you think there could be other correct solutions and support your position.

V. Reflect about What You Have Learned

- What mathematics was required to be able to solve the problem?
- What advanced mathematics helped you solve the problem in a more sophisticated way?
- How, if at all, does this problem relate to previous work you have done?
- How and when did collaboration with peers assist you in reaching a solution?
- What "real-world" applications might there be for problems like this?

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Crossing the River

You are on a camping trip with your family and your friend's family during a summer vacation. Your party consists of six adults and four children. While on a hike, you come upon a river that you will need to cross in order to continue on your hiking trail. At the edge of the river your friend's father finds a small boat. It can accommodate only one adult at a time. Or it can accommodate at most two children. It will not hold one adult and one child at the same time. Everyone is capable of rowing the boat.

Part A

1. How many one-way trips does it take for everyone to reach the other side of the river? A round trip counts as two one-way trips. Be sure to obey the safety rules about the number of passengers that can be in the boat at one time.
2. Draw a visual of the crossing operation so someone who looked at your diagram would understand how you arrived at your answer to question #1.
3. What if there were
   - 8 adults and 2 children?
   - 15 adults and 2 children?
   - 10 adults and 2 children?
   - Any number of adults ***a,*** and 2 children?
4. What if there were
   - 6 adults and 2 children?
   - 6 adults and 5 children?
   - 6 adults and 10 children?
   - 6 adults and any number of children, ***c***?
5. What happens if there were ***a*** adults and no children?
6. What happens if there are ***a*** adults and 1 child?
7. Write a rule that describes what happens if there are any number of adults, ***a,*** and any number of children, ***c***?

Part B

1. One group of adults and children required 27 one-way trips to cross the river. How many adults and children were in the group? Explain algebraically and by means of a diagram how you arrived at your answer.
2. Compare your result with that of a partner. Did you both reach the same conclusion? If not, could both answers be correct? How do you know?
3. Write your own question using the same conditions as those used in "Crossing the River." Exchange it with another team in the classroom. (Be sure you have answered the question correctly first!)

Part C

1. Add to your personal math glossaries any new words that you encountered in the problem.
2. Choose the one solution you like best.
3. Use *Student Handout 1.7a:* "Problem-Solving and Guide for a Solution Write-up" to guide you in writing up the solution.

Questions to Guide Students to the Equation for "Crossing the River"

1. Is there a pattern for adults?
2. What is the pattern?
3. Looking at the pattern, how many trips does it take to get one adult across the river?
4. If it takes four trips to move one adult to the other side of the river, how many trips would it take if you had five adults?
5. What if there were 2 adults?
6. How do you mathematically represent an unknown?
7. How can you represent the number of trips it takes for an unknown number of adults?
8. Now that we have the adult portion of the equation, let's focus on what has happened with the children. Once all the adults are across how many children have crossed?
9. How many children can cross at a time?
10. Each time two children go over, how many have to come back?
11. How could you mathematically represent some unknown number of children?
12. The first time two children cross the river one stays on the other side and does not return. How could you mathematically represent that one child stays on the other side of the river?
13. If you have an unknown number of children following that same pattern, what would that portion of the equation look like?
14. Keeping in mind that each child that crosses has to return with the boat to get another child across the river, how many trips is each child making to get the other children across?
15. Because each child has to cross twice we could multiply the children portion of the equation by 2 because each child takes two trips; what does that look like mathematically?
16. How many trips does the last child remaining on the other side of the river actually take?
17. Can the boat return by itself?
18. How can you represent mathematically one less trip or the boat not returning?
19. So what does the equation look like for unknown Adults and unknown Children?
20. Will this equation work with all combinations of adults and children? Are there any constraints that need to be mentioned?

Answer Key for Student Handout 1.7c

Questions to Guide Students to the Equation for "Crossing the River"

***Hint:* Have students think about patterns and function machines if they are struggling with the children portion of the equation.

1. Is there a pattern for adults?

   Yes! If you have someone clearly draw or model what is involved in getting one adult across the river the students will see the pattern.

2. What is the pattern?

   Looking at the diagram or recalling the model they should see that it takes four trips to get one adult across and with the one adult across the river all the children end up where they started.

3. Looking at the pattern, how many trips does it take to get one adult across the river?

   It takes four trips to move one adult to the other side of the river. All children will be where they originally started.

4. If it takes four trips to move one adult to the other side of the river, how many trips would it take if you had five adults?

   It will take four trips times the number of adults, five, or 20 trips. Keeping in mind that the children are assisting in getting the adults across the river. However, the children end up where they originally started.

5. What if there were 2 adults?

   It will take four trips times the number of adults, two, or eight trips.

6. How do you mathematically represent an unknown?

   By using a variable to represent the unknown.

   The variable "a" would be ideal to represent the unknown number of adults.

7. How can you represent the number of trips it takes for an unknown number of adults?

   4a

8. Now that we have the adult portion of the equation, let's focus on what has happened with the children. Once all the adults are across how many children have crossed?

   Based on previous representations nothing has happened to the children. All the children ended up where they originally started.

9. How many children can cross at a time?

   If the students re-read the original problem it will remind them that two children can cross at a time.

10. Each time two children cross the river, how many have to come back?

    Each time two children go over one has to return with the boat.

11. How could you mathematically represent some unknown number of children?

    The variable "c" would be ideal to represent the unknown number of children.

12. The first time two children cross the river one stays on the other side and does not return. How can you mathematically represent that one child stays on the other side of the river?

    You would have to subtract one, which would be represented mathematically by (2-1).

13. If you have an unknown number of children following that same pattern, what would that equation look like?

    (c-1)

14. Keeping in mind that each child that crosses has to return with the boat to get another child across the river, how many trips is each child making to get the other children across?

    If they are struggling with this, have them draw or model what the pattern looks like when all adults have crossed and only the children need to get across. The answer is 2 trips.

15. Because each child has to cross twice we could multiply the children portion of the equation by 2 because each child takes two trips; what does that look like mathematically?

    2(c-1)

16. How many trips does the last child remaining on the other side of the river actually take?

    One; at some point one of the children only takes one trip in order for all the children to be on the other side of the river.

17. Can the boat go back by itself?

    No.

18. How can you represent mathematically one less trip or the boat not returning?

    2(c-1) –1

19. So what does the equation look like for the number of Adults and Children?

    4a + 2(c-1) – 1

**Have students verify that this works for the scenarios they drew or modeled, by substituting numbers from the original problem.

20. Will this equation work with all combinations of adults and children? Are there any constraints that need to be mentioned?

    If there are no children only one adult can get across the river. There would be no way to return the boat to the other side. Hence there must be at least two children to make the transfer possible.

When we started the problem we identified the important information, such as the boat's maximum capacity of 1 adult or 2 children.

Then we drew pictures to help us understand the problem...

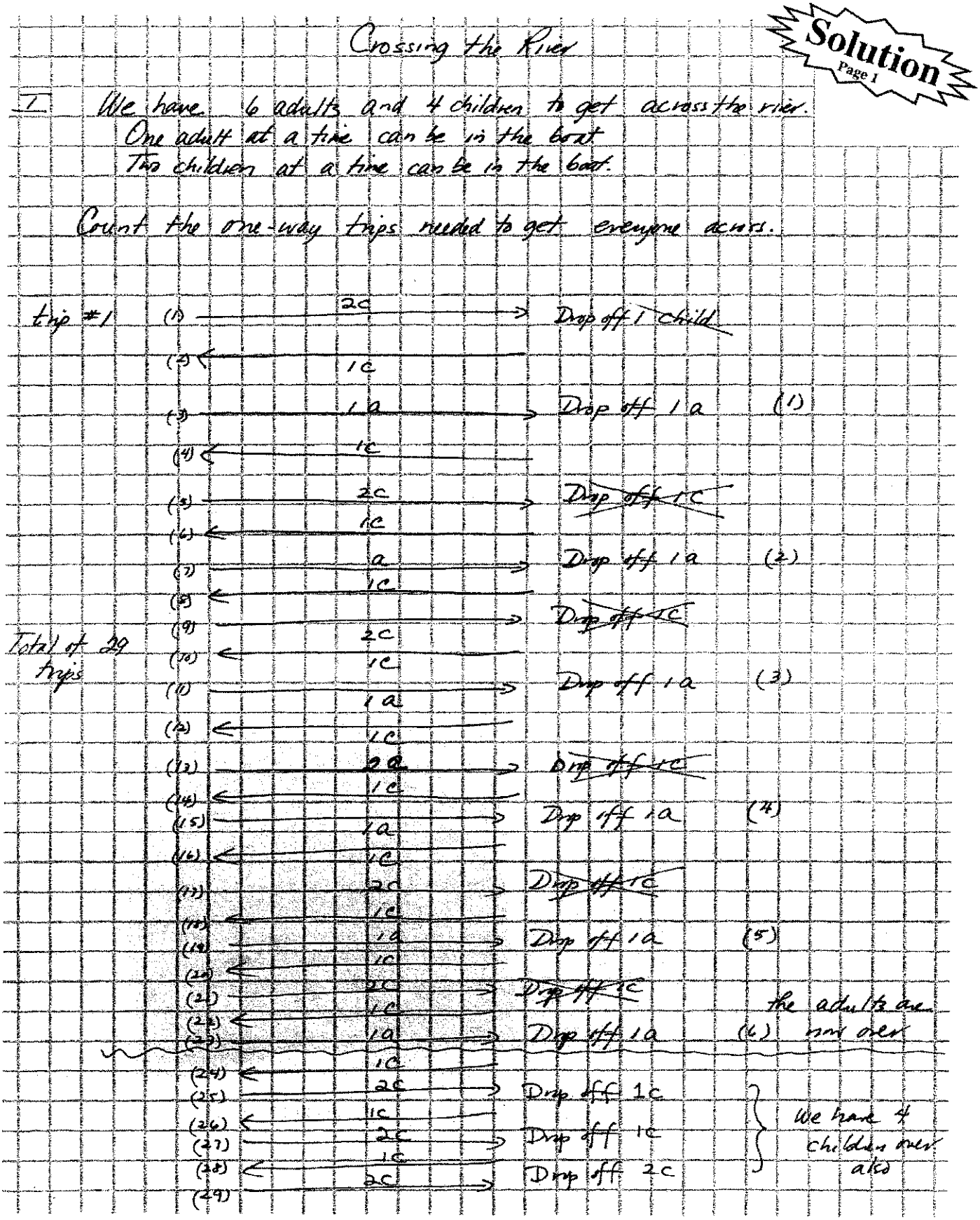
Trip 1
6 adults
2 children → 2 children

Trip 2
6 adults
3 children ← 1 children

Trip 3
5 adults
3 children → 1 adult
1 child

Trip 4
5 adults
4 children ← 1 adult

We found that this had to be done 6 times to get all the adults across. Which makes 24 trips, but 4 children are on the wrong side. It takes 5 additional trips to get all 4 children on the other side. This made a grand total of 29 trips. (It might have been faster to swim)


Solution
Page 1
Crossing the River
I We have 6 adults and 4 children to get across the river.
One adult at a time can be in the boat
Two children at a time can be in the boat.
Count the one-way trips needed to get everyone across.
trip #1
(1) 2c → Drop off 1 child
(2) ← 1c
(3) 1a → Drop off 1a (1)
(4) ← 1c
(5) 2c → Drop off 1c
(6) ← 1c
(7) a → Drop off 1a (2)
(8) ← 1c
(9) → Drop off 1c
(10) ← 2c
(11) → Drop off 1a (3)
(12) ← 1c
(13) 2a → Drop off 1c
(14) ← 1c
(15) → Drop off 1a (4)
(16) ← 1c
(17) 2c → Drop off 1c
(18) ← 1c
(19) → Drop off 1a (5)
(20) ← 1c
(21) 2c → Drop off 1c
(22) ← 1c
(23) → Drop off 1a (6) the adults are now over
(24) ← 1c
(25) 2c → Drop off 1c
(26) ← 1c
(27) 2c → Drop off 1c
(28) ← 1c
(29) 2c → Drop off 2c
We have 4 children over also
Total of 29 trips

Solution
Page 2

II Now I will explain what I did to find an answer to the question.

First I had to be sure I knew how to count the trips. I think a trip ⟶ counts for 1 trip ⟷ counts for 2.

I also think the boat holds 2 kids or 1 adult, but NOT 1 kid + 1 adult.

Then I made a "map" of all the crossings. I expected for fewer trips would be required, but as I worked on the picture I realized that the kids have to keep returning to their starting point in order to get the adults across. I "x"ed out each child when he/she returned to the left shore.

III Explain a rule that will work for other groups.

Judging from my diagram I think I need to expect 4 one-way trips for each adult, the last of which takes the boat back to get another adult over.

So Each child requires 2 trips except the last one who hops on the boat when the 2nd to last child goes across. In the first case we have 4 children and it takes $2(4-1)$ trips $= 6-1 = 5$

Total # of trips: $4a + 2(c-1) - 1$ ← we don't have to return on the last trip

If there are no children only 1 adult can get across. Then there is no way to get the boat back to the left shore. There have to be at least 1 children to make the transfer possible.

Solution Page 3

Now I will test my rule with 3 adults and 4 children
My rule suggests I will need $3 \cdot 4 + 2(4-1) + 1 = 18$ one-way trips

(1) 2c → Drop off 2c
(2) 1c ←
(3) 1a → 1a
(4) 1c ←
(5) 2c → Drop off 2c
(6) 1c ←
(7) 1a → 1a
(8) 1c ←
(9) 2c → Drop off 2c
(10) 1c ←
(11) 1a → Drop off 1a
(12) 1c ←
(13) 2c → Drop off 1c
(14) 1c ←
(15) 2c → Drop off 1c
(16) 1c ←
(17) 2c → Drop off 2c

= 17

3a

4c

So it works!
With this rule I can answer all the ~~rules~~ questions asked.

$6a + 2c$ children $= 6 \cdot 4 + 2(2-1) - 1 = 24 + 1 = 25$
$6, 5 = 4 \cdot 6 + 2(5-1) - 1 = 24 + 7 = 31$
$6, 10 = 4 \cdot 6 + 2(10-1) - 1 = 24 + 17 = 41$
$6, c = 4 \cdot 6 + 2(c-1) - 1 = 24 + 2(c-1) - 1$

I could also write the rule as: $4a + 2c - 3$
I prefer: $4a + 2(c-1) - 1$ because I understand why it works.

What size might the party be if they required 27 trips?

these are all equivalent expressions of the same question

$4a + 2(c-1) - 1 = 27$
$4a + 2(c-1) = 28$
$2a + c - 1 = 14$
$2a + c = 15$

Lots of answers are possible: 5 adults + 5 children
6 adults + 3 children
4 adults + 7 children

1.8: Technical Writing

Topic

- Area under the curve
- Technical Writing

Objectives

Students will:

- Practice technical writing skills
- Become acquainted with the writing process
- Explore alternative strategies for finding the area of a park

Timeline

- One 50-minute class period to find the area and begin the first draft, plus homework time to complete the draft
- One 50-minute class period to peer edit and begin a second draft, plus homework time to complete final draft

WICR Strategies

- Writing to Learn
  - — Write a business letter or a memo
  - — Write an invoice
  - — Practice technical writing skills
- Inquiry
  - — Discover alternative solution processes
  - — Decide between alternative solution processes
- Collaboration
  - — Work effectively with a partner for editing
- Reading to Learn
  - — Read personal writing
  - — Read and edit a peer's writing

NCTM Standards

Geometry

Instructional programs from pre-kindergarten through grade 12 should enable all students to use visualization, spatial reasoning, and geometric modeling to solve problems.

Communication

Instructional programs from pre-kindergarten through grade 12 should enable all students to use the language of mathematics to express mathematical ideas precisely.

Connections

Instructional programs from pre-kindergarten through grade 12 should enable all students to recognize and apply mathematics in contexts outside of mathematics.

Rationale

The *"Technical Writing"* activity provides a framework for developing and refining individual technical writing skills in a variety of modes. Students are given opportunities through formal writing assignments to practice and provide evidence of mastering technical vocabulary, technical writing, and key communication skills.

Vertical Alignment

- The specific content of the technical writing activity presented here is designed for students familiar with geometry concepts. However the concept of incorporating formal writing in the mathematics classroom can be introduced at any level.

Materials/Preparation

- Graph paper
- Computer for word processing
- *Student Handout 1.8a:* "Estimating the Number of Pumpkins"

Instructions

Day One

- Distribute *Student Handout 1.8a:* "Estimating the Number of Pumpkins."
- Divide the class into groups of two, three or four.
- Provide students time to read and understand the problem.
- Review alternative solution processes.
- Provide time for students to find the area of the pumpkin patch.
- Review the alternative writing prompts:
  - "Write a memo explaining the cost and how you arrived at it"
  - "Write a business letter explaining the costs and how you arrived at it."
  - "Write an invoice explaining the cost and how you arrived at it."
- Provide time or assign homework to complete the first draft of the writing.

Day Two

- Divide students into writing groups.

- Provide time to complete peer assessments and redrafting.
- Assign homework to complete the second draft of the writing.

Higher-Level Questions

Level Two

- What geometric shapes are the most effective in estimating the area of a pumpkin patch?

Level Three

- How many trapezoids need to be used to gain a close approximation of the area of a pumpkin patch?

Formative Assessment

- Did students get a close estimate of the area of the pumpkin patch?
- Did students use technical writing skills fluently?
- Did students work well together in groups?
- Did students give useful feedback during the peer editing?
- Was there a significant improvement between the first draft and the second one?

Estimating the Number of Pumpkins

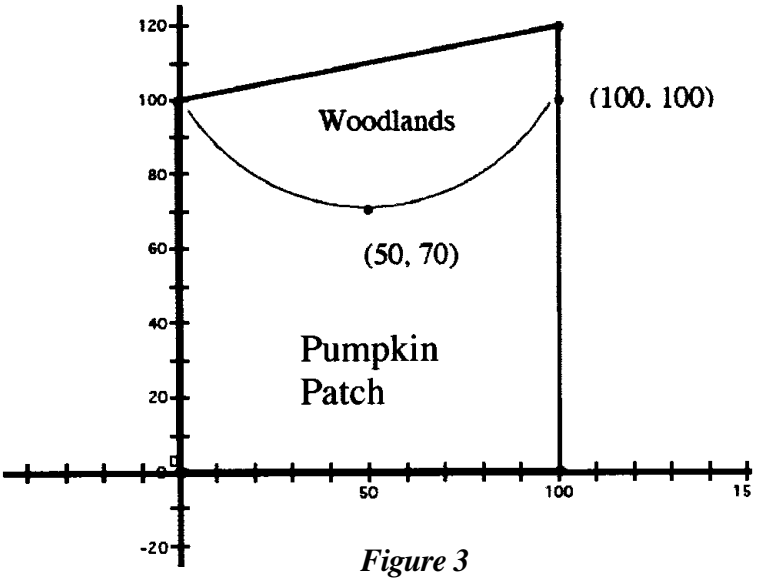


Figure 3

x	*f(x)*
5	94.3
10	89.2
15	84.7
20	80.8
25	77.5
30	74.8
35	72.7
40	71.2
45	70.3
50	70
55	70.3

* Not drawn to scale. All measurements are in yards.

Below are a variety of processes for finding the area of the pumpkin patch. Try each of the processes and decide which best estimates the true area of the pumpkin patch.

1. Using four squares/inch graph paper count the number of squares that approximate the area of the pumpkin patch.
2. Use graph paper with smaller squares than those used above and count the number of squares that approximate the area of the pumpkin patch.
3. Use four trapezoids, each with a height of twenty-five yards. Approximate the area of the pumpkin patch.
4. Use twenty trapezoids, each with a height of five yards. Approximate the area of the pumpkin patch.
5. **Challenge:** Find the equation of the parabola and then use your knowledge of the definite integral to find the area of the pumpkin patch.

Your local living history museum has planted pumpkins for the annual Harvest Festival. Two thousand pumpkins are needed on the day of the festival.

This past year was a dry year and some additional pumpkins may need to be purchased in time for the Harvest Festival. A museum volunteer estimated that the museum's pumpkin patch has an average of twenty-one pumpkins per 100 square yards. The cost of pumpkins is high this year, averaging 19 cents per pound. (An average pumpkin weighs fifteen pounds.)

Should the museum director order additional pumpkins? If so, how many should she order, and what will be the cost?

Make your recommendation in the form of a memo. Be sure to include enough information so as to convince the director that your estimation and recommendation are accurate.

1.9: Collaborative Drafting

Topic

- Writing to Learn

Objectives

Students will:

- Become acquainted with "Writing to Learn" as a topic in mathematics
- Work collaboratively to complete a first draft of a prompted writing
- Practice writing and editing skills
- Demonstrate their writing skills
- Understand that all writing is a draft that requires editing

Timeline

- One 50-minute class period to compose the first draft, plus homework time to write second draft
- One 50-minute class period to peer edit and compare second drafts, plus homework time to finish the final draft

WICR Strategies

- Writing to Learn
  - — Write a first draft based on a prompt
- Reading to Learn
  - — Read and edit the work or a partner or small group

NCTM Standards

Communication

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- organize and consolidate their mathematical thinking through communication; and
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others.

Rationale

Done individually, drafting produces writing that is ready for response and revision. Done collaboratively, drafting incorporates a variety of points of view forged into writing that is also ready for response and revision.

Vertical Alignment

- Collaborative drafting can be adapted to any level. At the higher grades the degree of complexity will increase.

Materials/Preparation

- Pen/pencil, markers, and highlighters
- Chart paper
- *Teacher Reference Sheet 1.9a:* "Sample Writing Prompts"
- Use *Teacher Reference Sheet 1.9a:* "Sample Writing Prompts" to prepare a selection of writing prompts.

Instructions

Day One

- Discuss the characteristics of first draft writing. Emphasize the importance of focusing on content and logical organization rather than mechanical correctness. Encourage the inclusion of diagrams, charts and graphs.
- Provide the students with a writing prompt along with the expected outcome and format of the final draft.
- Divide students into groups of two or three students each.
- Distribute materials.
- Explain that all the work in the first phase will be done silently.
- Encourage students to use a graphic organizer or other planning tool before they begin writing their first draft.
- Ask students to take turns silently writing their ideas on the chart paper.
- Provide students time to complete their collaborative first draft in class.
- Ask students to review the work on their first draft. In this phase they can speak aloud. Encourage them to ask questions to clarify the each other's writing and thinking.
- Ask students to select one or more sentences that they are most proud of and highlight them.
- Use a "Popcorn" activity or other group-share activity to provide an opportunity for students to share their favorite parts of their writing.
- Provide time for students to recopy the first draft ideas into their notebooks.
- Assign homework for students to write a second draft.

Day Two

- Upon returning to class, reconvene the groups.
- Review the guidelines for oral response groups. (See the *Introduction to Writing in Mathematics* section.)
- Ask each student to read his or her second draft to the group.
- Ask the group to provide specific editing suggestions for each other.
- Model accepting and giving editorial suggestions.
- Encourage students to take notes on the aspects of their partners writing that they would like to include in their final draft.
- Provide students with time to complete the editorial process.
- Ask students to complete the final draft as homework.

Higher-Level Questions

Level Two

- What are some of the best ways to begin organizing your thinking when completing a first draft?

Level Three

- What kinds of things can be incorporated into writing in mathematics that will help the reader understand the intent of the writer?
- What are some specific ways to give and receive editorial advice? Which are the most effective?

Formative Assessment

- Assess student collaboration skills.
- Assess the quality of student writing.
- Evaluate the complexity of student sentences.
- Assess the fluency of student writing.
- Do most students feel comfortable sharing their writing?
- Do most students improve their first draft through the editing process?

Sample Writing Prompts

Affective

- Describe a time when you were excited about a new thing that you learned in mathematics.
- Write a paragraph that begins with the statement, "People who are good at mathematics..."
- Write a paragraph that begins with the statement, "The hardest thing about mathematics class is..."
- Describe an activity in math class that you really enjoyed.
- Why is it important to study mathematics?

Mathematical Content

- The difference between ... and ... is ...
- Explain how the Pythagorean Theorem can be used to remember the distance formula?
- What are the patterns that you notice?
- Describe a place in life that you could use ...
- Write your own definition of ...
- Write everything that you know about...
- Write and solve an example word problem that uses the concepts we have been discussing.
- Compare and contrast ... and ...
- Describe practical uses of...
- Write a story that is represented by the graph/table or values ...

Process

- Write instructions that your younger brother or sister could use to ...
- How is what we learned today similar or different from what we have been doing?
- How can tables or graphs assist in ...
- How can you determine the best solution process for ...
- Describe the steps you would take to ...

"Today (AVID) is widely regarded as one of the most effective educational reforms ever created by a classroom teacher. The results have been extraordinary."

—Andrew Goldstein, *Time Magazine*

Unit Two: Inquiry in Mathematics

Introduction to Inquiry in Mathematics

AVID is based on inquiry, not lecture, because it is the process of posing and answering questions that teaches student to think. Many activities, such as tutorial and Cornell note-taking are built around asking questions and enable students to clarify, analyze, and synthesize material. Learning how to ask the right questions is a crucial skill, because many students have difficulty clarifying thoughts and asking the right questions to get the information and help that they need. Tutors and teachers are trained to ask questions that move students to successively higher levels of thinking.

Inquiry engages students with their own thinking processes. It teaches students to think for themselves instead of chasing the "right answer." The result is student ownership of the learning process and a better understanding of concepts and values. When Socrates encourages Crito in the Platonic dialogue to "examine the question together" and attempts to persuade him, he captures the essence of inquiry as an instructional method. In the AVID-inspired classroom, questioning takes many different forms: skilled questioning and writing questions (most often in collaborative learning groups), Socratic Seminars, Quickwrites, discussions, critical thinking activities, and open-mindedness activities.

Inquiry within the Math Tutorial

In the math tutorial, students engage in all levels of critical thinking, from recall to evaluation. Students pursue understanding with mutual respect and civility and are mindful of each other's dignity. They are willing to be persuaded by arguments/evidence more powerful than their own and to change their minds in light of fresh insights.

To begin tutorial, students bring questions for discussion. Guided by a teacher/tutor, students exchange responses and collaborate in search of understanding. By returning to notes and texts, students often gain a deeper understanding of the answers to the questions raised. This collaboration rests on the belief that the group can arrive together at some understanding that would not be arrived at independently.

There are several questioning strategies teachers/tutors can use to lead their groups. Two highly recommended methods outlined below are based on work in cognitive functions by Benjamin Bloom and Arthur Costa, respectively.

Using the revised Bloom's hierarchy of cognitive skills, teachers, tutors, and students can ask questions that follow along a continuum:

Remembering - Retrieving, recognizing, and recalling relevant knowledge from long-term memory.

Understanding - Constructing meaning from oral, written, and graphic messages through interpreting, exemplifying, classifying, summarizing, inferring, comparing, and explaining.

Applying - Carrying out or using a procedure through executing, or implementing.

Analysis - Breaking material into constituent parts, determining how the parts relate to one another and to an overall structure or purpose through differentiating, organizing, and attributing.

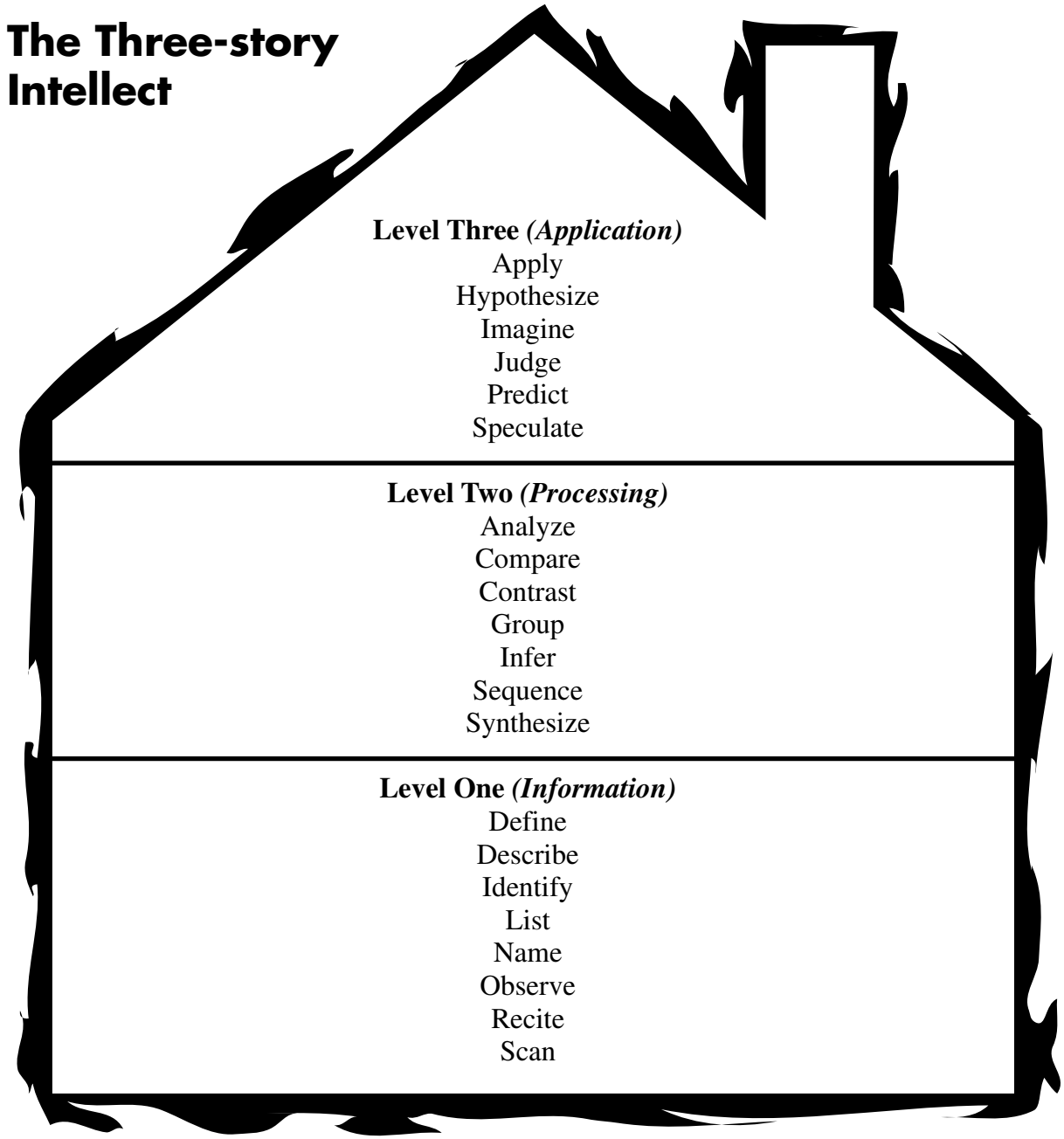
Evaluating - Making judgments based on criteria and standards through checking and critiquing.

Creating - Putting elements together to form a coherent or functional whole; reorganizing elements into a new pattern or structure through generating, planning, or producing.

(Anderson & Krathwohl, 2001, pp. 67–68)

Using Costa's Model of Intellectual Functioning in Three Levels, students ask three levels of questions:

1. **Level One** questions focus on gathering and recalling information.
2. **Level Two** questions focus on making sense of gathered information.
3. **Level Three** questions focus on applying and evaluating information.

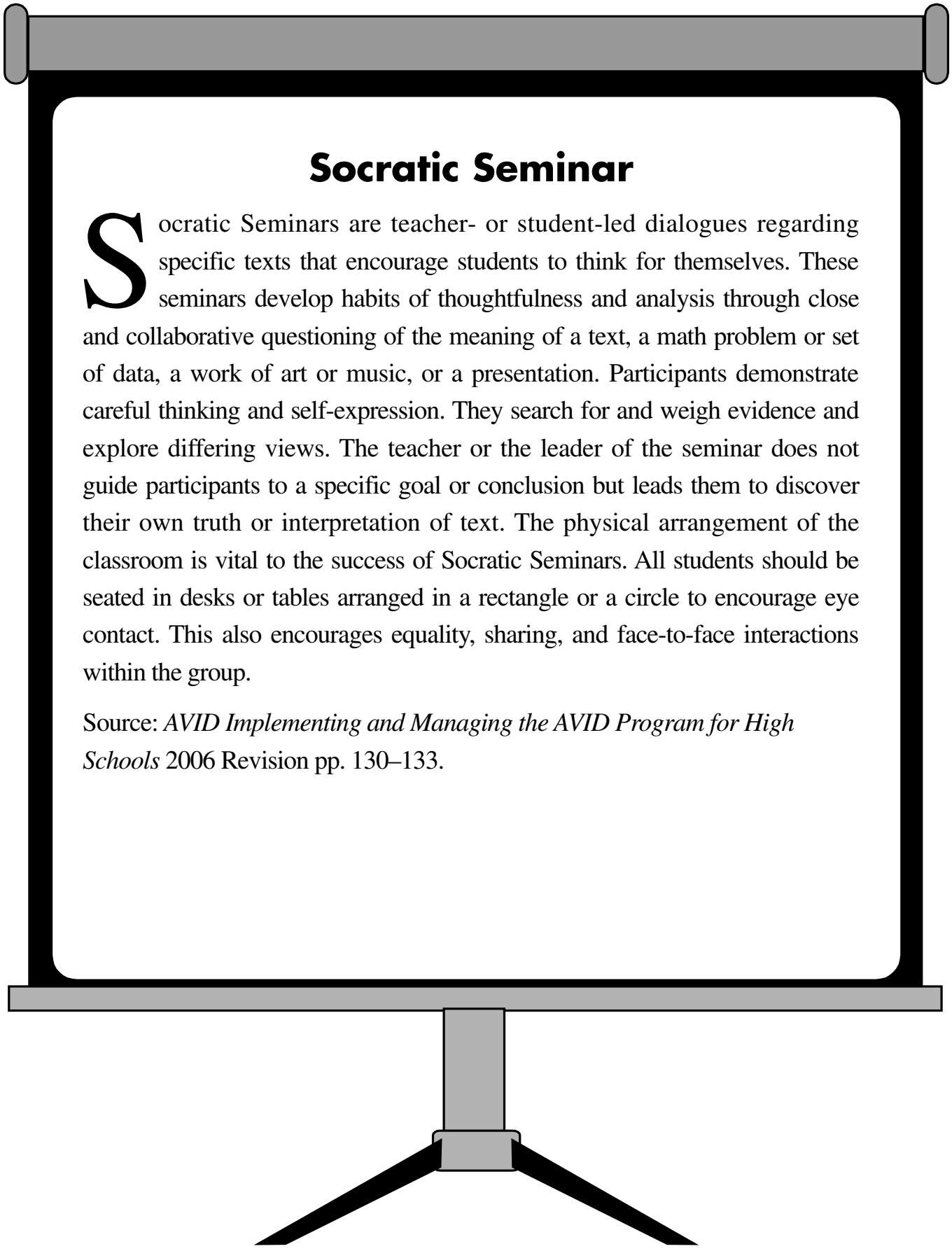


Costa's Levels of Questions

Level One	
defining	What is the definition of the slope of a line?
describing	What do all isosceles triangles have in common?
identifying	Identify the trig ratios of an acute angle in a right triangle.
naming	Name 5 different quadrilaterals.
listing	Make a table of ordered pairs that satisfy the function, $y = 3x + 1$.
observing	In a triangle, the longest side is opposite of which angle?
reciting	State the quadratic formula.
Level Two	
analyzing	In the proportion, $\frac{y}{5} = \frac{5}{x}$ what happens to the values of y as x increases?
comparing	How can you decide which graphic representation (line graph, box-and-whisker plot, stem leaf, etc.) to use with a given set of data?
contrasting	How are permutations and combinations different?
grouping	Group the following polygons according to a common characteristic: square, equilateral triangle, rectangle, scalene, triangle, regular pentagon, isosceles triangle, isosceles right triangle, trapezoid, rhombus, parallelogram
inferring	Given the first five members of a sequence below, find the sequencing rule that generates them and then find the next two members of the sequence: -5, -3, -1, 1, 3, ...
synthesizing	Consider the function of the form $f(t) = a(b)^t$ describe the function by means of a data table, its graph and an application. Choose any a and b you like.

Costa's Levels of Questions

Level Three	
applying a principle	Describe a use for the Commutative Property for real numbers.
hypothesizing	Under what conditions for x and y would $\frac{x}{y} > 1?$ or $\frac{x}{y} < |1|?$
imagining	What must be true about a set of data in which the median is larger than the mean?
judging	If the money and interest rates in two accounts are the same, will my principal earn more money in an account compounded quarterly or in a simple interest account? Why?
predicting	What happens to the value of a fraction if the numerator stays the same and the denominator keeps increasing?
speculating	In a normal distribution curve, can a data value ever be more than 3 standard deviations to the right of the mean? If you think the answer is yes, about how frequently would you expect that to happen?



Socratic Seminar

Socratic Seminars are teacher- or student-led dialogues regarding specific texts that encourage students to think for themselves. These seminars develop habits of thoughtfulness and analysis through close and collaborative questioning of the meaning of a text, a math problem or set of data, a work of art or music, or a presentation. Participants demonstrate careful thinking and self-expression. They search for and weigh evidence and explore differing views. The teacher or the leader of the seminar does not guide participants to a specific goal or conclusion but leads them to discover their own truth or interpretation of text. The physical arrangement of the classroom is vital to the success of Socratic Seminars. All students should be seated in desks or tables arranged in a rectangle or a circle to encourage eye contact. This also encourages equality, sharing, and face-to-face interactions within the group.

Source: *AVID Implementing and Managing the AVID Program for High Schools* 2006 Revision pp. 130–133.

2.1: Costa's Levels of Questions

Topic

- Costa's Levels of Questions

Objectives

Students will:

- Improve critical thinking
- Create higher-level questions
- Develop a model for translating lower-level questions into higher-level questions

Timeline

- One 50-minute class period to explore different levels of questions

WICR Strategies

- Writing to Learn
  - — Write sample test questions based on the Costa's Levels of Questions
- Inquiry
  - — Use tools of inquiry to examine test questions, and determine key words for types of questions
- Collaboration
  - — Discuss and make decisions about placement of key words and test questions according to Costa's Levels of Questions
- Reading to Learn
  - — Read questions from released exams with the purpose of categorizing them

NCTM Standards

Communication

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- analyze and evaluate the mathematical thinking and strategies of others; and
- use the language of mathematics to express mathematical ideas precisely.

Rationale

Student Handout 2.1c: "Sort It Out" provides a means to assess students on their understanding of different types of questions, as well as their ability to write questions for their Cornell Notes. This activity can be used as a review for upcoming exams while reinforcing inquiry models.

Vertical Alignment

- This activity can be done at any level beginning at a very young age. Prior to completing this activity, students should be provided an introduction to both Costa's Levels of Questions and Cornell Notes.

Materials/Preparation

- Copy of a state-released math exam
- Scissors, glue, and a large sheet of paper
- *Student Handout 2.1a:* "Costa's Levels of Questions"
- *Teacher Reference Sheet 2.1b:* "Costa's Levels of Questions Answer Key"
- *Student Handout 2.1c:* "Sort It Out"
- *Student Handout 2.1d:* "Vocabulary: Costa's Levels of Thinking and Questioning"
- *Student Handout 2.1e:* "Costa's Content Specific Questions"

Instructions

- Divide the class into groups of three or four students.
- Distribute *Student Handout 2.1a:* "Costa's Levels of Questions."
- Ask each group to separate the terms given into the three columns; a class discussion will follow about appropriate placement and differences between each category.
- Provide the students with an answer key for self-assessment.
- Distribute *Student Handout 2.1c:* "Sort It Out," as well as a copy of a teacher-created, district-, or state-released exam to each group.
- Distribute *Student Handouts 2.1d* and *2.1e* as reference tools to be used in sorting the exam questions.
- Ask students to cut out each question on the exam, and sort them according to Costa's Levels of Questions.
- Ask students to create a booklet of test questions; pasting the questions they have sorted in the appropriate category.

Higher-Level Questions

Level Two

- Sort problems from the state math exam into Costa's Levels of Questions categories.

Level Three

- Apply each level of Costa's questions to mathematics by writing your own math test question for each level.

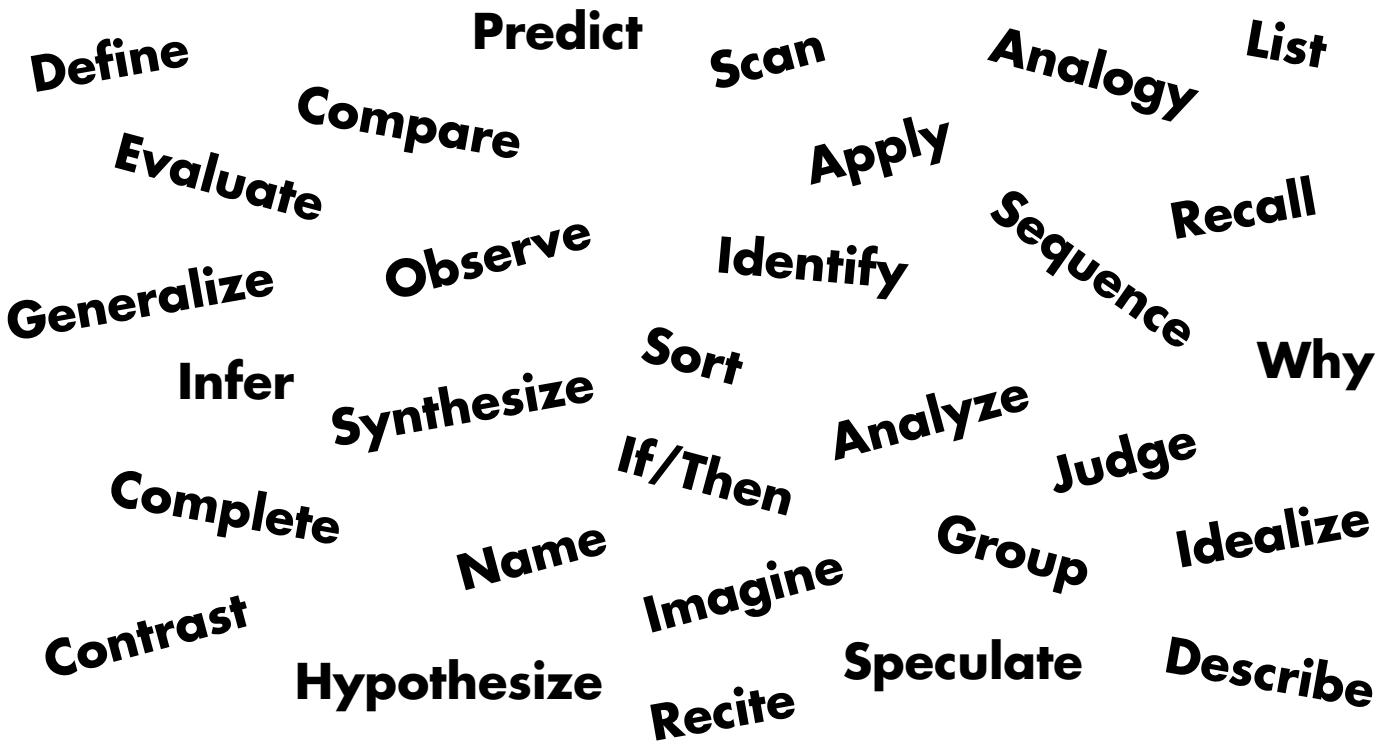
Formative Assessment

- Assess the accuracy with which students have sorted the questions provided into categories.
- Ask students to write questions for each level keeping in mind Costa's Levels of Questions.
- Ask students to submit questions for use on a formal assessment.

Costa's Levels of Questions

Organize the words below into the following categories:

Level One: Input	Level Two: Process	Level Three: Output



Costa's Levels of Questions Answer Key

Level One	**Level Two**	**Level Three**
Define	Analyze	Apply
Describe	Compare	Hypothesize
Identify	Contrast	Imagine
List	Group	Judge
Name	Infer	Predict
Observe	Sequence	Speculate
Recite	Synthesize	Generalize
Scan	Why	If/Then
Recall	Analogy	Idealize
Complete	Sort	

Sort It Out

Names: _____

_____

Materials Needed
Worksheet, Exam, Scissors (4 pairs), Glue Sticks, Hole Puncher

Procedures
Step 1: Cut out each question from the exam, making sure that you keep the problem number with the problem.
Step 2: Sort the problems according to the chart below, listing only the number of the problem:

LEVEL ONE	LEVEL TWO	LEVEL THREE

Step 3: Check with your teacher to make sure you have correctly placed the problems in their levels. Make corrections where appropriate.

Step 4: Once all your problems are correctly placed, take several sheets of colored paper, and glue each problem neatly, in numerical order, according to their category *(see example)*.

Step 5: Place the papers in order. Punch three holes down the left side of the colored papers. Use three pieces of string, one for each hole, to connect the papers into a book.

Vocabulary: Costa's Levels of Thinking and Questioning

LEVEL 1

Remember	Define	List	Recall	Match
	Repeat	State	Memorize	Identify
	Name	Describe	Label	Record
Show Understanding	Give examples	Rewrite	Review	Tell
	Restate	Recognize	Locate	Extend
	Discuss	Explain	Find	Summarize
	Express	Report	Paraphrase	Generalize

LEVEL 2

Use Understanding	Dramatize	Use	Translate	Interpret
	Practice	Compute	Change	Prepare
	Operate	Schedule	Pretend	Demonstrate
	Imply	Relate	Discover	Infer
	Apply	Illustrate	Solve	
Examine	Diagram	Question	Analyze	Criticize
	Distinguish	Inventory	Differentiate	Experiment
	Compare	Categorize	Select	Break down
	Contrast	Outline	Separate	Discriminate
	Divide	Debate	Point out	
Create	Compose	Draw	Plan	Modify
	Design	Arrange	Compile	Assemble
	Propose	Suppose	Revise	Prepare
	Combine	Formulate	Write	Generate
	Construct	Organize	Devise	

LEVEL 3

Decide	Judge	Justify	Assess	Summarize
	Value	Decide	Select	
	Predict	Measure	Estimate	
	Rate	Choose	Conclude	
Supportive Evidence	Prove your answer. Support your answer.	Give reasons for your answer.	Explain your answer. Why or why not?	Why do you feel that way?

Costa's Content Specific Questions

Costa's Levels of Questioning: Math

LEVEL 1	LEVEL 2	LEVEL 3
What information is given?	What additional information is needed to solve this problem?	Predict what will happen to _____ as _____ is changed.
What are you being asked to find?	Can you see other relationships that will help you find this information?	Using a math principle, how can we find...?
What formula would you use in this problem?	How can you put your data in graphic form?	Describe the events that might occur if...
What does _____mean?	What occurs when...?	Design a scenario for...
What is the formula for...?	Does it make sense to...?	Pretend you are...
List the...	Compare and contrast _____ to _____ .	What would the world be like if...?
Name the...	What was important about...?	How can you tell if your answer is reasonable?
Where did...?	What prior research/formulas support your conclusions?	What would happen to ____ if _____ variable were increased/decreased?
What is...?	How else could you account for...?	How would repeated trials affect your data?
When did...?	Explain how you calculate...	What significance is this formula to the subject you're learning?
Explain the concept of...	What equation can you write to solve the word problem?	What type of evidence is most compelling to you?
Give me an example of...		
Describe in your own words what _____ means.		
What mathematical concepts does this problem connect to?		
Draw a diagram of...		
Illustrate how _____ works.		

2.2: Philosophical Chairs: An Investigation of Cylinder Volume

Topic

- Cylinder volume

Objectives

Students will:

- Investigate the characteristics of cylinders
- Develop inquiry skills
- Develop oral and written language skills

Timeline

- 30 minutes to complete the Philosophical Chairs activity
- Extension activity will require an additional 50-minute class period

WICR Strategies

- Writing to Learn
  - — Write and support a prediction
  - — Write and self-evaluating participation in the class discourse
  - — Write about newly acquired understandings
- Inquiry
  - — Investigate the characteristics of a cylinder
  - — Develop a working hypothesis and supporting it
  - — Demonstrate a willingness to change position based on another's argument
- Collaboration
  - — Work together as a class to develop a deeper understanding of the characteristics of cylinders
- Reading to Learn
  - — Read and respond to a written prompt

NCTM Standards

Focal Point Grade 7

Measurement and Geometry and Algebra: Developing an understanding of and using formulas to determine surface areas and volumes of three-dimensional shapes

Focal Point Grade 8

Geometry and Measurement: Analyzing two- and three-dimensional space and figures by using distance and angles

Geometry

Instructional programs from pre-kindergarten through grade 12 should enable all students to analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.

Reasoning and Proof

Instructional programs from pre-kindergarten through grade 12 should enable all students to make and investigate mathematical conjectures.

Communication

Instructional programs from pre-kindergarten through grade 12 should enable all students to communicate their mathematical thinking coherently and clearly to peers, teachers, and others.

Rationale

This activity provides a framework for using and refining oral language and individual technical writing skills as well as building high-level questioning skills. Students working individually and collaboratively are given opportunities in *"An Investigation of Cylinder Volume"* to provide evidence of mastering mathematical content, technical vocabulary, technical writing, and key communication skills.

Vertical Alignment

- The "Philosophical Chair" activity enables students to discuss their conjectures in a non-threatening environment and lays the foundation for later Socratic discourse. The "Philosophical Chairs" activity can be used to explore mathematical content beginning at the middle level and extend through post-secondary discussions.

Materials/Preparation

- A transparency sheet (cut into two equal pieces)
- Tape
- An object to use as a "Talking Stick"
- M & Ms, Jelly Beans, Skittles or cereal (8–14 ounces)
- *Teacher Reference Sheet 2.2a:* "Using Philosophical Chairs"
- *Student Handout 2.2b:* "Philosophical Chairs: Rules of Engagement"
- *Student Handout 2.2c:* "Philosophical Chairs Report"
- *Student Handout 2.2d:* "Philosophical Chairs Written Evaluation Sheet"
- *Student Handout 2.2e:* "Philosophical Chairs Reflection"

Instructions

- Use the tape and two pieces of transparency to construct two cylinders. Make one with a circumference of 8.5 inches and the other with a circumference of 5.5 inches. See *Figure 1* below.

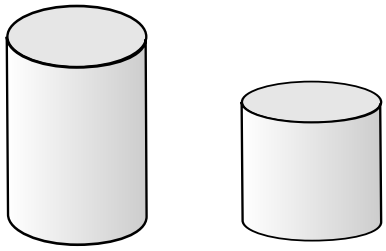


Figure 1

- Place the two cylinders on opposite sides of the classroom.
- Pose the Level One question: "Which of the cylinders will hold more candies, or will they hold the same amount?"
- Ask students to complete a "Two-Minute Quickwrite" predicting the correct solution and supporting their position.
- Arrange the chairs in the room into three areas representing the possible choices
  1. The cylinder with an 8.5-inch circumference holds more candy;
  2. The cylinder with a 5.5-inch circumference holds more candy; or
  3. The cylinders hold the same amount of candy).
- Ask students to move to the chairs that support their predictions.
- Distribute and review *Student Handout 2.2b:* "Philosophical Chairs: Rules of Engagement."
- Provide students with an object for use, such as a "Talking Stick." This will help students honor the commitment to one person talking at a time.
- Encourage students to have an open mind and be willing to be swayed by a good argument.
- When a student's mind is changed, he/she should physically move to the new position. Students may change their mind and move more than one time.
- Now offer to conduct an experiment to find the definitive solution.
- Place the tall cylinder inside the short cylinder. Fill the tall cylinder with small candies.
- Ask students to explain/predict what will happen to the candy when the tall cylinder is extracted.
- Slide the tall cylinder out and ask students to work with a small group to write an explanation of what the experiment shows and a proof that the same results can be shown for all cylinders with a fixed lateral surface area.
- Distribute *Student Handouts 2.2c, 2.2d,* and/or *2.2e* after the experiment is debriefed.
- Ask students to complete the recommended post-Philosophical Chair writing assignment (*Student Handout 2.2c, 2.2d,* and/or *2.2e*) or another writing assignment that will help students process their newly acquired understanding.
- *Challenge:* Investigate the relationships between volume and height and volume and radius with fixed surface area and/or fixed perimeter.

Higher-Level Questions

Level Two

- What are some "real world" applications of these circumference/volume relationships?

Level Three

- Will this hold true for all cylinders with the same lateral surface area?
- What is the relationship between volume and height and volume and radius with fixed lateral surface area and/or fixed perimeter?
- What is the general rule for geometric shapes and maximum volume?

Formative Assessment

- How well did students support their arguments in their "Quickwrite"?
- Were students able to follow the "Rules of Engagement" for the "Philosophical Chair" activity?
- Were students willing to change their position based on another student's argument?
- How well did students explain what they had learned?

Using Philosophical Chairs

Introduction

Philosophical Chairs is a format for classroom discussion and an activity that can be used in any content area. While this activity uses a format similar to debate, it is dialogue that we value. The benefits of this discussion activity include the development of students' abilities to give careful attention to other students' comments and to engage in dialogue with one another to gain a greater understanding of the topic presented.

Like Socratic Seminar, Philosophical Chairs exemplifies the use of WICR strategies in lesson planning. Inquiry and collaboration are inherent in Philosophical Chairs, and writing and reading are easily incorporated into a plan that results in the integration of the four components of WICR. Additionally, this activity makes a great prewriting activity as it allows students to gain and develop a variety of ideas about a topic.

Philosophical Chairs differs from Socratic Seminar in that it is not dependent on a text, but the reading of some text before engaging in the activity can only enhance the process. Philosophical Chairs focuses on a central statement or topic that is controversial.

Because the basic format for Philosophical Chairs remains the same from grade level to grade level, no explicit differentiations are included here. You will differentiate from grade level to grade level by choosing central statements or topics with increased complexity and by decreasing the level of teacher involvement in the process. In the middle school years, the teacher will almost always provide the topic and facilitate the discussion. By the junior and senior years in high school, students should be responsible for developing the central statement and for facilitating the discussions. Included in this unit are three activity sheets that may be used as part of the Philosophical Chairs activity. They provide varying degrees of structure. For middle level, you may want to provide more structure to the reflection after the activity. As students become more practiced at Philosophical Chairs and/or are in high school, you may use the activity sheets that are less structured.

Following are step-by-step guidelines for Philosophical Chairs and additional ideas for successful implementation of this activity in your classroom.

Guidelines for Philosophical Chairs

Classroom Setup

Chairs/desks are set up facing each other with about half facing one way and half facing the opposite way.

Directions

1. A statement is presented to the students. This statement might be based on a reading, a problem or it might be a stand-alone statement. Either way, the statement should be one that will divide the class into those who agree with the statement and those who disagree with the statement. Be sure that the statement is written on the board for reference during the activity. (*Note:* Allowing for a group of students who are undecided is addressed later in these guidelines.)

2. Those who agree with the central statement sit on one side and those who disagree sit on the other side.
3. A mediator who will remain neutral and call on sides to speak is positioned between the two sides. (The teacher usually fills this role in the beginning or middle school years. Eventually, students should take on this role.) In addition to facilitating the discussion, the mediator may at times paraphrase the arguments made by each side for clarification. It is important that the mediator always remains neutral.
4. The mediator recognizes someone from the side of the classroom that agrees with the central statement to begin the discussion with an argument in favor of the position stated. Next, the mediator will recognize someone from the other side to respond to the argument. This continues throughout the activity, and part of the job of the mediator is to ensure participation by as many students as possible and to keep just a few students from dominating the discussion. The mediator may also put a time limit on how long each side addresses the issue on each turn.
5. In addition to speaking in the discussion, students may express their opinions by moving from one side to other. Anyone may change seats at any time. Changing seats does not necessarily mean that a person's mind is changed, but rather that argument made is compelling enough to sway the opinions. Students may move back and forth throughout the discussion.
6. The discussion and movement go on for a designated period of time. The mediator may bring the discussion to a close at any time. Each side may be given an opportunity to make a final statement on the issue. If time allows, each participant states his/her final opinion and may also tell which arguments he/she found most convincing.
7. An additional piece to this activity can be to have a few students observe the process and take notes instead of participating. These students will debrief their observations to the class at the end of the activity. You may have students who were absent or unprepared to participate fulfill this role.

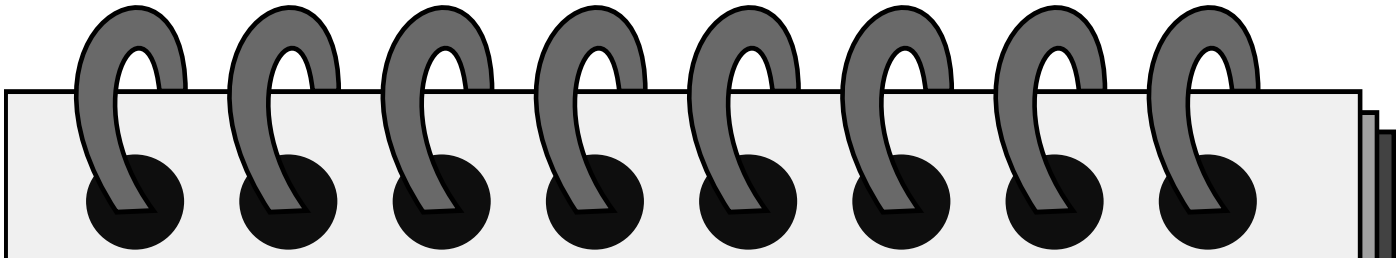
Evaluation

Leave time at the end of the period for students to reflect on the activity. Use one of the activities included in this unit. Students may begin the reflection in class and finish it for homework.

Modifications

It is recommended that you begin this activity with just two sides. If students experience difficulty choosing a side to begin the activity, encourage them to sit on the side that they agree with the most even if they do not completely agree. Once students are accustomed to this format, you may choose to add a third section of seats with a few chairs for students who are undecided. This section is placed between the two opposing sides.

During the discussion, you may allow students from the undecided section to participate or you may require that they take a position before participating. Students may move from the sides that agree or disagree with the statement to the undecided section if they wish. Before you end the discussion, require that all students still seating in the undecided zone move to one side or the other depending on which they believe made the most compelling arguments.



Philosophical Chairs: Rules of Engagement

1. **Be sure you understand the central statement or topic before the discussion begins. Decide which section you will sit in.**
2. **Listen carefully when others speak and seek to understand their arguments even if you don't agree.**
3. **Wait for the mediator to recognize you before you speak; only one person speaks at a time.**
4. **You must first summarize briefly the previous speaker's argument before you make your response.**
5. **If you have spoken for your side, you must wait until three other people on your side speak before you speak again.**
6. **Be sure that when you speak, you address the ideas, not the person stating them.**
7. **Keep an open mind and move to the other side or the undecided section if you feel that someone made a good argument or your opinion is swayed.**
8. **Support the mediator by maintaining order and helping the discussion to progress.**

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Philosophical Chairs Report

- **Central Statement/Topic**

- **My Original Position**

- **How many times did I change my seat? _____**
- **My Ending Position**

- **How open-minded was I as I listened to other people talk?**
  - ❑ Mostly open-minded
  - ❑ Partially open-minded
  - ❑ Not very open-minded
- **Use the space below to explain why your position did or did not change and the reasons for your thinking.**

Philosophical Chairs
Written Evaluation Sheet

Directions: Answer each of the following questions about today's Philosophical Chairs activity in a few sentences.

1. What was the most frustrating part of today's discussion?

2. What was the most successful part?

3. What statements led you to change your seat or to remain sitting in your original position?

4. What conclusions can you draw about how you form your beliefs based on today's discussion?

5. What would you change about your participation in today's activity? Do you wish you had said something that you did not? Did you think about changing seats but didn't? Explain.

Philosophical Chairs Reflection

Directions: Provide a written reflection of the philosophical discussion you heard in class. Be sure to include the following points in your reflection:

- the statement that was discussed;
- the arguments *for* the statement;
- the arguments *against* the statement;
- your position and the reasons for this position; and
- whether or not you changed your mind during the discussion, which arguments swayed your thinking, and why.

2.3: Socratic Seminar

Topic

- Socratic Seminar

Objectives

Students will:

- Improve critical thinking skills
- Strengthen problem-solving abilities
- Demonstrate an open mind and willingness to consider different opinions, perspectives and/or methods
- Develop a deeper understanding of complex ideas through rigorous thoughtful dialogue

Timeline

One 50-minute class period to introduce students to the purpose and procedures of the Socratic Seminar

WICR Strategies

- Writing to Learn
  - — Organize thinking with a quickwrite
  - — Summarize thinking with a written debrief
- Inquiry
  - — Explore new concepts and ideas through inquiry
- Collaboration
  - — Work in collaborative groups to investigate new thinking
- Reading to Learn
  - — Read math texts in preparation for thoughtful dialogue
  - — Read and assess peer writing

NCTM Standards

Reasoning and Proof

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- recognize reasoning and proof as fundamental aspects of mathematics;
- make and investigate mathematical conjectures;
- develop and evaluate mathematical arguments and proofs; and
- select and use various types of reasoning and methods of proof.

Communication

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- analyze and evaluate the mathematical thinking and strategies of others; and
- use the language of mathematics to express mathematical ideas precisely.

Connections

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- recognize and use connections among mathematical ideas;
- understand how mathematical ideas interconnect and build on one another to produce a coherent whole; and
- recognize and apply mathematics in contexts outside of mathematics.

Representation

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- create and use representations to organize, record, and communicate mathematical ideas;
- select, apply, and translate among mathematical representations to solve problems; and
- use representations to model and interpret physical, social, and mathematical phenomena.

Rationale

Socrates believed that enabling students to *think for themselves* was more important than filling their heads with *right answers.* Teaching students to have conversations about abstract mathematical concepts promotes critical thinking skills and allows students to see that math solutions are not always right or wrong. Students have the opportunity to see the real world connections to math. Because there is no right or wrong answer, students are motivated to take intellectual risks. Socratic Seminars provide students with an opportunity to practice skills in critical thinking, reading, and inquiry. As the students develop these skills, they will gain confidence in higher levels of inquiry and discourse and they will improve their problem-solving skills.

Materials/Preparation

- Copies of math texts or problems
- Individual white boards and markers
- *Student Handout 2.3a:* "Introduction to Socratic Seminar"
- *Student Handout 2.3b:* "Socratic Seminar Procedures"
- *Student Handout 2.3c:* "Dialogue vs. Debate"
- *Student Handout 2.3d:* "Guidelines for Developing Socratic Seminar Questions"
- *Student Handout 2.3e:* "Socratic Seminar Standards"
- *Student Handout 2.3f:* "Double Entry Diary"
- *Student Handout 2.3g:* "Socratic Seminar Process Debrief"
- *Student Handout 2.3h:* "Socratic Seminar Discussion Debrief"
- *Student Handout 2.3i:* "Socratic Seminar Observation Form"
- *Student Handout 2.3j:* "Fishbowl Observation Form"
- *Student Handout 2.3k:* "Socratic Seminar Rubric"
- *Student Handout 2.3l:* "Socratic Seminar Holistic Rubric"

- Select a variety of abstract ideas, articles, texts, or activities that lend themselves to open ended discussion and deeper understanding. Potential texts and problems include:
  - Historical accounts of mathematicians
  - Math anxiety
  - How mathematics vocabulary is unique?
  - Money and economics
  - The nature of zero
  - What is infinity?
  - Discussion of limits
  - Similar shapes
  - Explore the volume of solids with the same area and perimeter…
  - Newspaper articles and graphs
  - Questions about probability
  - Sports statistics
  - Research statistics
  - Business articles—scheduling of train routes, airline routes, etc. …

Instructions

- Explain Socratic Seminar guidelines using the following student handouts:
  - *2.3a:* Introduction to Socratic Seminar
  - *2.3b:* Socratic Seminar Procedures
  - *2.3c:* Dialogue vs. Debate
  - *2.3d:* Guidelines for Developing Socratic Seminar Questions
  - *2.3e:* Socratic Seminar Standards
  - *2.3f:* Double Entry Diary
  - *2.3g:* Socratic Seminar Process Debrief
  - *2.3h:* Socratic Seminar Discussion Debrief
  - *2.3i:* Socratic Seminar Observation Form
  - *2.3j:* Fishbowl Observation Form
  - *2.3k:* Socratic Seminar Rubric
  - *2.3l:* Socratic Seminar Holistic Rubric
- Discuss the differences between a dialogue and a debate.
- Remind students that we are teaching them critical thinking skills. Different concepts are perceived differently and diversity is okay; encourage them to open their minds to other possibilities and alternative solution methods.
- Ask students to arrange their desks in a circle. Students should be able to see everyone in the circle without having to lean forward or back. For large classes, consider using the Inner/Outer Circles (Fishbowl) with 15–20 students in the inner circle. When using a Fishbowl, provide a "Hot Seat" so outer circle students can "jump" into the conversation, or alternatively allow the outer circle students to pass written questions/comments to the inner circle. It is important to give outer circle participants a task. See the student handouts for observing and recording/mapping the inner circle discussion.

- Distribute individual white boards and markers.
- Once students are in a circle, direct their attention to the text, picture, or problem they will be discussing. Optimally, students would have reviewed this as part of their homework.
- Ask the students to review the text, picture or problem quietly and generate a related higher-level question. With some classes it may help to begin the discussion and collaboration in small groups prior to starting the Socratic seminar. The "Whip Around" strategy or another group sharing strategy can be used to ensure that all student voices are included. Utilizing a Quickwrite, Double Entry Diary or other text-processing tool will facilitate deeper thinking.
- Ask each student to share his/her question with the group.
- Listen to the questions carefully and choose one of the questions to start the conversation. Be prepared with a starter question in the event that the group questions do not generate the discourse that leads to deeper understanding of the topic or do not meet the overall goal for the discussion.
- Remind students that the discourse is between them not with the teacher. Also, remind students not to engage in side conversations. When necessary, refer students back to *Student Handout 2.3b:* "Socratic Seminar Procedures."
- Become involved in the discussion only to redirect the dialogue or to monitor process issues.
- Avoid a lot of eye contact because students may look to you for approval of their responses rather than engaging in genuine dialogue with their peers.
- Encourage students to use their white boards to show, illustrate, or explain their mathematical thinking.
- End the seminar on a high note, so the students maintain their engagement with the topic and continue their discourse outside the classroom.
- Debrief the Socratic Seminar by sharing your observations of the dialogue, or feedback you took notes on during the seminar. Point out things that need to be focused on as well as many positive things that occurred during the seminar.
- Provide students with an opportunity to have a last comment, to say something that they wish they had said or missed having the opportunity to say. This can be done one at a time or by debriefing with a partner or two.
- Ask students to write a reflection, a learning log, or complete *Student Handout 2.3h:* "Socratic Seminar Discussion Debrief."

  — Keys for success
    - Choose and read the text carefully
    - Craft an opening high-level question
    - Review seminar procedures frequently
    - Conduct Socratic Seminars regularly
    - Debrief the seminar

Higher-Level Questions

- Review *Student Handout 2.3d:* "Guidelines for Developing Socratic Seminar Questions."

Formative Assessment

- Review the Socratic Seminar rubrics.
- Assess the mapping charts.
- Debrief strengths and areas of focus.
- Assess adherence to the "Socratic Seminar Guidelines."

Introduction to Socratic Seminar

The Elements of Socratic Seminars

A good Socratic Seminar consists of four interdependent elements: (1) the text, (2) the questions raised, (3) the seminar leader, and (4) the participants. A closer look at each of these elements will help explain the unique character of a Socratic Seminar.

The Text

Socratic Seminar texts are chosen for their richness in ideas, issues, and values, and their ability to stimulate extended, thoughtful dialogue. A seminar text can be drawn from readings in literature, history, science, math, health, and philosophy or from works of art or music. A good text raises important questions in the participants' minds, questions for which there are no right or wrong answers. At the end of successful Socratic Seminar participants often leave with more questions than they brought with them.

The Question

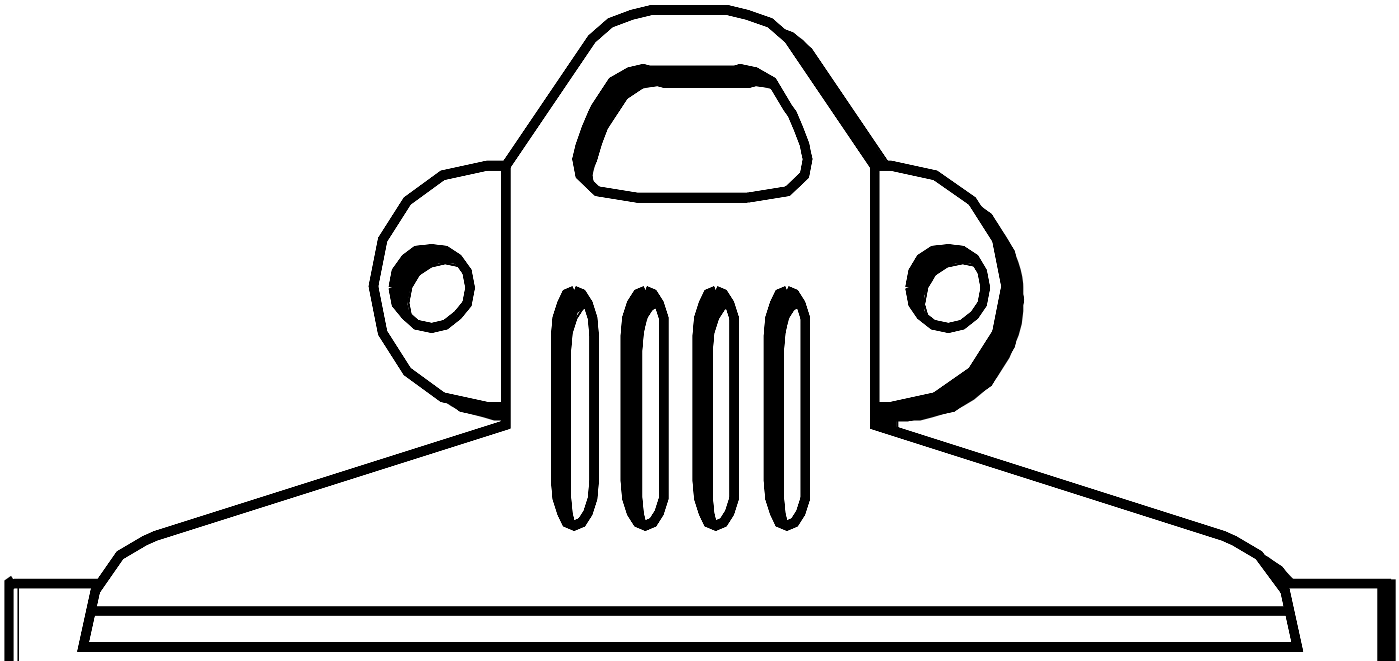
A Socratic Seminar opens with a question either posed by the leader or solicited from participants as they acquire more experience in seminars. An opening question has no right answer; instead, it reflects a genuine curiosity on the part of the questioner. A good opening question leads participants back to the text as they speculate, evaluate, define, and clarify the issues involved. Responses to the opening question generate new questions from the leader and participants, leading to new responses. In this way, the line of inquiry in a Socratic Seminar evolves on the spot rather than being pre-determined by the leader.

The Leader

In a Socratic Seminar, the leader plays a dual role as leader and participant. The seminar leader consciously demonstrates habits of mind that lead to a thoughtful exploration of the ideas in the text by keeping the discussion focused on the text, asking follow-up questions, helping participants clarify their positions when arguments become confused, and involving reluctant participants while restraining their more vocal peers. As a seminar participant, the leader actively engages in the group's exploration of the text. To do this effectively, the leader must know the text well enough to anticipate varied interpretations and recognize important possibilities in each. The leader must also be patient enough to allow participants' understandings to evolve and be willing to help participants explore non-traditional insights and unexpected interpretations. Assuming this dual role of leader and participant is easier if the opening question is one, which truly interests the leader as well as the participants.

The Participants

In Socratic Seminar, participants share with the leader the responsibility for the quality of the seminar. Good seminars occur when participants study the text closely in advance, listen actively, share their ideas and questions in response to the ideas and questions of others, and search for evidence in the text to support their ideas. Participants acquire good seminar behaviors through participating in seminars and reflecting on them afterward. After each seminar, the leader and participants discuss the experience and identify ways of improving the next seminar. Before each new seminar, the leader also offers coaching and practice in specific habits of mind that improve reading, thinking, and discussing. Eventually, when participants realize that the leader is not looking for the "right" answers but instead is encouraging them to think out loud and to openly exchange ideas, they discover the excitement of exploring important issues through shared inquiry. This excitement creates willing participants, eager to examine ideas in a rigorous, thoughtful manner.



Socratic Seminar Procedures

1. Refer to the text when needed during the discussion. A seminar is not a test of memory. You are not "learning a subject;" your goal is to understand the ideas, issues, and values reflected in the text.
2. It's okay to "pass" when asked to contribute.
3. Do not participate if you are not prepared.
4. Do not stay confused; ask for clarification.
5. Stick to the point currently under discussion; make notes about ideas you want to come back to.
6. Don't raise hands; take turns speaking. Do not engage in side conversations.
7. Listen carefully.
8. Speak up so that all can hear you.
9. Talk to each other, not just to the leader or teacher.
10. Discuss ideas rather than each other's opinions.

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Dialogue and Debate

Dialogue	**Debate**
Dialogue is collaborative with multiple sides working toward shared understanding.	Debate is oppositional with two opposing sides trying to prove each other wrong.
In dialogue, one listens to understand, to make meaning,and to find common ground.	In debate, one listens to find flaws, to spot differences, and to counter arguments.
Dialogue enlarges and possibly changes a participant's point of view.	Debate affirms a participant's point of view.
Dialogue creates an open-minded attitude developing an an openness to being wrong and an openness to change.	Debate defends assumptions as truth.
In dialogue, one submits one's best thinking, expecting that other people's reflections will help improve it rather than threaten it.	Debate creates a closed-minded attitude, a determination to be right.
Dialogue calls for temporarily suspending one's beliefs.	In debate, one submits one's best thinking and defends it against challenge to show that it is right.
In dialogue, one searches for strengths in all positions.	Debate calls for investing wholeheartedly in one's beliefs.
Dialogue respects all the other participants and seeks not to alienate or offend.	In debate, one searches for weaknesses in the other position.
Dialogue assumes that many people have pieces of the answer and that collaboration can lead to workable solutions.	Debate rebuts contrary positions and may belittle or deprecate other participants.
Dialogue remains open-ended.	Debate assumes a single right answer that someone already has.
In dialogue participants often leave with more questions than they brought with them.	Debate demands a conclusion and a winner.

Guidelines for Developing Socratic Seminar Questions

The learning in Socratic Seminars occurs as a result of the questions asked. Keep these guidelines in mind as you develop seminar questions.

- Be sure that your questions are void of judgment and derived from the text.
- Ask questions that raise questions.
- Avoid asking yes/no questions.
- Ask hypothetical and complex questions.
- Ask questions to which there are no right or wrong answers.
- Continue to ask "why" or to probe the responses of the participants with further questioning.
- Allow yourself to guide the discussion with your questioning, but to go with the discussion as well.

Listed below are examples of the types of questions you may want to develop and have ready for the Socratic Seminar.

1. By what reasoning did you come to that conclusion?
2. What would change your mind?
3. What are the assumptions, explicit or underlying, of this text?
4. Can you identify ideas in the text that seem to contradict one another?
5. What if _____ happened (or were true) instead of _____?
6. What might be some other good titles for this text?
7. What does the text say about the human race (or love, beauty, progress, etc.)? What do you think about it? What might other people say about it?
8. Do the ideas stated in this text seem to agree with or contradict this statement:_____
9. If _____ were writing (composing, painting, etc.) today, what might be different about this work?
10. In recent times, what well-known people are (were) like _____ in the text?
11. What does the term _____ mean in this text?
12. In what way would _____ change, if_____ happened differently?
13. How do you think something from the text was (would be) viewed by _____?
14. In what ways are _____ and _____ alike (or different)?
15. What part of this work provokes the most discussion? Least discussion? Why? What in the text supports that opinion?
16. What important conclusions can we draw from the text?
17. What does the writer/artist assume? What are you assuming?

Socratic Seminar Standards

Name _____

Instructions: Rate each standard on a scale of zero to four.

- **4** superior modeling of the standard
- **3** effective demonstration of the standard
- **2** some success at meeting the standard
- **1** little or no success at meeting the standard
- **0** no apparent attempt at meeting the standard

_____ Works to define abstract terms

_____ Develops and expands ideas

_____ Seeks enlightenment

_____ Takes intellectual risks

_____ Asks thoughtful questions

_____ Summarizes ideas of participants

_____ Demonstrates intellectual flexibility

_____ Asks for clarification

_____ Provides examples (and textual citations)

_____ Works to build collaborative thinking

_____ Contributes moderately and concisely

_____ Shows respect for others (and other ideas)

_____ Thinks before speaking

_____ Is polite and humble

_____ Promotes and maintains focus

_____ Is clear and articulate

Double Entry Journal

Instructions: Identify specific lines or passages from the reading and respond to those passages in order to understand their meaning and significance.

Statement from article/text/document or problem	Response—Initial thoughts about the statement

Socratic Seminar Process Debrief

Spending some time after the seminar to critique, debrief, and evaluate the process is critical. This reflection allows for the growth of the skills necessary to achieve quality seminars and high levels of thinking. The following questions may be asked of both participants and observers in the outer circle to help evaluate the seminar process.

Did the participants ...

- speak loudly and clearly?
- cite reasons and evidence for their statements?
- use the text to find support?
- listen to others respectfully?
- stick to the subject?
- talk to each other, and not just the leader?
- paraphrase accurately?
- ask for help to clear up confusion?
- support each other?
- avoid hostile exchanges?
- question each other in a civil manner?
- seem prepared?

Did the leader ...

- engage participants early? How?
- make sure that the questions were understood?
- ask questions that led to further questions?
- use answers as the basis for follow-up questions?
- allow for discussion of disagreements?
- listen carefully to participants' statements?
- accept participants' answers without judgment?
- keep attention on ideas in the text being discussed?
- correct misreadings of the text?
- allow time (pauses) for thinking?
- draw out reasons and implications?
- draw in all participants?

In the course of the seminar...

- what was the most interesting question?
- what was the most interesting idea to come from a participant?
- what was the best thing that you observed?
- what was the most troubling thing that you observed?
- what do you think should be done differently in the next seminar?

Socratic Seminar Discussion Debrief

Spending some time after the seminar to critique, debrief, and evaluate the discussion is critical. These following questions are designed to help participants and observers reflect on the content of the seminar dialogue. They may be used as a framework for discussion, or as a written debrief.

1. What was the best point made during the seminar?

2. What ideas did you agree with?

3. What ideas did you disagree with?

4. What questions were left unanswered?

5. What did you contribute to the discussion?

6. What do you wish you had said in the discussion?

7. Who were the top three contributors to the discussion?

8. What is your overall evaluation of the seminar?

Socratic Seminar Observation Form

Your name _____ Partner's name _____

Directions: Each time your partner does one of the following, put a check in the box.

Speaks in the discussion:

Looks at the person who is speaking:

Refers to the text:

Asks a new question: Literal questions (level 1) do NOT count.

Asks a clarifying questions:

Responds to another speaker:

Interrupts another speaker:

Engages in side conversation:

AFTER the discussion: What is the most interesting thing your partner said?

_____

_____

_____

AFTER the discussion: What would you like to have said in the discussion?

_____

_____

_____

Socratic Seminar Fishbowl

Directions: Choose three participants in the inner circle to observe during the seminar. Take careful notes and pay close attention to the dialogue, individual behaviors, and the group's dynamics.

Participant Name	New Idea	Asked a Question	Referred to Text	Positive Comment	Negative Behavior	Other Notes/ Observations
1.						
2.						
3.						

Socratic Seminar Rubric

Text Title: _____Name: _____

Rate on a scale of 0 = low and 3 = high

Academic Growth

Comments

1. Preparation of text	0	1	2	3
2. Speaking	0	1	2	3
3. Listening	0	1	2	3

Social Skills

1. Teamwork	0	1	2	3
2. Sensitivity/Good manners	0	1	2	3
3. Eye contact when speaking	0	1	2	3
4. Eye contact with speaker	0	1	2	3

Personal Skills

1. Willingness to accept other viewpoints	0	1	2	3
2. Responsibility and initiative	0	1	2	3

Comments about the group process:

Comments about the text:

Comments about the facilitator:

Comments about the Socratic question:

Socratic Seminar Holistic Rubric

This rubric gives a holistic assessment of a participant's behavior or in a seminar. It can be used by observers or a teacher to give individual feedback to students.

Exemplary

- ❑ Demonstrates patience with others' opinions
- ❑ Moves the conversation forward
- ❑ Speaks to all participants
- ❑ Thinks before answering
- ❑ Refers directly to the text
- ❑ Makes connections to other speakers
- ❑ Considers all opinions
- ❑ Builds on others' comments
- ❑ Asks for clarification when needed
- ❑ Identifies key words/phrases/details in the text

Commanding

- ❑ Comments often
- ❑ Responds to questions
- ❑ Refers to the text
- ❑ Offers interesting ideas
- ❑ Pays attention
- ❑ Asks a few questions

Competent

- ❑ Emphasizes only own ideas
- ❑ Ideas not always connected
- ❑ Refers to text
- ❑ Loses track of the conversation
- ❑ Judges others' ideas
- ❑ May ask questions

Developing

- ❑ Leans toward debate, not dialogue
- ❑ Disruptive or argumentative
- ❑ Mumbles or is silent
- ❑ Repeats some ideas
- ❑ Does not ask questions

Emerging

- ❑ Is not participating
- ❑ May be lost or overwhelmed with seminar

2.4: Mathematical Symbols, It's all Greek to Me!

Topic

- Reading and interpreting math symbols

Objective

- Students will develop an understanding of math symbols

Timeline

- One 50-minute class period to find the meaning of mathematical symbols

WICR Strategies

- Writing to Learn
  - — Provide a written explanation of mathematical sentences that use symbols
- Collaboration
  - — Work in small groups and agree upon the meanings of math symbols
- Reading to Learn
  - — Interpret math symbols when used in context of a math text

NCTM Standards

Communication

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others; • analyze and evaluate the mathematical thinking and strategies of others; and
- use the language of mathematics to express mathematical ideas precisely.

Representation

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- create and use representations to organize, record, and communicate mathematical ideas;
- select, apply, and translate among mathematical representations to solve problems; and
- use representations to model and interpret physical, social, and mathematical phenomena.

Rationale

Reading a math text is a lot like reading in a second language; both require the learning of new symbols. Without the knowledge of a symbol's specialized meaning in mathematics, a student's ability to understand and construct meaning from the text will be extremely limited. This activity will enable students to work collaboratively to translate and construct the meaning of mathematical symbols.

Vertical Alignment

- This activity can be adapted for every level of mathematics.

Materials/Preparation

- Select and make copies of the student handout that corresponds to the students' level of work.
- *Student Handout 2.4a:* "Mathematical Symbols *Example*"
- *Teacher Reference Sheet/Student Handout 2.4b:* "Mathematical Symbols"
- A textbook or other reference material

Instructions

- Distribute *Student Handout 2.4a:* "Mathematical Symbols *Example*."
- Ask students to complete the "First Guess" column independently, without any help from peers or reference materials.
- Have students form collaborative groups of between two and four students.
- Ask students to compare their interpretations and agree upon an interpretation for each symbol.
- Ask students to fill out the "Correct Meaning" column on the student handout as a group. Encourage students to use reference materials to find the meaning of any unknown symbols.
- Ask students to write a statement and explain the meaning of the symbols in their own words in the third and final column on the handout.
- *Challenge:* Distribute *Teacher Reference Sheet/Student Handout 2.4b:* "Mathematical Symbols." Ask groups of students to fill in the first column and then exchange and complete the form of a neighboring group.

Higher-Level Questions

Level Two

- Why is it important to use symbols in mathematical sentences?
- Are there different symbols that have the same meaning?

Level Three

- Are there any symbols that can have multiple meanings?
- Why might there be multiple symbols that have the same meaning?

Formative Assessment

- Assess the ability of students to translate written or verbal mathematical statements into symbolic representations and their ability to rewrite symbolic representations into sentences using the academic language of mathematics.

Mathematical Symbols *Example*

It is necessary to learn the meaning of mathematical symbols that are scattered throughout textbooks and technical writing. You know many of them already. Listed below are some frequently used symbols related to the math you are studying. Try to translate each symbol into a word or word phrase.

Symbol	First Guess	Correct Meaning	Write a Statement and Explain
$\sim$			
$\approx$			
$\cong$			
$m\angle ABC$			
${}_nP_r$			
(x,y)			
$p \leftrightarrow q$			
${}_nC_r$			
$\sum_{n=1}^{3} n^2 + 5$			

Symbol	**First Guess**	**Correct Meaning**	**Write a Statement and Explain**
$\therefore$			
$\langle a,b \rangle$			
$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$			
6^{-2}			
$b^2 - 4ac$			
$a^2 = b^2 = c^2$			

Mathematical Symbols

Symbol	First Guess	Correct Meaning	Write a Statement and Explain

2.5: Geometric Spaghetti

Topic

- Explore convex polygons

Objectives

Students will:

- Demonstrate an understanding of the characteristics of convex polygons
- Understand the necessary conditions for congruent triangles
- Learn through inquiry
- Work collaboratively to develop a general rule for the lengths of the sides of convex polygons

Timeline

- 10–20 minutes for students to generate a hypothesis and test it to form a rule

WICR Strategies

- Writing to Learn
  - — Write a hypothesis
- Inquiry
  - — Test a hypothesis
  - — Develop a general rule based on a tested hypothesis
- Collaboration
  - — Work in a small group to develop and test a hypothesis

NCTM Standards

Focal Point Grade 8

Geometry and Measurement: Analyzing two- and three-dimensional space and figures by using distance and angle

Algebra

Instructional programs from pre-kindergarten through grade 12 should enable all students to represent and analyze mathematical situations and structures using algebraic symbols.

Geometry

Instructional programs from pre-kindergarten through grade 12 should enable all students to analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.

Representation

Instructional programs from pre-kindergarten through grade 12 should enable all students to create and use representations to organize, record, and communicate mathematical ideas.

Rationale

Learning through inquiry is a critical skill. Students who interact with concepts will retain more for longer periods. In the *"Geometric Spaghetti"* activity students will be provided with the opportunity to generate hypotheses and quickly test them to form a general rule that will serve them well in their plane geometry courses.

Vertical Alignment

- This activity enables young students to discover the characteristics of polygons in a non-threatening way. The activity can be used when polygons are first introduced in the middle level and also used as students understanding of general rules of polygons and three-dimensional shapes expands in later math classes.

Materials/Preparation

- Paper
- Pencil
- Uncooked spaghetti

Instructions

- Divide the class into triads.
- Ask students to predict the number of pieces into which a piece of spaghetti will break if held at each end and bent.
  - *Note:* Physicists have reasoned that in nearly all cases the spaghetti will break into more than two pieces. (Many a sleepless night has gone into proving this mathematically.)
- Ask students to write their hypotheses.
- Survey the class for their ideas and then do the experiment.
- Ask students whose spaghetti broke into more than two pieces to use the pieces that they have to construct a convex polygon if possible.
- Ask the students who were unable to construct a convex polygon to display their pieces of spaghetti on an overhead/visual presenter for the class to see.
- Ask students to develop a rule that would explain why some students were able to construct the convex polygon and others were not.
- Ask the student groups to write a general rule for an n-sided polygon.
- Ask students to test their rule with a variety of lengths of spaghetti.
- Ask students to form triangles with three equal length sides of a given length of 10cm.
- Ask students to compare their triangles with those of other members in their group or with other groups.
- Ask students to construct triangles with side lengths of 10cm, 8cm and 15cm and then compare their triangles with those of other members in their group or other groups.

- Ask students to speculate why the all the triangles in the room are congruent.
- Ask students to speculate about how much must be known about two triangles to ensure that they are congruent.
- Divide the class into five groups and ask them to investigate the five remaining possible combinations of triangle sides and angles. (SAS, SSA, ASA, AAS and AAA)
- Ask each group to present a counterexample of why their combination does not work or a justification as to why it does.

Higher-Level Questions

Level Two

- What is the general rule for n-gons?
- What are the necessary conditions for congruent triangles?

Level Three

- Does the general rule for n-gons apply for concave polygons?
- Are there any generalizations that might be drawn for three-dimensional objects?
- Compare the characteristics of the congruent triangles and non-congruent triangles when using SSA.

Formative Assessment

- Were students able to work together to construct a general rule for an n-gon?
- Were students able to write and test a hypothesis?
- Were students able to apply their rule to a variety of polygons?
- Did students develop accurate triangle conjectures?

"...what AVID shows is that high minority achievement can be more ordinary when schools not only insist on academic rigor but also offer personal support. AVID offers a blueprint for this scaffolding."

—Richard Rothstein, *New York Times*

UNIT THREE: COLLABORATION IN MATHEMATICS

Introduction to Collaboration in Mathematics

Within an AVID-inspired classroom, groups are referred to as collaborative. The purpose of collaborative learning is to bring all students together to take responsibility for their own learning. In small groups, they ask, explore, and answer questions, become better listeners, thinkers, readers and writers, and discover ideas and remember them because they are actively involved. The teacher/tutor becomes a coach, carefully guiding students in their learning. Research shows that students learn best when they are actively manipulating materials through making inferences and then generalizing from those inferences. Collaborative groups encourage this kind of thinking.

Collaborative Learning Groups	*Traditional Learning Groups*
Positive interdependence	No interdependence
Individual accountability	No individual accountability
Heterogeneous	Homogeneous
Shared leadership	One appointed leader
Shared responsibility for one another	Responsibility only for self
Social skills necessary for task completion	Social skills ignored
Teacher/tutor observes and intervenes	Teacher ignores group functioning
Groups process their effectiveness	No group processing

Activities that encourage collaboration include: tutorials, jigsaws, group projects, read-arounds, and others.

Preparing for Collaborative Learning Groups

In collaborative learning groups, students experience the process of learning, the "how" as well as the "what" of learning. In order to achieve this, the teacher/tutor must carefully guide the group, thereby encouraging the members to share their ideas and explore and respect the ideas of others. The groups must constantly probe and define and redefine until the expression of ideas is precise and clear. The group task may have individual students share completed assignments or notes, as well as work together to brainstorm and problem solve.

Selection of Groups

In collaborative learning, there is no fixed way to group students. Depending upon the class and the assignment, the teacher may use teacher-determined, self-selected, spatial, or randomly selected groups.

Preparing Students

Students need to be prepared to work in groups, and, indeed, in the beginning, may shy away from group work because they are reluctant to share their work. Group work should begin with experiences that are non-threatening, gradually increasing in task demands and duration. Introduce collaborative group work by discussing group etiquette, stereotyping, and group dynamics with the students before they begin work. Spend time reviewing what a productive group "looks like" and "sounds like" and introduce students to "Socratic Seminar." Below are a few reasons why students should collaborate:

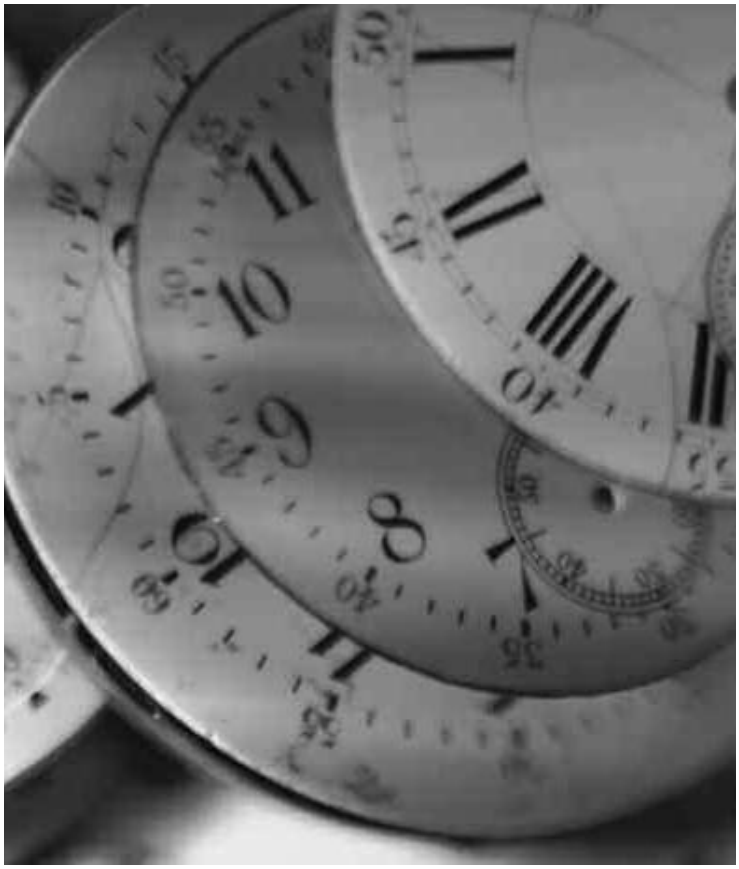
1. No one knows everything outlined in the Socratic Seminar lesson in the previous section.
2. Teachers expect analysis, synthesis, and evaluation of subject matter, which is the stuff of collaborative groups.
3. Students will move further faster and remember more.
4. Learning with other people is more fun than studying alone.

Reflecting on Collaborative Groups

Since learning to collaborate in groups is an ongoing process, after completing a group activity the students should write about and discuss what went well in their groups and what they need to improve for the next time.

Avoiding Mayhem

1. Provide the students with careful instructions and simple directions before they move into groups.
2. Establish a specific route for moving into groups.
3. Have students move their desks close together to prevent loud talking and to create a group atmosphere conducive to sharing ideas.
4. Establish a reasonable time limit. Allowing too much time for an activity can cause the groups to deteriorate. It is better for the groups to have too little time than too much. Remember, it takes time and practice for students to learn to work effectively in collaborative learning groups.



3.1: Math Tutorials

Topic

- Working in collaborative tutorials

Objectives

Students will:

- Work collaboratively to explore rigorous tutorial questions
- Demonstrate collaborative skills

Timeline

- One 50-minute class period to model and practice math tutorials

WICR Strategies

- Writing to Learn
  - — Write Cornell Notes
  - — Write a reflection and a summary
- Inquiry
  - — Investigate tutorial questions
- Collaboration
  - — Work in collaborative tutorial groups
- Reading to Learn
  - — Read tutorial questions
  - — Use a math text as a tutorial resource

NCTM Standards

Communication

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- analyze and evaluate the mathematical thinking and strategies of others; and
- use the language of mathematics to express mathematical ideas precisely.

Connections

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- recognize and use connections among mathematical ideas;
- understand how mathematical ideas interconnect and build on one another to produce a coherent whole; and
- recognize and apply mathematics in contexts outside of mathematics.

Rationale

Collaborative learning groups are the cornerstone of building successful tutorials. Students work together while taking responsibility for their learning, as well as the group's learning. By working in small groups, students have the opportunity to ask higher level questions and work together to explore and answer questions. Collaborative tutorial groups strengthen and enhance the students' listening, thinking, speaking, and writing skills. The collaborative process provides an opportunity to discover new ideas and take ownership of their learning because the students are actively involved.

Vertical Alignment

- Student collaboration for tutorials can be introduced at an early grade and refined as students mature.

Materials/Preparation

- *Student Handout 3.1a:* "Tutorial Learning Process"
- *Student Handout 3.1b:* "Inquiry in Tutorials"
- *Student Handout 3.1c:* "Roles in Collaborative Learning Groups"
- *Student Handout 3.1d:* "Tutorial Practice Problems"
- *Student Handout 3.1e:* "Sample Tutorial Notes"
- Make one set of "Tutorial Practice Problems" for each group of six students.

Instructions

- Model good, higher-level questions for tutorials.
- Provide students with an opportunity to practice writing tutorial questions.
- Encourage students to use their text books and other resources to assist them in writing higher-level questions. (Assist students in locating higher-level questions in their text books by identifying helpful icons and textual cues.)
- Distribute and review *Student Handouts 3.1a:* "Tutorial Learning Process," *3.1b:* "Inquiry in Tutorials" and *3.1c:* "Roles in Collaborative Learning Groups."
- Ask students to take Cornell Notes during the tutorial.
- Lead the students in a Fishbowl activity to demonstrate tutorials. You will play the role of the tutor. If you have tutors, they should observe you modeling this role as part of their training.
- Choose five or six students to take part in the fishbowl demonstration.
- Ask the students participating in the fishbowl to sit in a U-shape and face the white board. *Note:* Tutorial groups should never exceed seven students. The rest of the class will observe.
- Proceed with the fishbowl by selecting a student presenter. Ask the student come to the board and write down his/her tutorial question.
- Group members should take notes as the student presenter is at the board. The teacher/tutor should take notes for the student presenter.
- Model the inquiry questioning described on *Student Handout 3.1b:* "Inquiry in Tutorials." Take the lead in the questioning at the beginning of the demonstration.

- As you continue with the demonstration, coach the group members to ask the student presenters questions.
- Demonstrate how the role of the tutor/teacher or peer tutor is to encourage and promote the collaboration and the inquiry used in the tutorial process.
- Group members continue to take notes throughout the demonstration and they participate in the questioning as much as possible.
- Ask student presenters to finish up their question by reflecting on the process used to arrive at the solution.
- At the end of the demonstration, ask all group members to write a reflection and summary on their tutorial notes about the learning from the session.
- End the fishbowl demonstration by having a class discussion about the roles and responsibilities of each person in the group.
- Organize the students into groups of six students each. Select a group leader/peer tutor for each group to facilitate the group's process.
- Distribute *Student Handout 3.1d,* a set of six "Tutorial Practice Problems," to each group.
- Have the students follow the same process they saw in the fishbowl. Remind them to use questions to help the student presenter think about his/her question. Remind them that they should not give answers in tutorials.
- Move from group to group modeling good questioning and coaching the students with their collaboration and inquiry skills.
- Stop the groups at least 10 minutes before the period ends. Have one person from each group briefly share with the class some learning that took place in that group.
- Have all students write their reflective summaries showing the learning from the tutorial.
- Provide frequent opportunities for students to practice collaborative group skills in tutorials.
- *Challenge:* Distribute *Student Handout 3.1e:* "Sample Tutorial Notes." Ask students to improve the study questions and write a reflective summary for the sample tutorial notes.

Higher-Level Questions

Level Two

- How are Collaborative Tutorials the same or different than traditional tutorials?

Level Three

- What are the advantages of Collaborative Tutorials?

Formative Assessment

- Were student tutorial questions written at a higher level?
- Did students work collaboratively in finding solutions to tutorial questions?
- Did students take Cornell Notes on all tutorial questions?
- Were student reflections completed accurately?

Tutorial Learning Process

1. For homework the night before tutorials, write two questions from your classwork, text, or homework. *Note:* These questions should require higher-level processing and should not be "first level" questions.
2. The teacher/tutors or peer tutors should collect your tutorial questions as you enter the room.
3. The teacher/tutors or peer tutors should form tutorial groups based on the content of your questions. These groups should have four to seven students in them. The seating configuration should be a semi-circle and it will facilitate your communication if you face the board on which the student presenter illustrates their problem.
4. The tutor/teacher or peer tutor should be positioned with the seated members of the group and should facilitate the process of selecting a student presenter.
5. The student presenter should write his/her question on the board and explain to the seated members of the group the difficulty he/she is having. He/she should expect the seated members of the group to ask questions that clarify their own understanding of the question/problem, questions that check for understanding, and questions that probe deeper into possible approaches to solving the problem. When the presenter understands the problem with greater clarity, he/she should then communicate to the group this understanding.
6. The seated members of the group are responsible for helping the presenter think about the problem by asking questions. They are not responsible for finding the solution or necessarily leading the presenter to a solution. They should however, ask questions to clarify their understanding and to push the thinking of the presenter.
7. The teacher/tutor or peer tutor should facilitate the inquiry aspect of this tutorial process by guiding the seated members of the group with questions and modeling questions that they might ask the student presenter. The teacher/tutor or peer tutor should remind the seated members of the group to focus on the presenter's thinking, not the solution to the problem.
8. The teacher/tutor or peer tutor should remind the seated members of the group to take Cornell Notes on all questions.
9. The teacher/tutor should take notes for the student presenter.
10. Near the end of the tutorial session, all members of the group should write a summary/reflection of their learning (content and/or process). You may share these short writings with another member of your group if time permits.

Inquiry in Tutorials

You will often be asked to serve in the role of a peer tutor. The following processes and questions will help guide you in your role as a peer tutor and also in your role as a participant or student presenter.

The text for inquiry may come from ideas and notes in your learning logs and notes or materials from your math class.

You will be expected to come to your tutorial group with questions already formulated.

You will be provided with an opportunity to exchange responses and collaborate in a search for understanding. The strength of the group process rests on the belief that the group can arrive together at some understanding that would not be arrived at independently.

The following is a list of general questions that you may ask in your role as a peer tutor or as a tutorial group participant to help guide the learning of the student presenter:

Understanding the Problem

- What is this problem about?
- What level question have you asked? (If a level one question has been offered, assist the group in raising the level of the question. Use the text book and other resources to ensure that a higher-level question is being addressed.)
- What can you tell us about it?
- Can you explain the problem in your own words?
- What do you know about this part?
- Is there something that we can eliminate or that is missing?
- What assumptions do we have to make?
- How would you explain what we know right now?

Strategies: Thinking It Through

- What have you tried? What steps did you take?
- Do you have a system or strategy?
- What information do you have?
- How did you organize the information?
- What didn't work?
- Have you tried ... (guess and check, list, diagrams, etc.)?
- Where could we find out more information about that?
- Let's look at your notes.
- Let's see if we can break it down. What would the parts be?
- Have you tried making a guess?

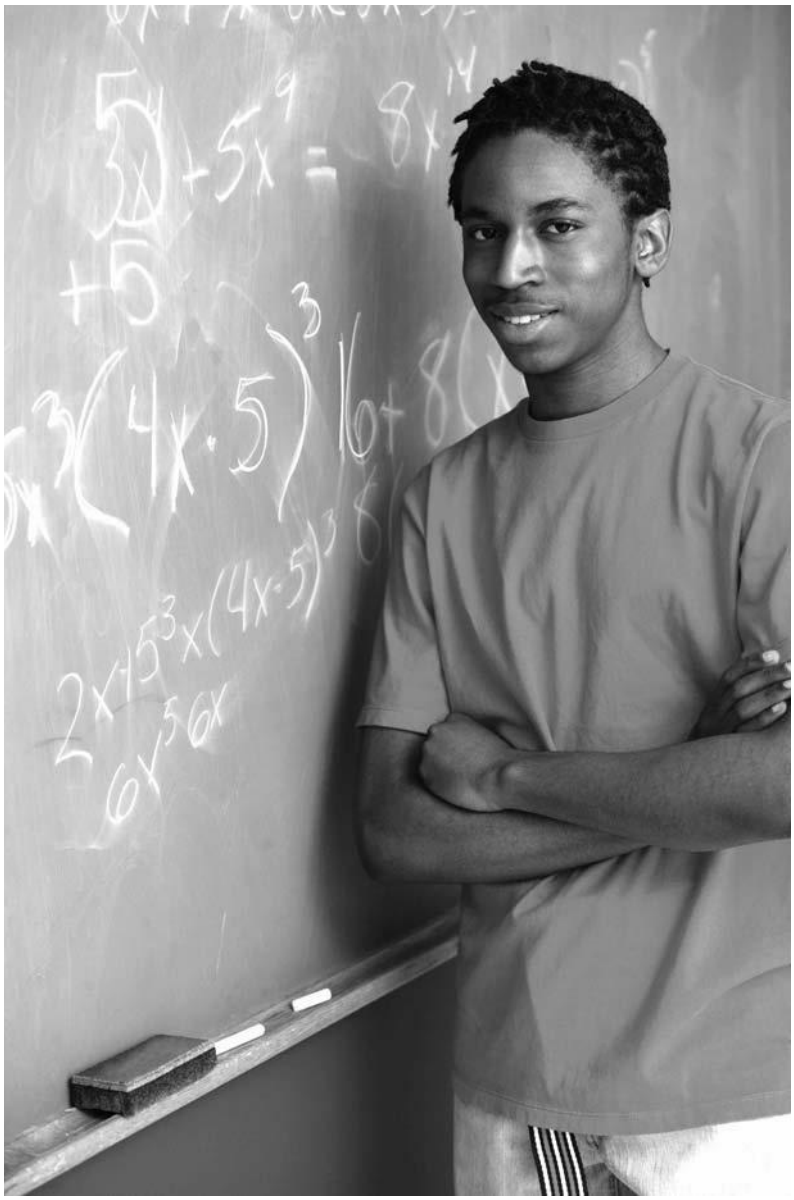
- Can you think of an easier but similar problem?
- What do you think comes next?
- What do you need to do next?

Checking the Solution

- Is your solution reasonable?
- How could you check your answer?
- Is that the only possible answer?
- Is there another way to do this problem?
- How do you know you have completed the problem?

Presenting the Solution

- Is your explanation clear and concise?
- Is there a general rule?
- Did you include charts, graphs, or diagrams in your explanation?
- Can anyone explain it in a different way?
- Is there a real-life situation where this could be used?
- Could your method of solving the problem work for the other problems?
- What were some things you learned from this problem?



Roles in Collaborative Learning Groups

Collaborative learning groups are the cornerstone of building successful tutorials. You will be working together while taking responsibility for your learning, as well as the group's learning. By working in small groups, you will have the opportunity to ask higher level questions as you work together to explore and answer questions. Collaborative groups will strengthen and enhance your listening, thinking, speaking, and writing skills. The collaborative process will provide an opportunity to discover new ideas and take ownership of your learning because you will be actively involved. For true collaboration, it is not essential that all members of the group master the same concepts at the same time. The members of the group will have strengths in a variety of different areas. Depending on the strengths of the individual group members, the collaborative group will create a positive interdependence and productiveness. The teacher/tutor or peer tutor will serve as a facilitator and coach. It is important that all members of the tutorial group understand their role as an active participant in the collaborative tutorial process.

Teacher/Tutor or peer tutor's role in the collaborative process:

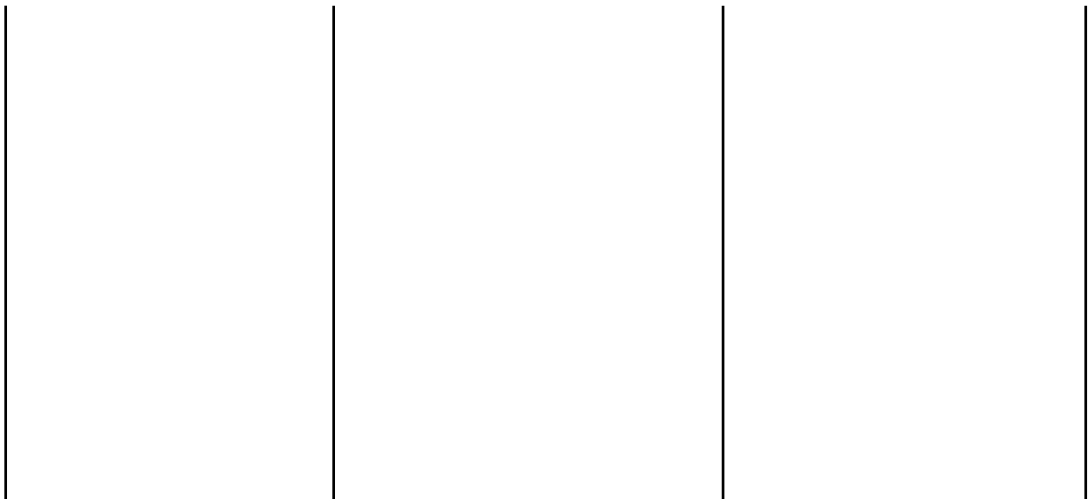
- Encourage group members to respect the ideas/thinking of others
- Model use of inquiry to allow group members to gain a deeper understanding
- Facilitate a balance of shared participation among group members
- Prompt members of the group to use WICR to summarize learning
- Coach members of the group to ask higher level questions of each other in order to gain a deeper understanding of their rigorous content
- Ensure a safe environment where members of the group are free to ask for clarification of the content

Student's role in the collaborative process:

- Formulate and write higher-level questions in preparation for the tutorial group.
- Respect ideas/thinking of others in the group
- Use inquiry to gain a deeper understanding of the content being discussed
- Actively participate in the group by listening, asking questions, answering questions, and taking Cornell notes
- Use WICR in the collaborative process
- Create an environment where group members feel comfortable and safe to ask questions and seek clarification of content
- Communicate openly about the group experience.

Five Marks Problem

Question: How can you add five more marks to make ten?

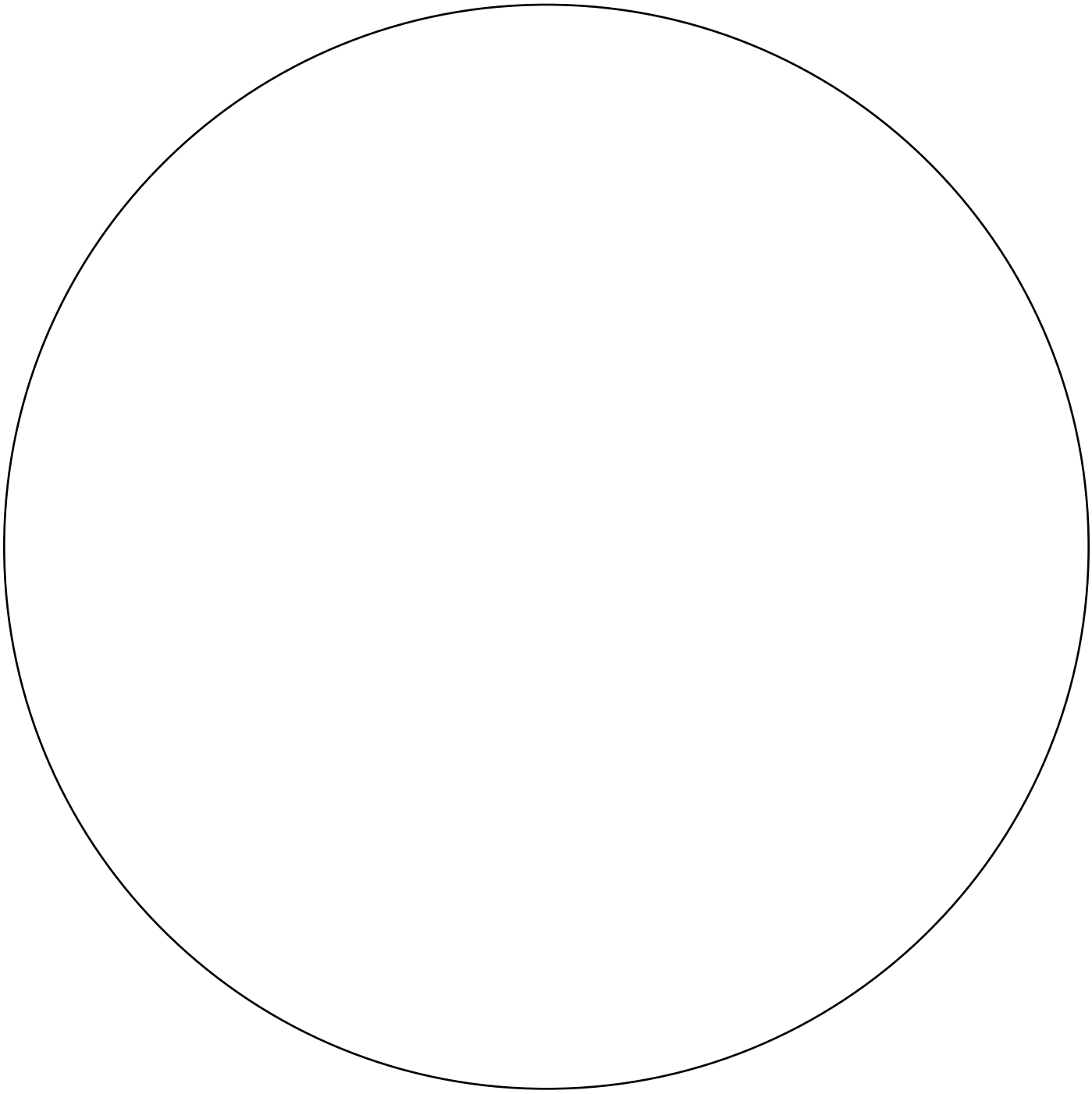


Answer to Amoeba Question

It will take the single amoeba three hours and three minutes to fill the jar. Once the amoeba in the first jar has reproduced itself (a process that takes three minutes), the jar is at the same point at which the second jar started. The only difference is that the amoeba in the first jar is three minutes behind the amoebas in the second jar.

Circle Problem

Question: What is the maximum number of parts into which a circle may be divided by drawing four straight lines?



Answer to the Water Lily Question

The lake is half covered on the fifty-ninth day. Since the water lilies double each day, the lake is half covered the day before it is fully covered.

Amoeba Problem

There are two jars of equal capacity. In the first jar there is one amoeba. In the second jar there are two amoebas.

An amoeba can reproduce itself in three minutes. It takes the amoebas in the second jar three hours to fill the jar to capacity.

Question: How long does it take the one amoeba in the first jar to fill the jar to capacity?

Answer to the Jamais/Toujours Question

1. Make the single question a nonsense question, such as, "Are you a rhinoceros?" Clearly, the individual who claims to be a rhinoceros is from Jamais.

OR

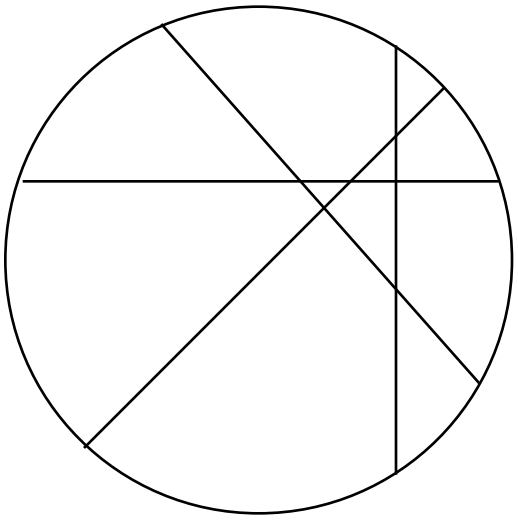
2. Ask any question that you can verify, such as, "Is it raining?"

Jamais/Toujours Problem

You know that the inhabitants of Jamais always lie, while the inhabitants of Toujours always tell the truth. You meet a man who you know comes from either Jamais or Toujours. You want to know which village he comes from.

Question: How can you find out by asking him only one question?

Answer to the Circle Question



Eleven parts may be formed with the four lines. The key is that each successive line must divide as many parts as possible.

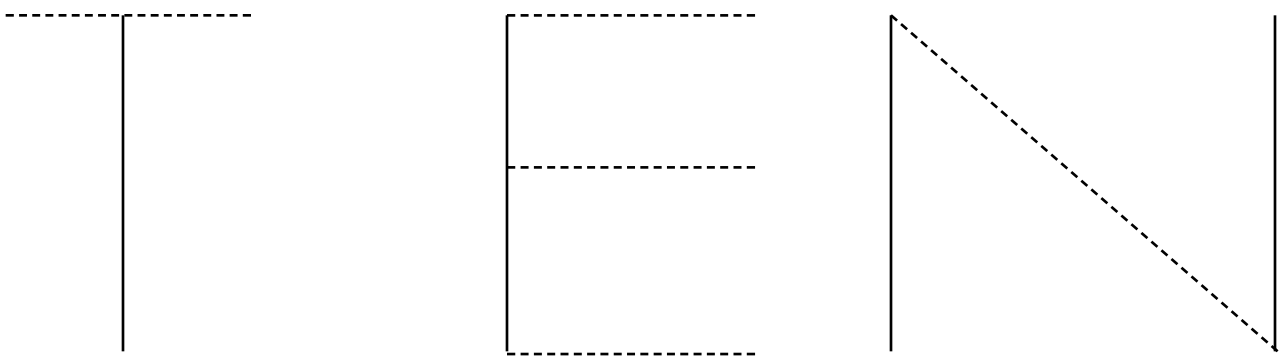
Rope Ladder Problem

A ship is at anchor. Over its side hangs a rope ladder with rungs a foot apart.

The tide rises at the rate of 8 inches per hour.

Question: At the end of 6 hours, how much of the rope ladder will remain above the water, assuming that 8 feet were above the water when the tide began to rise?

Answer to the Five Marks Question



(Two other solutions are also possible. Can you find them?)

Water Lily Problem

Water lilies on a certain lake double in area every twenty-four hours. From the time the first water lily appears until the lake is completely covered takes sixty days.

Question: On what day is the lake half covered?

Answer to the Rope Ladder Question

Since the ship is afloat, the water level in relation to the ship is always the same. Therefore, eight feet of the rope ladder are above the water at the end, just as at the beginning.

Sample Tutorial Notes

Tutorial oct 25,

How can you solve a function by grouping?

$f(x) = 2x^3 - 3x^2 - 8x + 12$

$= (2x^3 - 3x^2) - (8x + 12)$

$x^2(2x-3) - 4(2x+3)$

$(2x-3)(x^2-4)$

$(2x-3)(x-2)(x+2) = 0$

$2x - 3 = 0$

$+3 \quad +3$

$\frac{2x}{2} = \frac{3}{2}$

$x = 3/2, -2, 2$

x is interception

The multiples principles can be used if theyr is a percentage decrease suppose these is a 20 percentage discount. Again iet x be the original cost.

What u the new price in term of x?

A percentage put into an equation to get a number.

new price $= x - (.20x)$

Ex: new shoe = $ 101.00

discount 20%

old shoe $= 101.00 - [.20(101.00)]$

101.00

× .20

00000

20200

019.0

101.00

- 20.2

80.8

old shoe = $80.8

3.2: Families of Functions

Topic

- Characteristics of parent functions

Objective

- Students will investigate the domain, range, graphs, equations, and zeroes of linear, quadratic, square root, absolute value and cubic functions

Timeline

- One 50-minute class period to explore the characteristics of functions

WICR Strategies

- Writing to Learn
  - — Record knowledge of parent functions
- Collaboration
  - — Use the parent function information sheet with study partners during review and tutorials

NCTM Standards

Focal Point Grade 7

Number and Operations and Algebra: Developing an understanding of operations on all rational numbers and solving linear equations

Focal Point Grade 8

Algebra: Analyzing and representing linear functions and solving linear equations and systems of linear equations

Algebra

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- understand patterns, relations, and functions;
- represent and analyze mathematical situations and structures using algebraic symbols;
- use mathematical models to represent and understand quantitative relationships; and
- analyze change in various contexts.

Reasoning and Proof

Instructional programs from pre-kindergarten through grade 12 should enable all students to make and investigate mathematical conjectures.

Communication

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- analyze and evaluate the mathematical thinking and strategies of others; and
- use the language of mathematics to express mathematical ideas precisely.

Representation

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- create and use representations to organize, record, and communicate mathematical ideas;
- select, apply, and translate among mathematical representations to solve problems; and
- use representations to model and interpret physical, social, and mathematical phenomena.

Rationale

Understanding the characteristics of functions is a foundation of secondary mathematics requiring extensive review and practice. This activity provides an opportunity for students to use collaboration to explore the characteristics of functions and teachers to assess student understanding of functions.

Vertical Alignment

- This activity can be used to explore and/or assess the characteristics of functions beginning in Algebra 1 and extending into higher-level mathematics and complex functions.

Materials/Preparation

- *Student Handout 3.2a:* "Characteristics of Functions"
- *Teacher Reference Sheet 3.2b:* "Characteristics of Functions Answer Key"
- Graphing calculators

Instructions

- Introduce each parent function.
- Ask students to complete *Student Handout 3.2a:* "Characteristics of Functions."
- Direct student attention to the domain, range, *x* - intercepts and symmetrical properties of the graphs.
- The student handout can be used in several ways:
  - — Use the student handout for guided notes while introducing parent functions;
  - — Use the student handout as a cumulative homework or class work assignment; and
  - — Fill in parts of the chart and have students complete the missing information.

Higher-Level Questions

Level Two

- Why is the domain of a linear function all real numbers?
- Which functions have domains that are all real numbers?

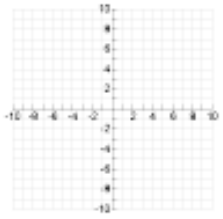
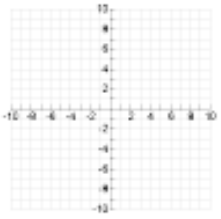
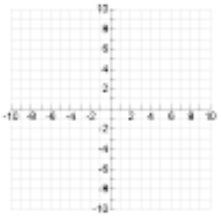
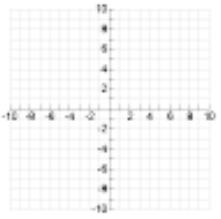
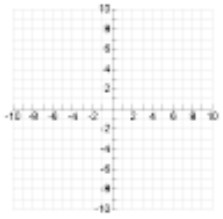
Level Three

- Predict the range of the function $y = 3x^2 + 5$.

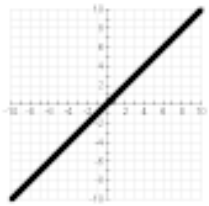
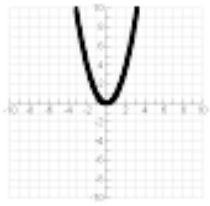
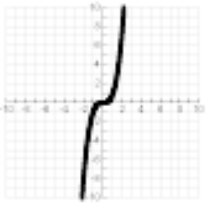
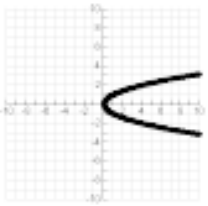
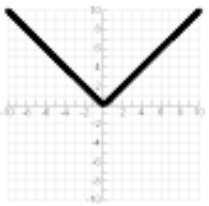
Formative Assessment

- Review the student handout formally and informally to assess student mastery.

Characteristics of Functions

Function	Linear	Quadratic	Cubic	Square Root	Absolute Value
Parent					
Graph					
Domain					
Range					
Symmetric					
Zeroes					

Characteristics of Functions Answer Key

Function	Linear	Quadratic	Cubic	Square Root	Absolute Value
Parent	$y = x$	$y = x^2$	$y = x^3$	$y = \sqrt{x}$	$y = \lvert x \rvert$
Graph					
Domain	All real numbers	All real numbers	All real numbers	$x \geq 0$	All real numbers
Range	All real numbers	$y \geq 0$	all real numbers	$y \geq 0$	$y \geq 0$
Symmetric	$y = x$	y axis	Origin	None	y axis
Zeroes	(0 , 0)	(0 , 0)	(0 , 0)	(0 , 0)	(0 , 0)

3.3: A "Dicey" Game of Numbers

Topic

- Multiple representations of functions, numbers, and vocabulary

Objectives

Students will:

- Develop skills using four representations of functions, numbers, processes, etc.
- Improve technical vocabulary

Timeline

- One 50-minute class period to create and play "A "Dicey" Game of Numbers"

WICR Strategies

- Writing to Learn
  - — Write a verbal representation of a function, number, etc.
  - — Write equivalent examples using various processes
  - — Write multiple equivalent expressions
  - — Write a description of a game
- Inquiry
  - — Investigate alternative uses for a tetrahedron
- Collaboration
  - — Work in small groups to construct and play a game
- Reading to Learn
  - — Read and follow directions for constructing a tetrahedron
  - — Read and follow peer-written game directions

NCTM Standards

Focal Point Grade 7

Number and Operations and Algebra: Developing an understanding of operations on all rational numbers and solving linear equations

Focal Point Grade 8

Algebra: Analyzing and representing linear functions and solving linear equations and systems of linear equations

Number and Operations: Understanding numbers, ways of representing numbers, relationships among numbers, and number systems

Representation

Instructional programs from pre-kindergarten through grade 12 should enable all students to create and use representations to organize, record, and communicate mathematical ideas.

Rationale

Developing facility with multiple representations of mathematical concepts is a keystone to delving deeply into the language of mathematics. Games and manipulatives can greatly aid in the learning process. Vocabulary and language usage can only be developed through regular practice. This activity provides a framework for practicing various representations of functions, numbers, processes, etc. In addition, students are given an opportunity to practice technical vocabulary and hone their math communication skills.

Vertical Alignment

- This activity can be done at any level where multiple representations are introduced. This activity is designed to enable students to practice and review multiple representations of functions, numbers, processes, and vocabulary.

Materials/Preparation

- *Student Handout 3.3a:* "A Dicey Game of Numbers"
- Check-size envelopes (Business letter size envelopes may be used; however, the tetrahedron created will be obtuse and irregular.)
- Color markers
- Straight edges (Rulers)
- Scissors

Instructions

- Form groups of two, three, or four.
- Distribute an envelope, straight edge, scissors, and color markers to each group.
- Instruct groups to read and follow the directions in the student handout.

Higher-Level Questions

Level Two

- Are the four representations on the tetrahedron mutually exclusive?
- Is the game that you created a fair game?
- What are the characteristics of a fair game?

Level Three

- What are some other uses for the tetrahedron that you have constructed?
  - *For example:* Write alternative process choices on each face. One possibility is that each face could contain an example of factoring process, and a game could be constructed requiring the teams to throw

the tetrahedron and use the hidden method while factoring a given problem. The team that solves the problem fastest using their method would win a given number of points and a Socratic discourse would naturally follow about the best methodology to use in various situations.

Formative Assessment

- Were the groups able to follow the directions unaided and construct a tetrahedron?
- Were the groups' representations accurate?
- Were the written directions for the games clear and concise?
- What was the level of participation during the class discourse using higher level questions?

A "Dicey" Game of Numbers

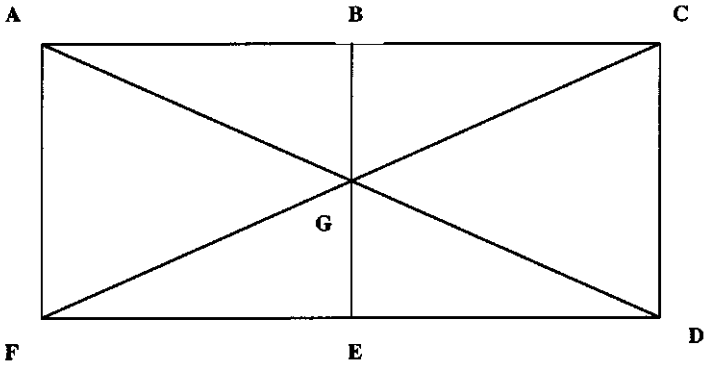


Figure 1

1. Seal the envelope.
2. Use the straight edge to construct lines as shown in Figure 1.
3. Make *crisp folds* along all the constructed lines. (Fold back and forth, and crease sharply.)
4. Use scissors to cut out Δ AGC and set it aside.
5. Open up the modified envelope, by pulling apart the sides until a base in the shape of a rhombus with edges FG and GD is created.
6. Insert segment CD inside the pocket formed along segment AF.
7. The construction should result in a tetrahedron similar to that shown in *Figure 2* below.

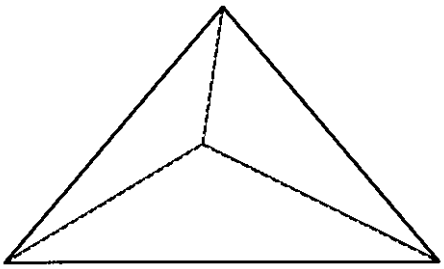


Figure 2

1. Use the color markers to write one of the four representations of a function, a number, a process, or a vocabulary word on each of the four faces of the tetrahedron. For example, you could place one of the following on each of the four faces of the tetrahedron: (1) a graph, (2) a table of values, (3) a symbol, (4) a written description.
2. Create a game involving rolling the tetrahedron like a die, possibly divining the hidden face of the tetrahedron.
3. Write a description of the game that you created.
4. Exchange your game description and tetrahedron with a neighboring group.
5. Read the game description your peers created.
6. If necessary, meet with your neighboring group to ask clarifying questions.
7. Play the game.

3.4: Algebra Aerobics

Topic

- Transformations for linear, quadratic, absolute value, square root, cubic, logarithmic, exponential and rational parent functions

Objective

- Students will learn parent functions and their transformations through a hands-on activity

Timeline

- 10–15 minutes

WICR Strategies

- Collaboration
  - — Work in teams to create physical representations of parent functions and their transformations

NCTM Standards

Focal Point Grade 7

Number and Operations and Algebra: Developing an understanding of operations on all rational numbers and solving linear equations

Focal Point Grade 8

Algebra: Analyzing and representing linear functions and solving linear equations and systems of linear equations

Algebra

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- understand patterns, relations and functions; and
- analyze change in various contexts.

Geometry

Instructional programs from pre-kindergarten through grade 12 should enable all students to apply transformations and use symmetry to analyze mathematical situations

Communication

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers and others;
- analyze and evaluate the mathematical thinking and strategies of others; and
- use the language of mathematics to express mathematical ideas precisely.

Rationale

"Algebra Aerobics" provides an alternate, kinesthetic assessment and practice for students learning parent functions and their transformations. Visual monitoring will provide clear evidence of students' mastery of graphs of functions.

Vertical Alignment

- This activity can be done at any level where graphs are introduced. Students should first be introduced to a parent function and be well versed in transformations. This activity is meant to be an assessment and review of graphing parent functions and their transformations.

Materials/Preparation

- A list of several parent equations and the equations of their transformations
- *Teacher Reference Sheet 3.4a:* "Algebra Aerobics"

Instructions

- Write the equations on an overhead/visual presenter and reveal them one at a time.
- After introducing a parent function from those listed, and discussing the transformations of that parent function, ask students to demonstrate the functions and their transformations physically.
- Begin with vertical and horizontal transformations then introduce compressions and stretches.
- A version of the game "Simon Says" can be used for variety.
- Ask two or more students to work together to represent the same function. This will require that they discuss which quadrant they will represent and what the behavior of the function is in that quadrant.

Higher-Level Questions

Level Two

- Compare the graph of $y = x^2$ to the graph of $y = 2x^2$.

Level Three

- Predict the equation of a function that has been translated and/or reflected.

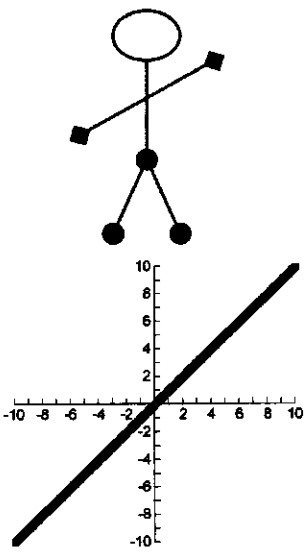
Formative Assessment

- While students are performing these transformations, you will be able to detect any material on which students may need extra practice.
- Monitor student conversation while working in groups to represent rational functions or other functions requiring two or more students.

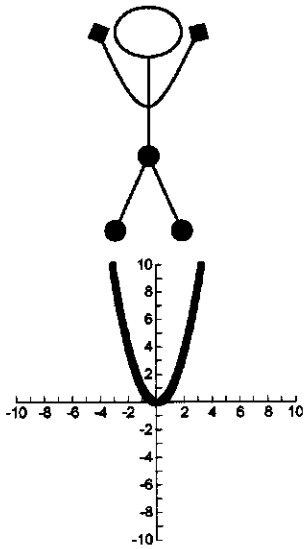
Algebra Aerobics

What You Should See

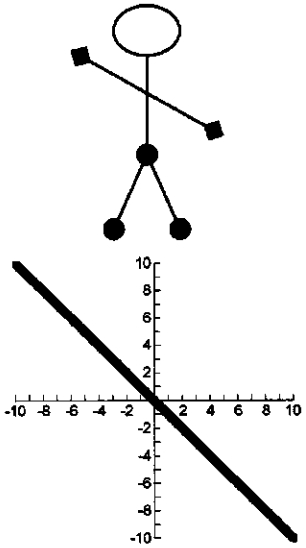
y = x



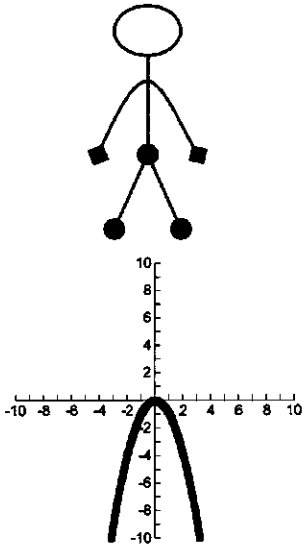
y = x^2



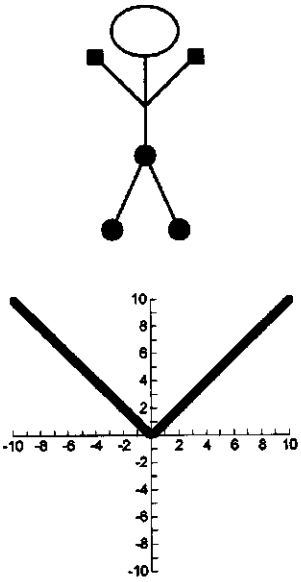
y = -x



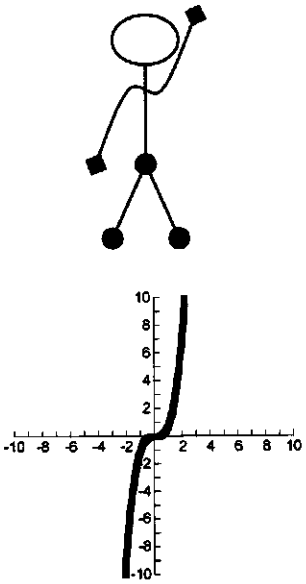
y = $-x^2$



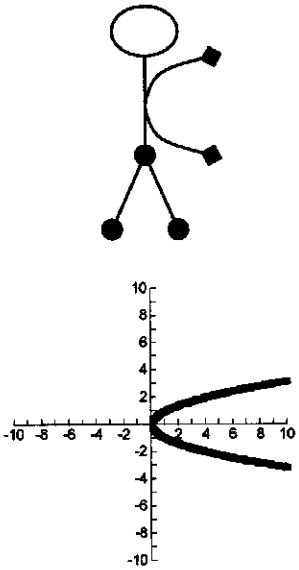
y = | x |



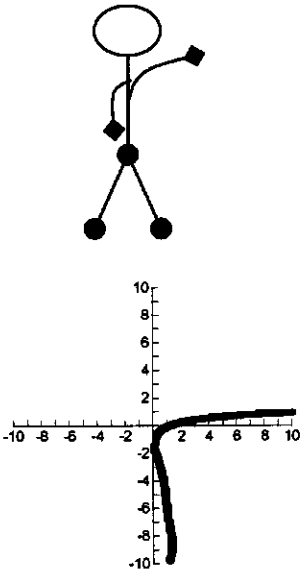
y = x^3



$y = \pm\sqrt{x}$



y = log x



Linear Functions

$y = x$	$y = x$	$y = x$	$y = x$	$y = x$
$y = -x$	$y = x + 2$	$y = 5x$	$y = \frac{1}{2} x$	$y = -5x - 2$
	$y = -x + 2$	$y = -5x$	$y = -\frac{1}{2} x$	$y = 5x + 2$

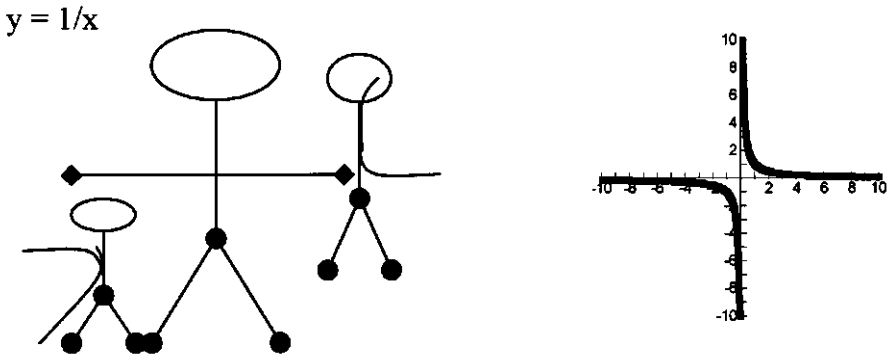
Quadratic Functions

$y = x^2$	$y = x^2$	$y = x^2$	$y = x^2$
$y = -x^2$	$y = x^2 + 2$	$y = 5x^2$	$y = 3x^2 + 2$
	$y = -x^2 + 2$	$y = \frac{1}{2} x^2$	$y = -3x^2 - 2$
		$y = -5x^2$	
		$y = -\frac{1}{2} x^2$	

Advanced Algebra Aerobics

Algebra Aerobics can also be used for rational functions, but it will require students to be in groups of up to four.

What you will see with three students



One student acts as the *x* and *y* axes, while two other students each represent part of the function.

Other functions to try:

$y = -1/x$

$y = 1/x^2$

$y = -1/x^2$

$y = x^3 + 2$

$y = \dfrac{x+4}{x^2 + 9x + 20}$

3.5: Four-Color Activity for Statistics: Describing Distributions

Topic

- Exploring the relationships between mean, medium, mode, range, and different representations of data

Objectives

Students will:

- Understand different measures of central tendency
- Explore the correspondence between data sets and their graphical representations

Timeline

- One 50-minute class period to complete the "Four-Color Activity for Statistics" and write a reflection

WICR Strategies

- Collaboration
  - — Work in teams to investigate measures of central tendency
- Writing to Learn
  - — Complete a written reflection as part of a formative assessment
  - — Complete an alternative data set based on a histogram, box-and-whisker plot or stem-and-leaf plot

NCTM Standards

Focal Point Grade 8

Data Analysis and Number and Operations and Algebra: Analyzing and summarizing data sets

Data Analysis and Probability

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them;
- select and use appropriate statistical methods to analyze data;
- develop and evaluate inferences and predictions that are based on data; and
- understand and apply basic concepts of probability.

Communication

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- analyze and evaluate the mathematical thinking and strategies of others; and
- use the language of mathematics to express mathematical ideas precisely.

Representation

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- create and use representations to organize, record, and communicate mathematical ideas;
- select, apply, and translate among mathematical representations to solve problems; and
- use representations to model and interpret physical, social, and mathematical phenomena.

Rationale

Students' academic and personal lives are filled with a need to summarize and evaluate data. The *"Four Colors"* lesson will give students an opportunity to work collaboratively to investigate and evaluate multiple representations of data.

Vertical Alignment

- Alternative methods for representing and evaluating data are introduced in middle level and expanded upon at every level of students' secondary and post-secondary experience. The "Four Colors" activity can be modified for any concept that includes multiple representations. Examples are equivalent representations of numbers and functions.

Materials/Preparation

- *Teacher Reference Sheet 3.5a:* "Activity Master—Data Cards"
- *Student Handout 3.5b:* "Four-Color Activity for Statistics"
- *Teacher Reference Sheet 3.5c:* "Four-Color Activity for Statistics Answer Key"
- Copy the four pages of *Teacher Reference Sheet 3.5a:* "Activity Master" onto colored paper. Each page should be a different color. Laminate the pages for reuse if possible. Make enough copies so that each student group will have a complete set of cards from each page. Cut out the individual cards.
- Place a complete set of cards (all four colors) in an envelope or plastic bag for each student group.
- Make one copy of *Student Handout 3.5b:* "Four-Color Activity for Statistics" for each group.

Instructions

- Distribute the *Student Handout 3.5b:* "Four-Color Activity for Statistics" and a set of four-color cards to each group.
- Provide time for groups to discuss and match the cards.
- Debrief informally and with a written reflection.

Higher-Level Questions

Level Two

- How are histograms alike and different from stem-and-leaf plots or box-and-whisker plots?
- When is a histogram a better representation of data than a stem-and-leaf plot or a box-and-whisker plot?

Level Three

- What could be a possible data set for D7, if there are 9 items total in the set?
- Which gives a more accurate representation of a specific data set, a histogram or a box-and-whiskers plot?

Formative Assessment

- Ask students to complete a written reflection of their observations and record what they learned.
- Ask students or student groups to create alternative representations from a selected data card.

Activity Master—Data Cards

D2

6, 6, 6, 7, 7, 8, 8, 8

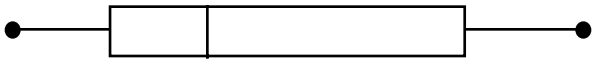
D4

1, 2, 3, 8, 9, 10, 11, 11

D6

2	0 1 1 3 4 6
3	1 2

D8



1.2 3.9 11.5 17.5 26.4

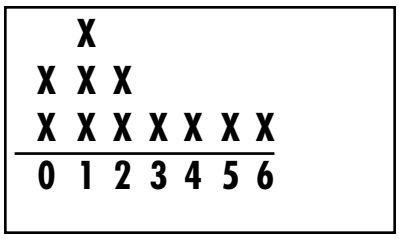
D1

0 (L)	1 1 3 4 4
0 (H)	5 6 8
1 (L)	0 2

D10

22, 23, 23, 24, 25, 28, 30

D3



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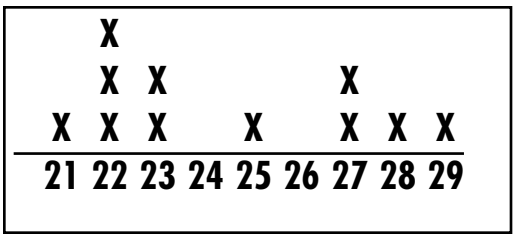

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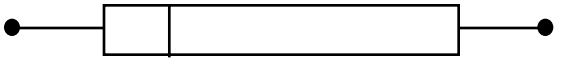


3 11 17 19 27

D9

7, 7, 7, 7, 7, 7, 7

Activity Master—Mean Cards

N1 7	**N2** 6.875
N3 24.75	**N4** Cannot be determined from data given
N5 5.4	**N6** 25
N7 2.27	**N8** 24.455
N9 Cannot be determined from data given	**N10** 7

Activity Master—Mode Cards

M3 6 and 8	**M5** 11
M7 21	**M9** Cannot be determined from data given
M10 4 and 1	**M8** 23
M6 1	**M4** 22
M2 Cannot be determined from data given	**M1** 7

Activity Master—Range Cards

R4 2	**R3** 10
R2 12	**R1** 25.2
R6 11	**R7** 8
R8 6	**R9** 8
R10 24	**R5** 0

Four-Color Activity for Statistics

Describing Distributions

Data	Mean	Mode	Range
D1	_____	_____	_____
D2	_____	_____	_____
D3	_____	_____	_____
D4	_____	_____	_____
D5	_____	_____	_____
D6	_____	_____	_____
D7	_____	_____	_____
D8	_____	_____	_____
D9	_____	_____	_____
D10	_____	_____	_____

Four-Color Activity for Statistics Answer Key

(Describing Distributions)

Data	Mean	Mode	Range
D1	N5	M10	R6
D2	N1/N10	M3	R4
D3	N7	M6	R8
D4	N2	M5	R3
D5	N8	M4	R7 or R9
D6	N3	M7	R2
D7	N4/N9	M2 or M9	R10
D8	N4/N9	M2 or M9	R1
D9	N1/N10	M1	R5
D10	N6	M8	R7 or R9

Unit Four: Reading in Mathematics

Introduction to Reading in Mathematics

Reading to Learn in Mathematics

The mathematics classroom should incorporate strategies that can help students become more effective readers of mathematics. In the same way that a student needs to be taught the special skills needed to read poetry, fiction and non-fiction, the skills needed to comprehend a math textbook must also be explicitly taught. The math content teacher is often in a better position to teach the skills needed for reading a math textbook than any other teacher in the school. Math teachers have, by the nature of their work, developed effective skills for decoding the mathematics text. The skills that they have identified and honed over the years can and should be taught to students. Students need to be taught how to "read like a mathematician." It has become far too common for teachers to translate the text into graphs, tables, algorithms, or verbal explanations rather than giving students the tools and the opportunity to practice using those tools to decipher their texts. Rather than making the students dependent on the teacher to construct meaning from the textbook, the teacher can equip the students to be self-reliant and in the process provide them with powerful reading comprehension tools which they can use in furthering their understanding of mathematics.

The math text cannot be simply read. It must be "worked through." To do this, students need to understand critical structural characteristics of the math text. Math textbooks are written in a very terse or compact style. Every word counts. If a concept is missed, there is little chance of picking it up later. An "elegant" explanation, derivation, or proof in mathematics is the one that uses the fewest words and uses the words in the most precise way, making vocabulary acquisition critical to comprehension. Identifying new vocabulary and utilizing specific strategies such as concept maps, word walls, graphic organizers, semantic feature analysis, picture vocabulary cards, etc., will provide students with the tools they need for constructing meaning on their own.

Since each section in a mathematics text makes the assumption of having mastered the previous sections, there is no chance of just "catching the drift." There is very little redundancy, each word, symbol, or sentence has to be decoded prior to moving on to the next. While a student may read thirty to sixty pages of a novel in thirty minutes, in the same time the student may dwell on two to three lines in the math text. When reading mathematics, "reading slow *is* fast."

While every sentence in a math text is logically linked to a previous section and those sections that follow, it is not a linear reading experience. Math textbooks must be read in all directions, top to bottom, bottom to top, right to left, left to right, front to back, and back to front. There is usually something very wrong if an explanation, problem, or example is read only once. Each sentence and section must be thoroughly understood before moving ahead. The process of making sense of the text may include many iterations of scanning, rereading, cross referencing, attempting solutions, pausing and revisiting explanations, examples, illustrations, and glossaries.

Reading instruction must be scaffolded so that students develop strategies that help them become more confident with comprehension skills. Three factors are most helpful for ensuring successful comprehension: connecting to prior knowledge, understanding text structure, and using text-processing strategies.

Prior Knowledge

All readers bring what they already know to the piece they are reading. Readers compare information with their own experiences to assist in comprehension. When they encounter something new, they can make inferences based on their prior knowledge. Readers who have a greater range of prior knowledge will find comprehension easier than readers whose knowledge is more limited. For struggling readers, it is essential that the teacher provide some prior knowledge with new topics. Good teaching would suggest that prior to reading a teacher should ask questions that evoke anything a student might already know about the concept. Also, brainstorming as a whole class or in groups helps pool the information that students possess.

Text Structure

Understanding the pattern or structure of the math text can greatly improve the students' ability to construct meaning from the text. Most texts follow the prototypical pattern of statement, example(s), explanation, and practice. In addition, most include additional information in the margin or offset in colorfully illustrated boxes meant to engage student interest, activate background knowledge, and help make essential connections. Without direct explicit instruction and guidance, students often overlook and/or ignore these critical structural cues and vital reading aids.

Text Processing Strategies

Students who use strategies to help them make sense of their reading while in the act and to synthesize their understanding at the end of their reading will have greater comprehension of and greater satisfaction with a text. Likewise, understanding how these strategies work and becoming aware of their own mental processes while reading (metacognition) can help students make informed and purposeful choices about how they read. They become aware of how reading a novel or a history text is different from reading a chapter in a math textbook. They recognize that what they do during reading and what they do to make sense of their understanding after they've read differs based on their purposes and the kind of text it is. Students also become aware that they need different processing strategies based on the difficulty or density of the text.

BEFORE READING	DURING READING	AFTER READING
• Think about prior knowledge related to the subject • Know the purpose for reading • Preview the text: look at the title, pictures, graphics	• Focus full attention on the material • Think aloud • Predict • Ask questions • Take notes/Draw Diagrams	• Create visuals to clarify meaning (tables, Venn diagrams, graphs, charts, etc.) • Summarize • Evaluate • Apply and practice what has been read

4.1: General Reading Strategies

Topic

- Reading textbook material

Objectives

Students will:

- Develop reading comprehension strategies
- Utilize different reading comprehension strategies for casual reading and technical reading

Timeline

- 10–20 minutes to practice one or more reading strategies

WICR Strategies

- Reading to Learn
  - — Listen to and practice reading comprehension strategies

NCTM Standards

Communication

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- analyze and evaluate the mathematical thinking and strategies of others; and
- use the language of mathematics to express mathematical ideas precisely.

Rationale

Reading a math text requires unique and specialized skills. Helping students recognize and practice those skills will enable them to better utilize their textbooks as an important tool for learning. The mathematics classroom should incorporate strategies that can help students become more effective readers of mathematics. The skills needed to comprehend a math textbook must also be explicitly taught. The math content teacher naturally utilizes a variety of successful comprehension and decoding strategies and is in a unique position to make these strategies transparent.

Vertical Alignment

- General reading strategies can be adapted to any level. At the higher grades the degree of complexity will increase.

Materials/Preparation

- Math text
- *Student Handout 4.1a:* "Solving Quadratic Equations by Factoring"

Instructions

Select one of the following General Reading Strategies or create one of your own

- Turn the titles, headings, and subheadings into questions
- Create new titles, headings, and subheadings for each section
- Develop questions from the illustrations
- Prepare multiple representations of data presented in the section
- Provide new examples that fit the pattern
- Make connections to another section or prior knowledge
- Create a visual representation of the information in the section
- Create a concept map
- Illustrate a word for the word wall
- Evaluate a new vocabulary word based on its semantic features
- Develop a graphic organizer
- Develop "Picture Vocabulary/Concept Cards"
- Write questions in the margin or on sticky notes
- Take Cornell Notes while you read
- Employ a partner reading strategy like "Reciprocal Reading"
- Develop a visual or physical model
- Draw and label a diagram
- Make "real world" connections

Provide students with frequent opportunities to practice general reading strategies

- Use *Student Handout 4.1a:* "Solving Quadratic Equations by Factoring" or another text to practice one or more of the strategies.

Higher-Level Questions

Level Two

- Compare the strategies utilized for comprehension in mathematics and in other content areas?

Level Three

- How can the reading skills developed in mathematics be utilized in other reading content?
- What are some creative ways to demonstrate that the general reading strategies have been utilized?

Formative Assessment

- Monitor students as they demonstrate the use of general reading strategies in small groups.
- Assess the level of discourse during the class discussion.
- Ask students to provide suggestions for constructing meaning when they are "stuck."
- Design and review self-assessment rubrics.

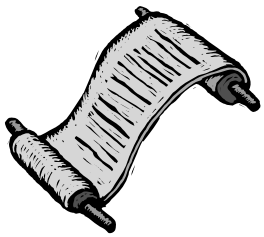
Solving Quadratic Equations by Factoring

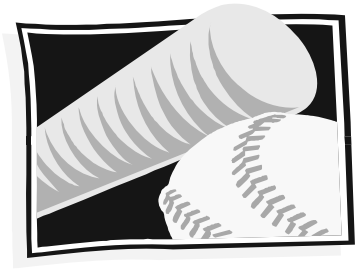
Goal 1:

Solve quadratic equations by factoring.

Goal 2:

Use the zero product property to find the roots of a function.





Text Sample: History

Between 1800 BCE and 1600 BCE the ancient Babylonians left evidence of the discovery of quadratic equations. They even gave methods for solving them.

In India the mathematician Baudhayana first used quadratic equations in the form $ax^2 = c$ and $ax^2 + bx = c$ and gave methods for solving them in the 8th century BC.

In the 9th century Shridhara an early mathematician gave a general rule for solving quadratic equations:

> *Multiply both sides of the equation by a known quantity equal to four times the coefficient of the square of the unknown; add to both sides a known quantity equal to the square of the coefficient of the unknown; then take the square root.*

In 1963 Mickey Mantle, the great New York Yankee, hit a home run that some say was the longest ball ever hit in the major leagues. The problem with the claim is that the ball hit the stadium roof 352 feet from home plate. Since that day, baseball fans have argued about the distance the ball would have traveled. Today the home team provides a chart that projects the expected path of the ball. That chart was not available in 1963 and the debate continues with some saying that Mantle's ball would have traveled 620 feet.

Conservatively, the path of Mantle's baseball has the equation*:

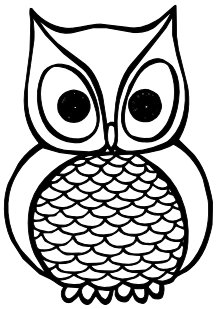
$-0.00213x^2 + 1.08x = 0$

$-0.00213x\ (x - 507) = 0$

The positive solution of the equation is 507 feet.**

* The early estimations were made with the assumption that Mantle's ball hit the stadium roof near the peek of its travel. This is currently thought to be unlikely.

** Based on Hit Tracker analysis, <http://www.hittrackeronline.com/'historic.php?id=1963_3> retrieved January 1, 2007.



The problem-solving strategy of looking for a pattern is one of the most-used.

Investigation: Follow the pattern in the first two examples to find the missing term in the rest.

	-20			18			-21			x	
5		-4	6		3	3		-7	4		-3
	1			9			x			1	
	-18			-6			–				
9		-2	2		x	a		c			
	x			-1			–				

Write a rule for finding the missing terms.

Zero Product Property: For any real numbers a and b, if ab=0 then either a=0, b=0, or both a and b are equal to zero.

Investigation Answers: (-4, -12, 7, -3, ac, a+c)

Example 1

$(x-4)(x+5)=0$

$(x-4)=0$ ***Zero Product Property***

or

$(x+5)=0$

$x=4$ or $x=-5$

Example 2

When an object like a baseball is hit straight up into the air or dropped, the height of the object is given by the function $h(t) = v_0t - 16t^2$, where $h(t)$ represents the height of the object, v_0 represents the initial velocity, and t represents the time that the object has traveled. If a pop-up fly is hit straight up with an initial velocity of 64 feet per second, approximately how much time does the catcher have to get in place under it?

Since the height when the ball returns to its original position is zero, then by substitution you get:

$0 = 64t - 16t^2$

$0 = 16t(4-t)$

$\therefore 16t = 0$ or $(4-t) = 0$ ***Zero Product Property***

The catcher has approximately 4 seconds to get under the ball.

In the previous lesson, you learned how to write $(x + 2)(x + 5)$ as $x^2 + 7x + 10$.
The expressions $(x + 2)(x + 5)$ are ***binomials*** because they have two terms.
The expression $x^2 + 7x + 10$ is a ***trinomial*** because it has three terms.
You can use ***factoring*** to write a trinomial as a product of binomials.

To factor $x^2 + bx + c$, find the integers m and n such that:

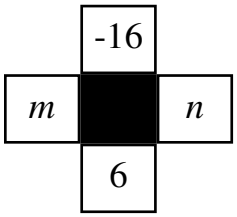
$x^2 + bx + c$
$= (x + m)(x + n)$
$= x^2 + (m + n)x + mn$

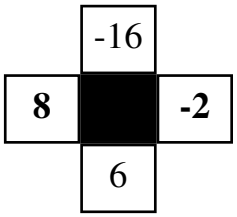
The ***sum*** of m and n must equal b and the ***product*** of m and n must equal c.

Example 3

Solve $3x^2 + 18x - 48 = 0$ by factoring.

$3(x^2 + 6x - 16) = 0$
$x^2 + 6x - 16 = 0$





Since $-2(8) = -16$ and $-2 + 8 = 6$
$m = 8$ *and* $n = -2$

$(x + 8)(x-2) = 0$
$(x + 8) = 0$ or $(x-2) = 0$ ***Zero Product Property***
$\therefore x = -8$ or $x = 2$

Example 4

The product of two consecutive negative integers is 240. What are the numbers?

Let the first number be n.

Since consecutive integers are one unit apart, the second number can be represented by $n + 1$

$n(n + 1) = 240$
$n^2 + n = 240$
$n^2 + n - 240 = 0$
$(n-15)(n+16) = 0$

The solutions are $n = 15$ and $n = -16$.
We are looking for the consecutive negative integers. Since $-16 + 1 = -15$, the solutions are -15 and -16. We can check our solutions by confirming that the product is equal to 240. $(-15)(-16) = 240$

Exercises

Solve each equation.

1. $(x - 3)(x + 4) = 0$
2. $(a + 6)(a - 8) = 0$
3. $y(y - 5) = 0$
4. $x^2 - 3x = 0$

Solve each equation by factoring.

1. $x^2 + 5x + 4 = 0$
2. $x^2 + 9x + 14 = 0$
3. $4y^2 + 12y - 40 = 0$
4. $x^2 + 6x - 27 = 0$
5. $a^2 - 3a - 10 = 0$
6. $b^2 - 8b + 12 = 0$

Write and solve an equation for the following.

Your friends are planning a new sunning area around their swimming pool. They would like to have a uniform width around the pool. They are limited by the building code to a total surface area for the pool and sunning area of 3036 square feet. The pool is 30 feet by 50 feet. What is the maximum width for the sunning area?

4.2: Text Structure

Topic

- Reading textbook material

Objectives

Students will:

- Develop reading comprehension strategies
- Utilize different reading comprehension strategies for casual reading and technical reading

Timeline

- 10–15 minutes to explore text structure as organized meaning

WICR Strategies

- Reading to Learn
  - — Practice reading comprehension strategies

NCTM Standards

Communication

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- analyze and evaluate the mathematical thinking and strategies of others; and
- use the language of mathematics to express mathematical ideas precisely.

Rationale

Understanding the pattern or structure of the math text can greatly improve the students' ability to construct meaning from the text. Most texts follow the prototypical pattern of statement, example(s), explanation, and practice. In addition, most include icons and additional information in the margin or offset in colorfully illustrated boxes meant to engage the reader's interest, activate background knowledge, and help make essential connections. Without direct explicit instruction and guidance, students often overlook and/or ignore these critical structural cues and vital reading aids.

Vertical Alignment

- The "Text Structure" activity can be adapted to any level. At the higher grades the degree of complexity will increase.

Materials/Preparation

- Math text
- *Student Handout 4.2a:* "Text Structure"
- *Student Handout 4.1a:* "Solving Quadratic Equations by Factoring" or another sample text

Instructions

- Identify a text selection or topic for students to consider during the activity.
- Ask students to complete *Student Handout 4.2a:* "Text Structure" using *Student Handout 4.1a:* "Solving Quadratic Equations by Factoring" or another sample text to practice understanding the text structure strategy.
- Provide students with frequent practice utilizing their text.

Higher-Level Questions

Level Two

- In what ways does the "Text Structure" activity help students improve their reading comprehension?

Level Three

- How can the reading skills developed in mathematics be utilized in other reading content?
- In what ways is the text structure of a math text unique?
- Which elements of the math text's structure assists readers the most in constructing meaning from the text?

Formative Assessment

- Monitor students as they demonstrate utilizing understanding text structure in small groups.
- Assess the level of discourse during the class discussion.
- Review the completed *Student Handout 4.2a:* "Text Structure."

Text Structure

Understanding the pattern or structure of a math text can greatly improve your ability to construct meaning from the text. Most texts follow the prototypical pattern of statement, example(s), explanation, and practice. In addition, most include icons and additional information in the margin or offset in colorfully illustrated boxes meant to engage your interest, activate your background knowledge, and help you make essential connections.

Practice

Use the *assigned text selection* to identify:

The outcomes of the section:

The pattern of the text:

The meaning of the icons:

Other text structure cues:

4.3: KWL

Topic

- Reading textbook material and activating background knowledge

Objectives

Students will:

- Develop reading comprehension strategies
- Utilize different reading comprehension strategies for casual reading and technical reading

Timeline

- 10–15 minutes to complete the KWL worksheet

WICR Strategies

- Reading to Learn
  - — Listen to and practice reading comprehension strategies

NCTM Standards

Communication

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- analyze and evaluate the mathematical thinking and strategies of others; and
- use the language of mathematics to express mathematical ideas precisely.

Rationale

The "*KWL*" worksheet provides one of the common formats for "activating background knowledge." Activating background knowledge is critical in providing a framework for reading comprehension.

Vertical Alignment

- The "KWL" strategy can be adapted to any level. At the higher grades the degree of complexity will increase.

Materials/Preparation

- Math text
- *Student Handout 4.3a:* "KWL Worksheet"
- *Student Handout 4.1a:* "Solving Quadratic Equations by Factoring" or another sample text

Instructions

- Draw a KWL chart on chart paper or distribute *Student Handout 4.3a:* "KWL Worksheet."
- Identify a text selection or topic for students to consider during the activity.
- Ask students to brainstorm and enter information in the columns indicating what they know and want/need to know.
- Provide students with frequent practice utilizing their text.
- Use *Student Handout 4.1a:* "Solving Quadratic Equations by Factoring" or another sample text to practice the KWL strategy.

Higher-Level Questions

Level Two

- In what ways does the KWL activity help in reading comprehension?

Level Three

- How can the reading skills developed in mathematics be utilized in other reading content?
- Why is it important to list what you "Want/Need to Know" at the beginning of a lesson or reading activity?

Formative Assessment

- Monitor students as they demonstrate utilizing general reading strategies in small groups.
- Assess the level of discourse during the class discussion.
- Review student-generated KWLs.
- Review the KWLs at the end of a lesson or reading activity to assess if the "Want/Need to Know" questions were addressed.

KWL Worksheet

K What Do I Know?	**W** What Do I Want/ Need to Know?	**L** What Did I Learn? How Am I Going to Use It?

4.4: Concept Definition Map

Topic

- Graphic organizer for vocabulary development

Objective

- Students will use a "Concept Definition Map" as a graphic organizer as they acquire the academic language of mathematics

Timeline

- 10–15 minutes to brainstorm ideas and write a definition for a target term or concept

WICR Strategies

- Collaboration
  - — Students will work with partners, table groups, and in classroom activities
- Reading to Learn
  - — Develop fluency in the academic language of mathematics

NCTM Standards

Communication

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- analyze and evaluate the mathematical thinking and strategies of others; and
- use the language of mathematics to express mathematical ideas precisely.

Rationale

Developing fluency in the academic language of mathematics is a critical foundation skill. The *"Concept Definition Map"* is a graphic organizer designed to give students an important tool for the acquisition of the academic language. It provides students with the opportunity to explore a new concept or vocabulary word and to visually record the results. By asking students a series of guiding questions, the teacher facilitates a process by which students create their own (mathematically correct) definitions for target vocabulary and important concepts. Students can then keep a reference glossary of terms they have defined with the *"Concept Definition Map"* for future reference.

Vertical Alignment

- "Concept Definition Maps" can be adapted to any level. At the higher grades the degree of complexity will increase.

Materials/Preparation

- *Student Handout/Overhead Transparency 4.4a:* "Concept Definition Map"
- *Teacher Reference Sheet/Overhead Transparency 4.4b:* "Concept Definition Map *Example"*
- Electronic versions of the "Concept Definition Map" can be made using the drawing tools and text box tools in Microsoft Word®, or with graphic organizer software tools such as Inspiration®.

Instructions

- Distribute the blank *Student Handout/Overhead Transparency 4.4a:* "Concept Definition Map."
- Ask students to write down the target vocabulary term or concept in the middle box with the darkest sides.
- Use "Popcorn" or other group-sharing activity to brainstorm ideas for the *"What is it like?"* boxes and the *"Examples"* ovals.
- Record the student responses on the overhead/visual presenter or board.
- Repeat the process for the *"What is it not like?"* boxes and the non-linguistic illustrations.
- Ask students to work collaboratively to write a definition of the target term or concept.
- Ask students to share their definitions with the class. Guide the discussion to ensure that a group consensus is reached and the definition is mathematically accurate.
- Ask students to write the agreed upon definition in the *"What is it?"* box.
- Ask students to write the definition in their Cornell Notes and add it to their glossary for future reference and review.

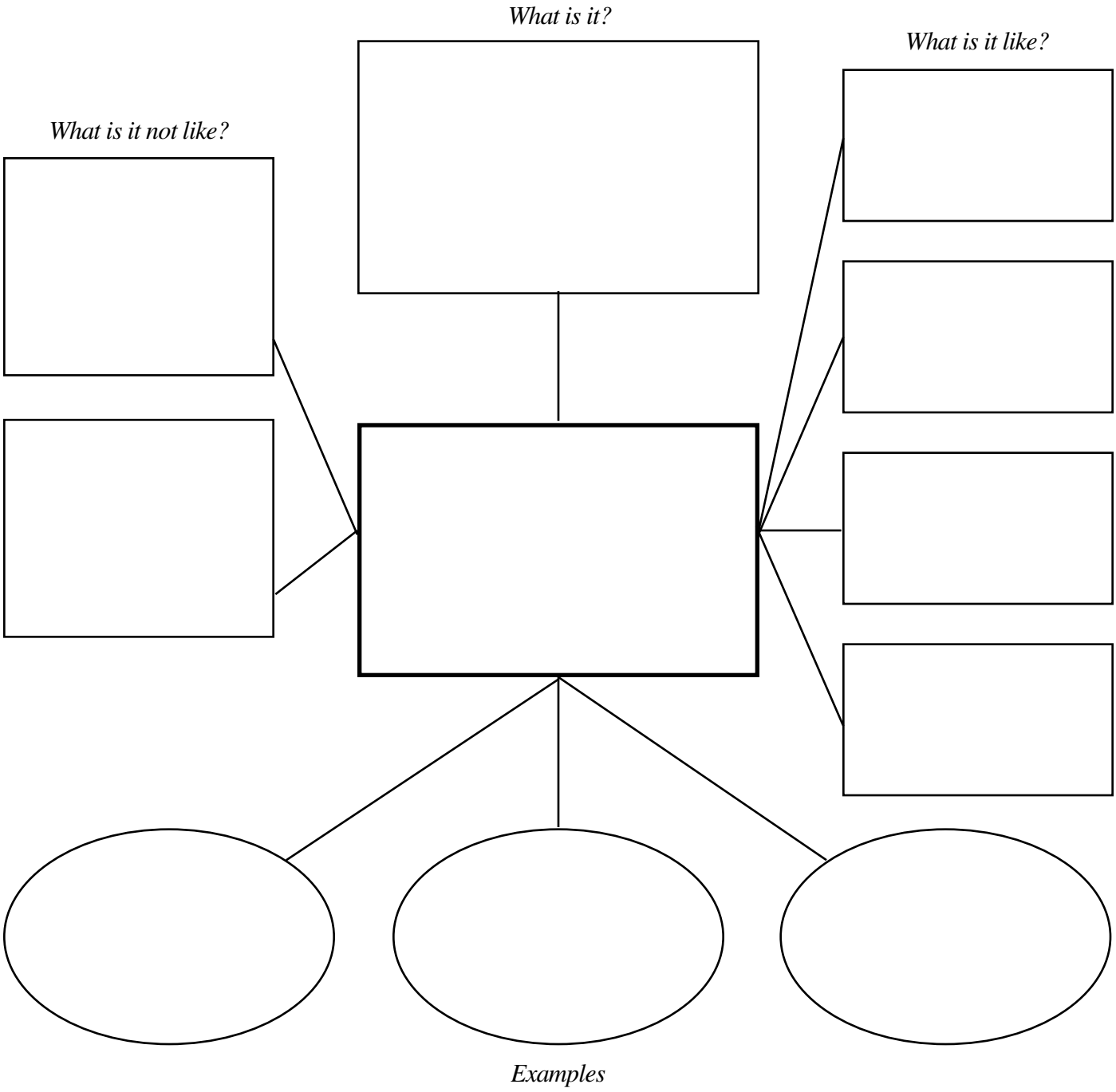
Higher-Level Questions

- How is the class consensus definition similar or different from the definition in the text?

Formative Assessment

- Ask students to compare and contrast their definition to the textbook definition for the target vocabulary term or concept.
- In their Cornell Notes' summary, ask students to write down any additional information they learned about the target vocabulary or concept during the lesson or activity.

Concept Definition Map



Concept Definition Map *Example*

What is it?

A quadrilateral is a polygon with four sides.

What is it like?

- Picture
- Piece of Paper
- White Board
- Photo

What is it not like?

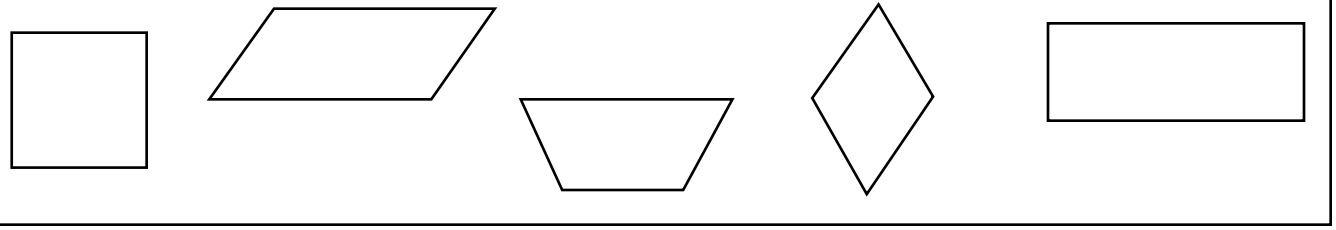
- Triangle
- Prism
- Pentagon

Quadrilateral

- Square
- Trapezoid
- Parallelogram

Examples

Illustration



4.5: PQ5R

(Preview, Question, Read, 'Rite, Review, Recite, Reflect)

Topic

- Reading and comprehending math topics

Objective

- Students will utilize a template to assist in reading comprehension

Timeline

- 10–30 minutes to complete *Student Handout 4.5a:* "PQ5R"

WICR Strategies

- Collaboration
  - — Work in collaborative groups to construct meaning from a text
- Reading to Learn
  - — Practice reading comprehension strategies

NCTM Standards

Communication

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- analyze and evaluate the mathematical thinking and strategies of others; and
- use the language of mathematics to express mathematical ideas precisely.

Rationale

One of the best-known methods for reading and retaining textbook information is *"PQ5R."* This systematic reading strategy developed by Francis P. Robinson at Ohio State University during World War II is adapted here for the mathematics' text. Providing opportunities for students to practice reading comprehension strategies in collaborative groups enables them to capitalize on each other's strengths. Furthermore, teaching concepts to a peer provides the best opportunity to master the concept.

Vertical Alignment

- The "PQ5R" reading strategy can be adapted to any level. At the higher grades the degree of complexity will increase.

Materials/Preparation

- Math text
- *Student Handout 4.5a:* "PQ5R"
- *Student Handout 4.1a:* "Solving Quadratic Equations by Factoring" or another sample text

Instructions

- Select a text or problem.
- Divide students into groups of three to five students.
- Distribute *Student Handout 4.5a:* "PQ5R."
- Encourage students to work collaboratively as they complete the student handout.
- Allow time for reading and preparation of the presentation.
- Ask students to share information they have recorded on their student handout.
- Encourage students to take notes on the material and to question each other about the portions of the text.
- Use the math text or *Student Handout 4.1a:* "Solving Quadratic Equations by Factoring" to practice PQ5R.

Higher-Level Questions

Level Two

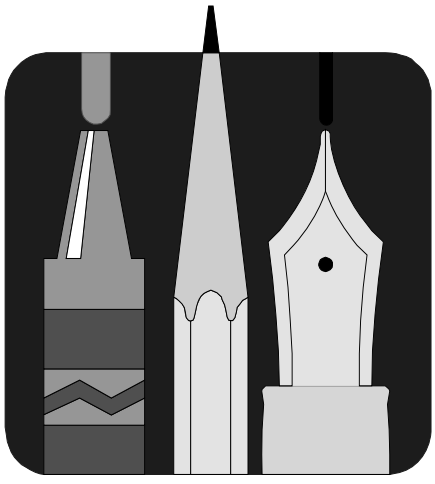
- Which general reading strategies were used to construct meaning?

Level Three

- Which of the PQ5R strategies were the most useful? Why?

Formative Assessment

- Assess the level of discourse during the class activity.
- Assess the fluency of student language when expressing math concepts.
- Review student handouts and notes.



PQ5R

(Preview, Question, Read, Record, Recite, Review, Reflect)

"Preview" the material. Skim or scan the text.

1. *Look at* the title, main headings and subheadings.
2. *Look at* the structure of the text. Note bold print, statements in boxes, etc.
3. *Look at* the pictures, graphs, charts.
4. *Read* the introduction including the background information if provided. Pay special attention to margin notes, examples and the chapter summary if provided.
5. *Identify* and look up any new terms, symbols or theorems.
6. *Review* any previously learned terms, symbols, theorems or equations that you might need.
7. *Look at* any review or summary questions at the end of text.

Practice

Record your thinking below:

_____

_____

_____

_____

"Question" the material. Develop questions about the text.

1. Turn headings or subheadings into questions and write them down.
2. Use the introduction, examples and information highlighted by a box, color or bold printing to help write questions.
3. Develop a purpose for your reading. It could be to identify places where the logic is not clear, where one or more steps are implied or an explanation of why a particular example is included.
4. Write a brief version of any extended questions that appear in the practice section.

Practice

Write some of your questions below:

_____

_____

_____

_____

"**R**ead" the material. Read to answer the questions developed.

1. Read with a pencil, paper, and calculator at hand.
2. Think about the pictures, graphs, and tables.
3. Know what all the vocabulary and symbols mean.
4. Read and reread slowly with concentration.

Practice

Read the material. Record your thinking below:

_____

_____

_____

_____

"**R**ecord" key ideas found during reading. Write:

1. Cornell Notes as you read.
2. Key ideas, vocabulary, symbols, and concepts found during reading.
   a. Construct vocabulary/concept cards.
   b. Construct a physical model or illustration of the concepts.
3. Answers to questions you posed.
4. New information that seems significant.
5. Reflections about the material.
6. Any new questions.

Practice

"Record" key ideas found during reading. Record your notes below or on a separate paper:

_____

_____

_____

_____

"**R**ecite" from the text. Try to talk about the information. Think about:

1. Differences or similarities between the examples.
2. Concepts, formulas, and rules that were used to solve the example problems.
3. Connections to previous work or problems.
4. How do these problems compare with others from the text?
5. Cover and rework the example problems. *Include all the steps!*

Practice

Recite from the text. Record your thinking below:

_____

_____

_____

_____

"Review" the examples and the practice problems.

1. Have any steps been combined?
2. Is the author using a particular method?
3. Work the example problems.
4. Identify steps that are unclear or implied.
5. Write an explanation for why each of the examples is included. Explain how they are the same or different.
6. If the author suggests looking at another section or deriving an equation—do it!
7. Solve the practice problems. (Choose the ones with answers in the back of the text so you can verify your work.)

Practice

Review the examples and the practice problems. Annotate the text and/or record your work below:

_____

_____

_____

_____

8. Follow the steps below for word problems.
   a. Read the problem for the "big picture."
   b. Find the question mark(s). What are you looking for?
   c. Reread the problem for detail.
      - Write down the details (use a graphic organizer or chart if possible.)
      - Draw and label a sketch or construct a table or graph.
      - Give unknown quantities a variable name.
      - Write the facts in the problem in symbols.
      - Write any relationships in symbols. *(Don't skip this step.)*
   d. Make an estimate of a reasonable answer. (Be sure to include the units or the form of the answer.)
   e. If you "get stuck," following are some helpful suggestions for working through the problem:
      - Have you used all the data? Have you used data that you should not have?
      - Can you draw a picture or make a table or chart?

- Can you solve part of the problem? Try working backward. What is needed for this part of the problem?
- Have you solved a similar problem?
- Is there an example problem that could help?
- Can you rearrange the data?
- Can you collaborate with a partner or study group?

9. Don't skip any problem-solving steps!
10. Check the answer!
    a. Is this answer even reasonable?
    b. Is the solution close to the estimate that you made?
    c. Is it in the proper form or in the proper units?
    d. Does the answer make the statement true?

Practice

Use the word problem in the text to apply the steps for solving word problems. Record your work below:

_____

_____

_____

_____

"Reflect," reread text and notes, and think about the significance of the material.

1. Look back over the chapter and your notes.
    a. Cover the right side of your Cornell Notes and quiz yourself.
    b. Ask a peer to quiz you on the material.
2. Recite how you solved each example problem.
3. Review the vocabulary, symbols, and formulas.
4. Answer the practice questions at the end of the section.
5. What previously learned strategies, methods, formulas, or skills are you applying in this chapter?
6. How can the information in this chapter be applied?
7. How is this information related to what is already known?
8. Would other sources have different versions of the material?

Practice

Reflect, reread text, and notes. Record your thinking below.

_____

_____

_____

_____

4.6: Jigsaw

Topic

- Reading and comprehending math topics

Objective

- Students will work collaboratively to construct meaning of a selected text

Timeline

- 10–20 minutes for students to read the text, take notes, present informally to their peers, and receive feedback

WICR Strategies

- Collaboration
  - — Work in collaborative groups to construct meaning from a text
- Reading to Learn
  - — Practice reading comprehension strategies

NCTM Standards

Communication

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- analyze and evaluate the mathematical thinking and strategies of others; and
- use the language of mathematics to express mathematical ideas precisely.

Rationale

Providing opportunities for students to practice reading comprehension strategies in collaborative groups enables them to capitalize on each other's strengths. Teaching concepts to a peer provides the best opportunity to master the concept.

Vertical Alignment

- "Jigsaw" activities can be adapted to any level. At the higher grades the degree of complexity will increase.

Materials/Preparation

- Math text

Instructions

- Select a text or problem and divide it into portions that are equal in length and content.
- Assign a portion of the text or problem to each student in the group. Have each student read the text, take notes, and decide on the most important points to teach others in the group who've not read the section.
- Allow time for reading and preparation of the presentation.
- Following the order of the passage, have students share information about their sections aloud. Encourage students to take notes on the material and to question each other about the portions of the text.

Higher-Level Questions

Level Two

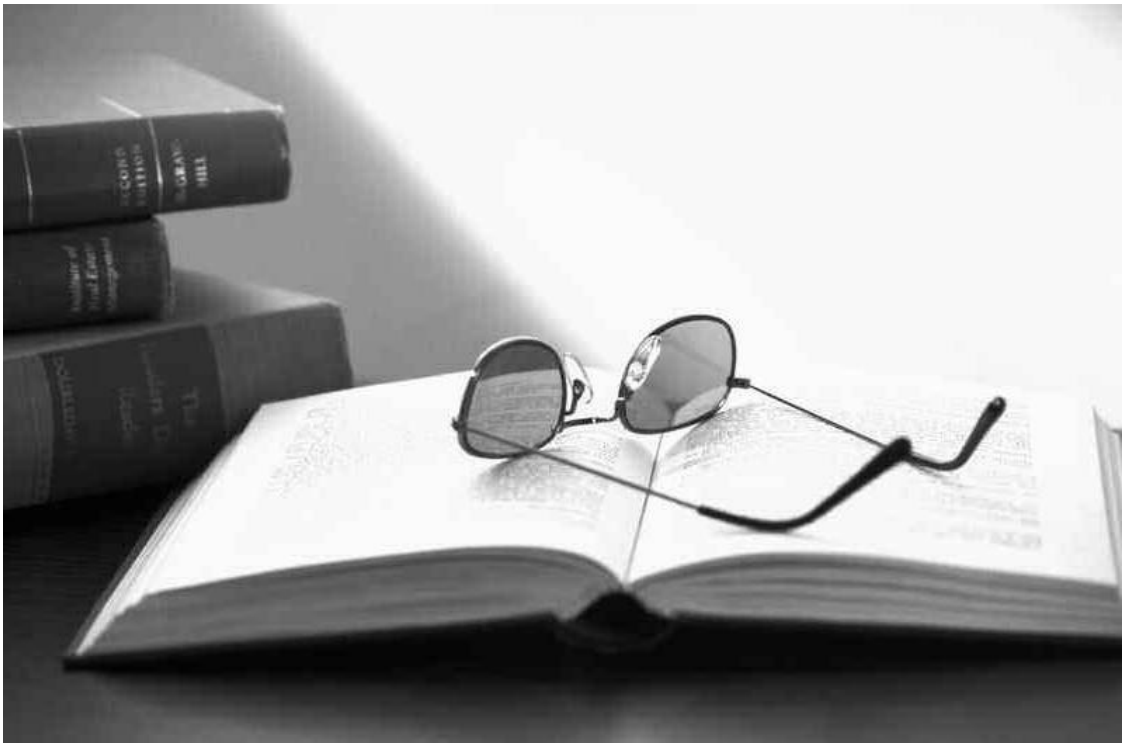
- Which general reading strategies were used to construct meaning?

Level Three

- What is the best way to teach the concepts to members of the group?

Formative Assessment

- Assess the level of discourse during the class activity.
- Assess the fluency of student language when expressing math concepts.
- Review student notes.



4.7: Sentence Frames

Topic

- Acquisition and use of academic language

Objectives

Students will:

- Create and practice using academic language orally
- Acquire academic language through aural practice

Timeline

- 5–15 minutes to complete sentence frames of increasing complexity and to check for correctness

WICR Strategies

- Collaboration
  - — Work in a small group and whole group activity
- Reading to Learn
  - — Practice reading and speaking the using the academic language of mathematics

NCTM Standards

Communication

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- analyze and evaluate the mathematical thinking and strategies of others; and
- use the language of mathematics to express mathematical ideas precisely.

Rationale

Fluency in the academic language of mathematics is required for accessing rigorous text and assessment items. Students rarely have oral practice with new concepts and vocabulary. *"Sentence Frames"* can be adapted for any content and are utilized for the following:

- Acquisition of the academic language of mathematics
- Oral language practice in context
- Development of logical connectors
- Integration of concepts
- Aural practice

Vertical Alignment

- "Sentence Frames" can be adapted to any level. At the higher grades the degree of complexity will increase.

Materials/Preparation

- *Teacher Reference Sheet 4.7a:* "Sentence Frames"
- Prepare sentence frames electronically or as a visual artifact in advance so that students see only one sentence at a time. Sentence frames should be developed as follows:
  - — Each frame starts easy and increases in complexity with each iteration.
  - — Include two or more sentences.
  - — Fill in the blanks with numbers to work on new vocabulary.
  - — Fill in the blanks with vocabulary to review concepts.
  - — Focus on the academic language of mathematics. (See *Teacher Reference Sheet 4.7a:* "Sentence Frames.")

Instructions

- Show students the first sentence frame and give them time to fill in the blanks individually.
- In pairs or larger table groups, ask students to read their sentence frame to check for correctness and fluency.
- Ask **each** student to read his or her sentence frame. Students may repeat responses if they do not have a unique response. The goal is to have each student say the sentence once and to hear the sentence 25 or more times.
- Repeat the process with each sentence frame in the series.
- Keep it moving, but be prepared to discuss any sentences that are mathematically incorrect or problems with academic language fluency.

Higher-Level Questions

- What higher-level questions could be written for each series of sentence frames?

Formative Assessment

- Ask a colleague to sit in on your class and record the ways in which students use the target academic language.

Sentence Frames

Example 1

1. "The square root of _____ is _____."
2. "The square root of _____ is _____, because _____ squared is _____."
3. "If the side of a square measures _____, the area of the square is _____. Therefore, the square root of _____ is _____."

Higher-Level Questions

Why do we use the term "squared" to describe multiplying a number by itself?

Where do you think the term "cubed" comes from?

Example 2

Please complete this sentence frame with a linear equation of the form $y=mx+b$.
Example: $y=2x+5$

1. "The slope of the linear equation ($\underline{y=mx+b}$) is _____."

Please complete this sentence frame with a linear equation of the form $ax+by=c$.
Example: $2x+y=5$

2. "The slope of the linear equation ($\underline{ax+by=c}$) is _____."

Higher-Level Questions

Why is the second sentence frame more difficult than the first?

What would happen in the second sentence frame if $a = 0$? If $b = 0$?

4.8: Think Aloud

Topic

- Reading textbook material

Objectives

Students will:

- Develop a variety of reading comprehension strategies
- Utilize different reading comprehension strategies for casual reading and technical reading

Timeline

- 15 minutes to practice reading skills and perform a self-assessment

WICR Strategies

- Reading to Learn
  - — Listen to and practice reading comprehension strategies

NCTM Standards

Communication

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- analyze and evaluate the mathematical thinking and strategies of others; and
- use the language of mathematics to express mathematical ideas precisely.

Rationale

Most students arrive in the mathematics class with few if any math-specific reading strategies and fewer strategies to apply when they get "stuck." They often rely on rereading and/or asking the teacher to decode the text for them. The *"Think Aloud"* activity will provide students with a wide variety of reading strategies that are specific to making sense of the mathematics text.

Vertical Alignment

- "Think Alouds" can be adapted to any level. At the higher grades the degree of complexity will increase.

Materials/Preparation

- Math text
- *Student Handout 4.8:* "While I Was Reading, What Did I Do?"

Take a moment to jot down the reading comprehension strategies that a good reader and an AVID reader of mathematics uses while reading a math text. Review the selected text for the "Think Aloud" activity and identify a few exemplars of the strategies that "good" readers of mathematics use. They may include:

- Read slowly (Forget speed reading)
- Reread often
- Decode unknown vocabulary and symbols
- Reread the section prior to and after the current section
- Use *Writing to Learn* strategies: write notes, draw diagrams, write definitions, rewrite the explanations in your own words...
- Expect gradual understanding
- Spend extra time "reading" diagrams, charts and graphs
- Workout derivations and proofs for yourself
- Read ahead
- Create a task for yourself; look for other examples that fit the pattern, look for logical connections, look for missing or implied steps in examples and insert them, look for reasons to do it "this" way...
- Get the big picture before progressing
- Use highlighting strategies
- Write questions and notes in the margin or on sticky notes
- Fill in the gaps with words, symbols or steps that are missing
- Make it your own—explain it to somebody else
- Learn by doing. Don't just push numbers at the example or homework. Use the practice problem(s) to understand the concepts and connections.
- Talk to a peer or teacher about the material
- Look closely at the labels and captions of illustrations, graphs and charts
- "This is not a spectator sport!" Explore examples and patterns. Cover the example and "do it."
- There is no substitution for hard work and time
- Follow the idea back to its origin. Rediscover it for yourself.
- Phrases like "It is easy to see..." or "It follows that..." are cues that much is left out and should be annotated. It may not be easy. Finding the connections will be instructive.
- Read it, Write it and then Teach it to somebody else
- Create Picture Vocabulary/Concept Cards
- Expect to read sections more than once
- Read with a partner

Instructions

- Read an excerpt from a textbook aloud in the way a typical student would read the passage, leaving out pre-reading activities, references to graphs, tables, equations, and definitions. Demonstrate few, if any, reading comprehension skills.
- Ask students to listen carefully and jot down notes as they follow along in their own book.
- Read the excerpt a second time modeling a **few** strategies that a "good" reader would use to construct and monitor comprehension.
- Pause during the reading to explain to students what thinking is going on while reading. Describe the specific comprehension strategies that are being utilized. For example, pause to explain how the headings were reviewed to develop an advanced organizer and purpose, how a particular vocabulary word was decoded, or how an illustration was used to solidify understanding of a concept.
- Ask students to take notes about the various strategies that are revealed.
- Discuss and debrief the two different readings.
  - — Brainstorm reading comprehension strategies that could be utilized when reading a math text. For example:
    - Read slowly
    - Always have a pencil and paper ready when reading a math text
    - Draw a picture/illustration
- Distribute *Student Handout 4.8a:* "While I Was Reading, What Did I Do?" Provide opportunities individually and in groups for students to regularly practice the various strategies that were discussed.
- Work with students to develop a poster for the classroom enumerating what "good" readers do to make sense of the math text.
- Compare the strategies that a "good" reader of mathematics utilizes to construct meaning out of math texts with those a "good" reader of a narrative text utilizes to understand fiction.
- Encourage students to "not give up" when it seems impossible. Successful readers roll-up their metaphorical sleeves and try another strategy.
- Design an instructional "mini-lesson" to teach one of the reading strategies that were illustrated in the "Think Aloud" activity.

Higher-Level Questions

Level Two

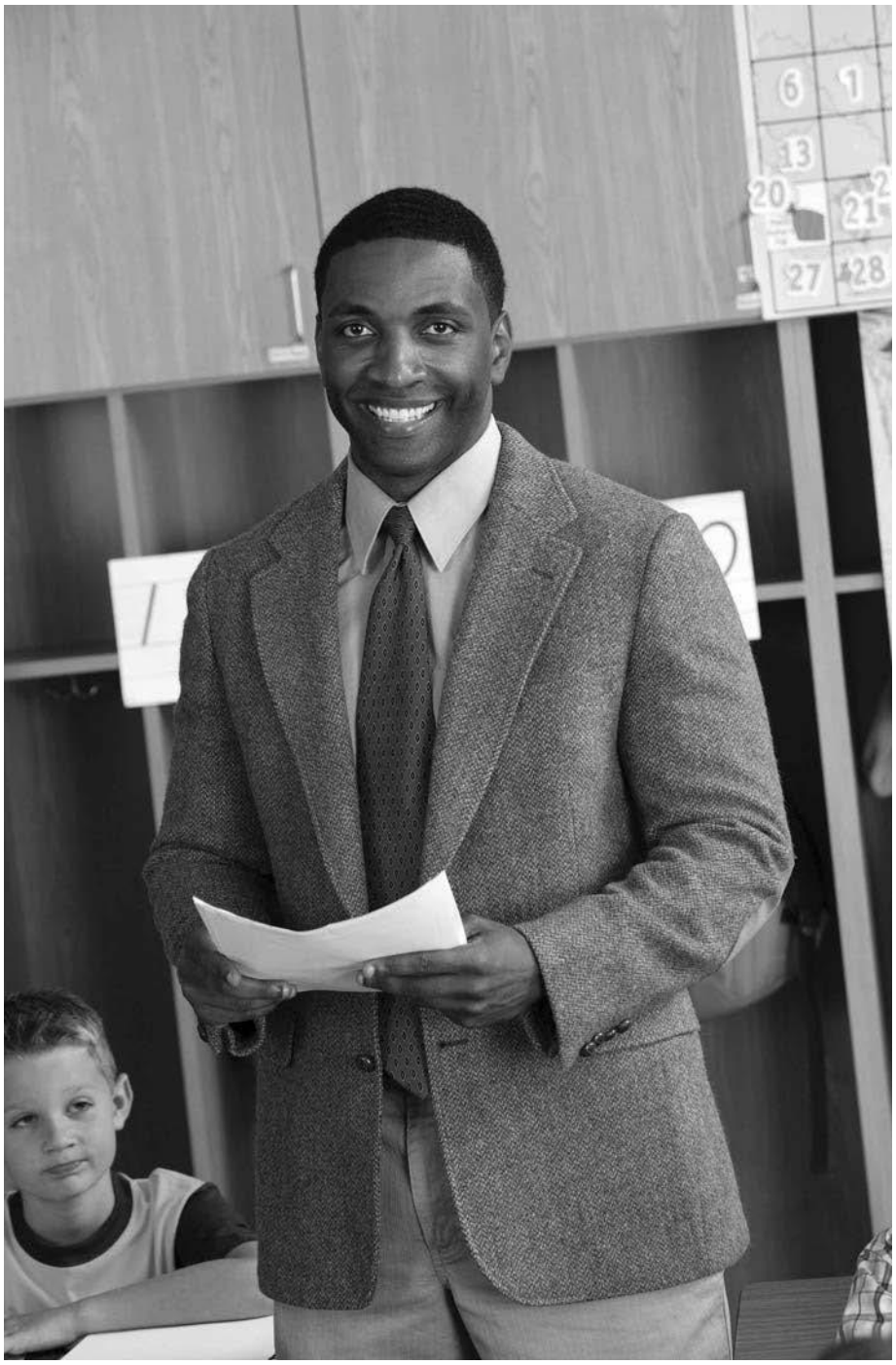
- How is reading a math textbook different from reading a novel?

Level Three

- Why do some people believe math textbooks are difficult to read?
- Which are the most effective reading comprehension strategies in mathematics?
- In some countries it is common for many people to read a math book for fun. Why is that less common in North America?

Formative Assessment

- Monitor students as they demonstrate the "Think Aloud" activity in small group.
- Assess the level of discourse during the class discussion.
- Review the accuracy and inclusiveness of the reading strategies poster that groups design.
- Ask students to provide suggestions for constructing meaning when they are "stuck."
- Review the self-assessment matrices.



While I Was Reading, What Did I Do?

Instructions: Put an √ in the appropriate column.

Strategy (brainstorm strategies below)	**Not very much**	**A little**	**Much of the time**	**All the time**
Drew a picture/illustration				

4.9: I Have - Who Has

Topic

- Reading and comprehending math topics

Objectives

Students will:

- Work collaboratively to construct meaning of individual questions and answers
- Review mathematical concepts

Timeline

- 10–15 minutes to complete the I Have - Who Has activity

WICR Strategies

- Collaboration
  - — Work in collaborative groups to organize game cards
- Reading to Learn
  - — Practice reading comprehension strategies

NCTM Strategies

Focal Point Grade 7

Number and Operations and Algebra and Geometry: Developing an understanding of and applying proportionality, including similarity

Measurement and Geometry and Algebra: Developing an understanding of and using formulas to determine surface areas and volumes of three-dimensional shapes

Number and Operations and Algebra: Developing an understanding of operations on all rational numbers and solving linear equations

Focal Point Grade 8

Algebra: Analyzing and representing linear functions and solving linear equations and systems of linear equations

Geometry and Measurement: Analyzing two- and three-dimensional space and figures by using distance and angle

Data Analysis and Number and Operations and Algebra: Analyzing and summarizing data sets

Communication

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- organize and consolidate their mathematical thinking through communication;

- communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- analyze and evaluate the mathematical thinking and strategies of others;
- use the language of mathematics to express mathematical ideas precisely.

Rationale

Students rarely have the opportunity to practice math vocabulary aurally. Hearing and practicing math terms and vocabulary will facilitate student comprehension and fluency. Providing varied and energetic review activities aids in student learning outcomes. The *"I Have - Who Has"* activity can be adapted for any math content.

Vertical Alignment

- The "I Have - Who Has" strategy can be adapted to any level. At the higher grades the degree of complexity will increase.

Materials/Preparation

- *Teacher Resource Sheet 4.9a:* "I Have - Who Has Example."
- Construct a set of "I Have - Who Has" cards for the concepts that are being reviewed, or have students construct the cards.

Instructions

- Randomly distribute the "I Have - Who Has" cards to students.
  - — *Option one:* One student reads his/her question and the student with the answer on his or her card reads it and then reads his/her question. Thus daisy-chaining around the room.
  - — *Option two:* Ask the students to organize themselves in a circle so that the person with the answer to their question is on their right. Then have students read their questions and answers around the circle.

Higher-Level Questions

Level Two

- Which questions were easier or harder?
- Which questions should be included on a formal assessment to ensure mastery of the content?

Level Three

- How can games such as "I Have - Who Has" aid in test preparation?
- What questions should have been included on a card?

Formative Assessment

- Assess the level of discourse during the class activity.
- Assess the fluency of student language when expressing math concepts.
- Are there fluency problems with the academic language?

I Have - Who Has *Example*

I have a triangle with two congruent sides. Who has a quadrilateral with only two sides that are parallel?	I have 360° Who has a trapezoid with congruent legs?
I have a trapezoid. Who has the formula for the area of a circle?	I have an isosceles trapezoid Who has the altitude of a lateral face of a pyramid?
I have $A = \pi r^2$ Who has the angle inscribed on a 60° arc?	I have the slant height. Who has a linear pair?
I have a 30° angle. Who has complementary angles?	I have two adjacent angles whose noncommon sides are opposite rays. Who has a major arc?
I have two angles whose sum is 90° Who has a pentagon?	I have an arc that measures between 180° and 360° Who has a segment that connects the midpoints of two sides of a triangle?
I have a five-sided convex polygon. Who has the sum of the exterior angles of a polygon?	I have a midsegment that is parallel to the base of the triangle. Who has perpendicular lines?

I have two lines that intersect at a right angle. Who has a rhombus?	I have a hexagon. Who has the sine of 30˚?
I have a parallelogram with four congruent sides. Who has a point that bisects a segment?	I have $\frac{1}{2}$ Who has the complement of 65˚?
I have a midpoint. Who has the ratio of the side opposite to the hypotenuse?	I have 25˚ Who has the distance formula?
I have the sine ratio. Who has two lines that do not intersect and are not parallel?	I have d = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Who has concentric circles?
I have skew lines. Who has a triangle with three 60˚ angles?	I have circles that have a common center. Who has congruent segments?
I have an equilateral triangle. Who has Supplementary angles?	I have segments that are the same length. Who has the geometric mean between 4 and 6?

I have two angles whose sum is 180 degrees. Who has the length of the hypotheses of a right isosceles triangle with a 6cm leg?	I have $2\sqrt{6}$ Who has a kite?
I have $6\sqrt{2}$ Who has the area of a triangle?	I have a quadrilateral that has two pairs of consecutive congruent sides but the opposite sides are not congruent. Who has the coordinates of the midpoint of a segment?
I have $A = \frac{1}{2}b * h$ Who has a six sided polygon?	I have $\left(\frac{(x_1 + x_2)}{2}, \frac{(y_1 + y_2)}{2}\right)$ Who has an isosceles triangle?

4.10: Math Dictionary

Topic

- Developing the academic vocabulary of mathematics

Objectives

Students will:

- Increase the familiarity and fluency of their mathematical vocabulary
- Create a personalized mathematics dictionary

Timeline

- 10–20 minutes for students to set up their personal math dictionary and make the first entries

WICR Strategies

- Writing to Learn
  - — Verbalize and write the definition of terms and concepts
  - — Represent terms and concepts pictorially
- Collaboration
  - — Construct a dictionary of study terms and concepts in collaborative groups
- Reading to Learn
  - — Read class notes, the word wall, a dictionary, or textbook to clarify the definition of terms and concepts

NCTM Standards

Communication

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- analyze and evaluate the mathematical thinking and strategies of others; and
- use the language of mathematics to express mathematical ideas precisely.

Rationale

Clear understanding of the academic language utilized in mathematics is a foundation skill. Providing students with opportunities to master new vocabulary and concepts is critical. The *"Math Dictionary"* activity provides students with a variety of representations and engages students at a range of different modalities.

Vertical Alignment

- "Math Dictionaries" can be constructed and adapted to any level. At the higher grades the degree of complexity will increase.

Materials/Preparation

- *Student Handout 4.10a:* "Create Your Own Math Dictionary"
- Index cards
- 1.5-inch metal ring
- Hole puncher
- Optional:
  - — Dictionaries
  - — Glossary from text
  - — Spanish/English Dictionaries
  - — Word Wall

Instructions

- Distribute index cards and metal rings.
- Distribute *Student Handout 4.10a:* "Create Your Own Math Dictionary."
- Ask students to create a Cover Card for their dictionary. The card should include their name and class period. Allow students time to creatively decorate their cover card.
- Ask students to hole punch their Cover Card and several blank cards in the top left-hand corner.
- Show students how to attach the hole-punched cards to the metal ring or to a ring in their notebook.
- Provide students with a list of vocabulary words and concepts.
- Encourage students to incorporate humor into illustrations and definitions. Humor, color, and creativity will improve memory.
- Model the creation of a vocabulary card. Use creativity, humor, and color.
- Ask students to work collaboratively or individually to construct their dictionary cards.
- Encourage students to include time in their planning agenda to study and construct cards weekly.
- Ask students to alphabetize their cards for easy access.
- Display exemplar cards on a class Word Wall.
- Collect cards and grade them once every one to two weeks.

Higher-Level Questions

Level Two

- How are the cards in each of the groups similar and different?

Level Three

- What are the characteristics of vocabulary cards that aid learning?
- What are some ways to use vocabulary cards to help with the mastery of math vocabulary and concepts?

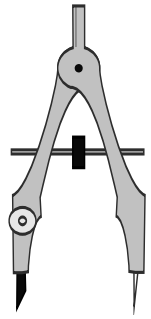
Formative Assessment

- Assess cards regularly for completeness and accuracy.
- Provide opportunities for students to design and play informal games with their cards.
- Vote on cards to include as exemplars for the class "Word Wall."

Create Your Own Math Dictionary

Cover Card

Lynn Garza



Geometry

Period 4

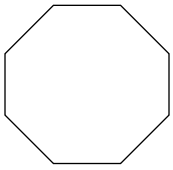
Front of Card

Octagon

Incorporate humor, color, and creativity. They will help you learn the term or concept.

Back of Card

Octagon



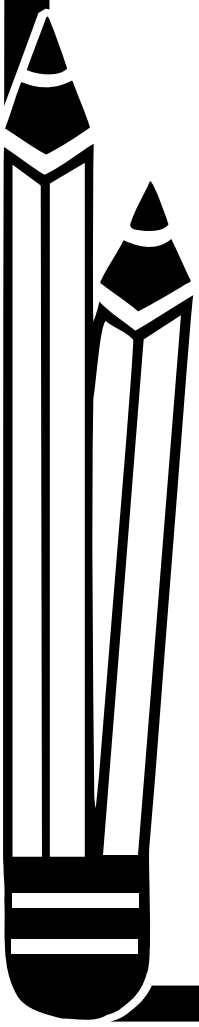
An octagon is a polygon with eight sides.

A stop sign has eight sides so it is an octagon. A stop sign is octagonal.

The measure of the exterior angles of a regular octagon can be found by dividing 360˚ by 8.

Each exterior angle of a regular octagon is 45 degrees.

Resources for Use During Write Path Training



The Write Path I mathematics training introduces you, the practitioner, to a wide variety of activities designed to illustrate the use of Writing, Inquiry, Collaboration, and Reading (WICR) in the mathematics classroom. The goal of the training is to stimulate your thinking, and to spark new and creative applications for use in your daily practice. We hope that you will personalize the lessons by substituting the Active Learning Methodologies and content of your choice.

During training, use the following Training Resources to record your innovative ideas:

Cornell Notes: Three Formats.

Provided so that you can practice the note-taking strategy on different forms during the training and in the process, find the one that will work best for you and your students.

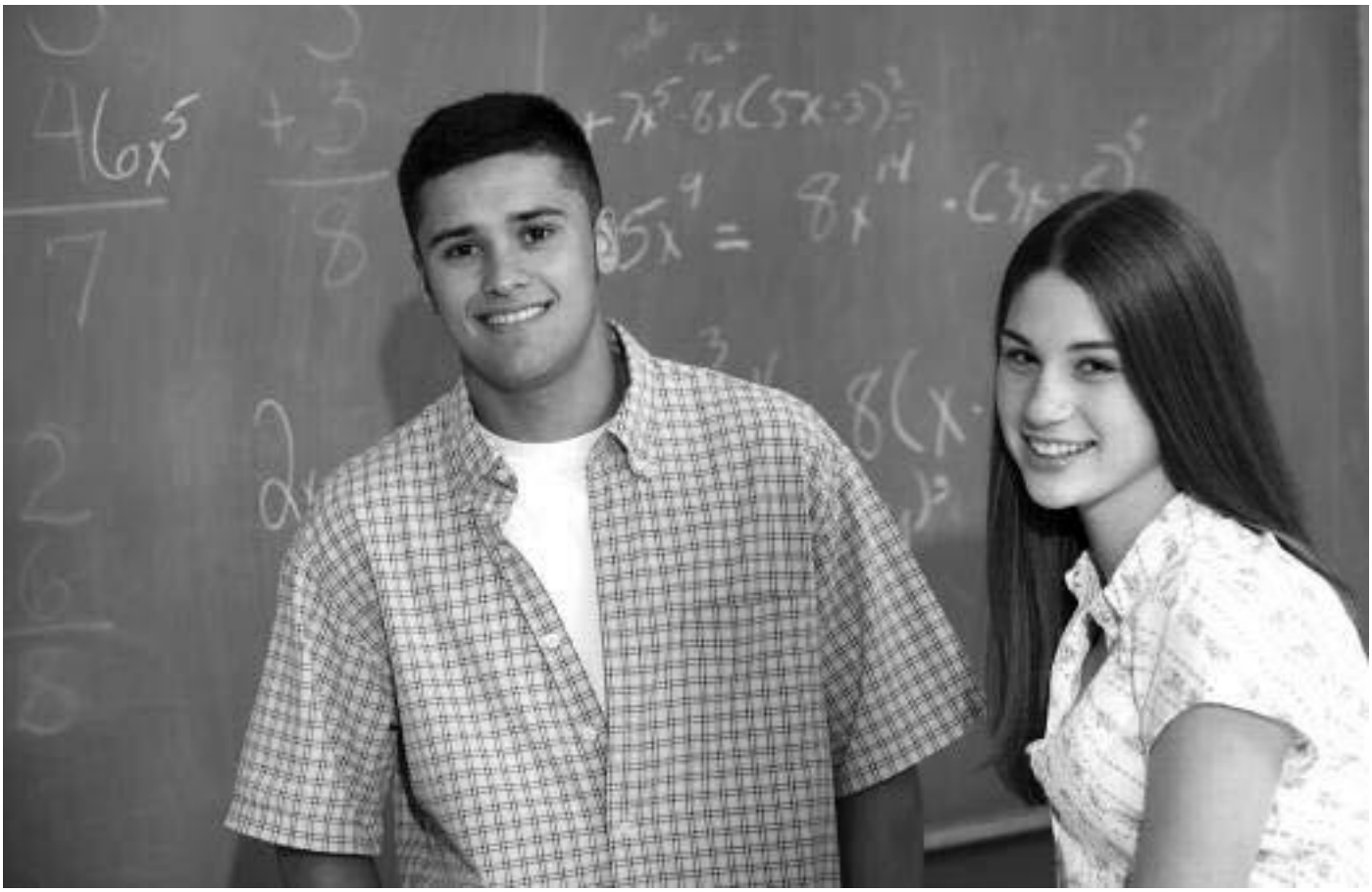
The Learning Log and Reflective Journal.

In addition to the *Cornell Notes'* forms, a *Learning Log* and a *Reflective Journal* have been provided so that you can practice using them before introducing them to your students.

The Action Lesson Plan.

During training, you will be given an opportunity to develop an *Action Lesson Plan;* a blank template has been provided for this purpose.

The Write Path I mathematics training will be an active training. You will not only learn more about WICR lessons and Active Learning Methodologies, you will have the opportunity to actively engage them. To facilitate this active involvement, several training handouts are provided in the following section. The original handouts are embedded in the lesson materials of the *Write Path I: Mathematics'* text and will remain copy-ready.



Action Lesson Plan Template

Title _____

Topic

-

Objectives

Students will:

-
-

Timeline

-

WICR Strategies

- Writing to Learn
  - —
  - —
  - —
- Inquiry
  - —
  - —
  - —
- Collaboration
  - —
  - —
  - —
- Reading to Learn
  - —
  - —
  - —

NCTM Standards

-
-
-
-

Rationale

Vertical Alignment

•

Materials/Preparation

•
•
•
•

Instructions

Higher-Level Questions

Level Two

•
•

Level Three

•
•

Formative Assessment

•

Reflective Journal

Name: _____ Date: _____

Name of the Course: _____

In class today we…

(Describe what topics were covered, what problems were worked on, what presentations were made by students and teachers, or how otherwise you used your time.)

I learned…

(Sum it up in a few sentences using standard English. Be specific, include examples as evidence of your understanding.)

One or two questions or comments I still have are…

(You may start this sentence with, "I don't understand how to…," or "I didn't understand the difference between…," or "I still don't know why…," or "When am I supposed to…," or "At last I understand…,")

Question #1:

Question #2:

My plan for getting help with my homework, should I need it, is to…

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Class Notes/Learning Logs/Textbook Notes

Level 2: sort, infer, analyze, sequence, organize, solve, explain, compare, contrast, classify, isolate, characterize, make analogies.

Level 3: conclude, criticize, reorganize, justify, judge, estimate, predict, speculate, make a model, extrapolate, apply a principle, interpret, hypothesize, if/then

Name: _____

Class: _____

Period: _____ Date: _____

Topic

Study/Review Questions	
Connections, Summary, Reflection, Analysis:	

Class Notes/Learning Logs/Textbook Notes

Level 2: sort, infer, analyze, sequence, organize, solve, explain, compare, contrast, classify, isolate, characterize, make analogies.

Level 3: conclude, criticize, reorganize, justify, judge, estimate, predict, speculate, make a model, extrapolate, apply a principle, interpret, hypothesize, if/then

Name: _____

Class: _____

Period: _____ Date: _____

Topic

Study/Review Questions

Connections, Summary, Reflection, Analysis:

Class Notes/Learning Logs/Textbook Notes

Level 2: sort, infer, analyze, sequence, organize, solve, explain, compare, contrast, classify, isolate, characterize, make analogies.

Level 3: conclude, criticize, reorganize, justify, judge, estimate, predict, speculate, make a model, extrapolate, apply a principle, interpret, hypothesize, if/then

Name: _____

Class: _____

Period: _____ Date: _____

Topic

Study/Review Questions	
Connections, Summary, Reflection, Analysis:	

DLIQ Summary

Include the following in a DLIQ summary:

A. What did we ***do*** in class?

B. What did we ***learn*** in class?

C. What was ***interesting*** in class?

D. What ***questions*** do I have?

E. Check your summary to be sure the details support the topic and the concept in your notes.

Topic: _____

_____

Concept: _____

_____

DLIQ

D _____

L _____

I _____

Q _____

Write three to five sentences using DLIQ:

Cornell Note-taking Checklist

Name_____ Period _____

Do your notes have the following characteristics?

1. Consistent Cornell physical format, notes dated and titled, readable — 3 pts
2. Use of abbreviations, key words/phrases, underlining, starring — 1 pt
3. Main ideas are easily seen; correct sequencing of information — 1 pt
4. Questions are completed on left hand side; Level 2 and 3 questions — 3 pts
5. An accurate, complete summary follows the notes — 2 pts

Characteristics **Date**					
1. Consistent Cornell physical format, notes dated and titled, readable					
2. Use of abbreviations, key words/phrases, underlining, starring					
3. Main ideas are easily seen; correct sequencing of information					
4. Questions are completed on left hand side; Level 2 and 3 questions					
5. An accurate, complete summary follows the notes					
Total Points					

Rubric

Consistent Cornell physical format, notes dated and titled, readable

3. Vertical line drawn 2.5 inches from the left margin. Heading is complete with name, date, subject. The notes are titled. Notes are adequate in length.
2. Minor problem with format
1. No date or no title; short
0. Fails to use Cornell note-taking format or date and title are missing or notes are inadequate in length

Use of abbreviations, key words/phrases, underlining, starring

1. Techniques used throughout
0. Too much verbiage

Main ideas are easily seen; correct sequencing of information

1. Information is complete and in correct order
0. Notes confusing

Questions are completed on left hand side; Level 2 and 3 questions

3. A substantive number of higher order thinking questions are noted in the left margin which are answered in the notes to the right
2. Level 1 questions are many; level 2 and 3 questions minimal
1. Level 1 questions only
0. No questions in the left hand margin

An accurate, complete summary follows the notes

2. Detailed summary covers the main topics of the notes
1. Summary is generic or incomplete
0. Summary missing

Name: _____

Date:_____ Period: _____

Systems of Equations Review

Name all of the different ways to solve a system of linear equations.

Solve this system two different ways:

$x + 2y = 13$

$x - y = 10$

Example problem:

You want to have business cards printed. The executive style card will cost $25 plus 2¢ per card. The economy style card will cost $10 plus 5¢ per card. Write a system of equations to represent the pricing plan for the two different card styles. Solve the system in two different ways. Explain how the solution will inform your decision on which cards to purchase.

Connections, Summary, Reflection, Analysis:

Phone Company Problem

Two competitive cell phone companies, General Telephone and Western Regional, provide special service rates to families who have more than two phones. For their family plan, General charges $40 per month plus $0.15 per minute. Western charges $45 per month plus $0.10 per minute for their family plan.

- Create two equations to describe the two plans.

- Which service would be the best choice?

- Explain how you arrived at your answer.

- How did your notes benefit you as you worked through the problem?

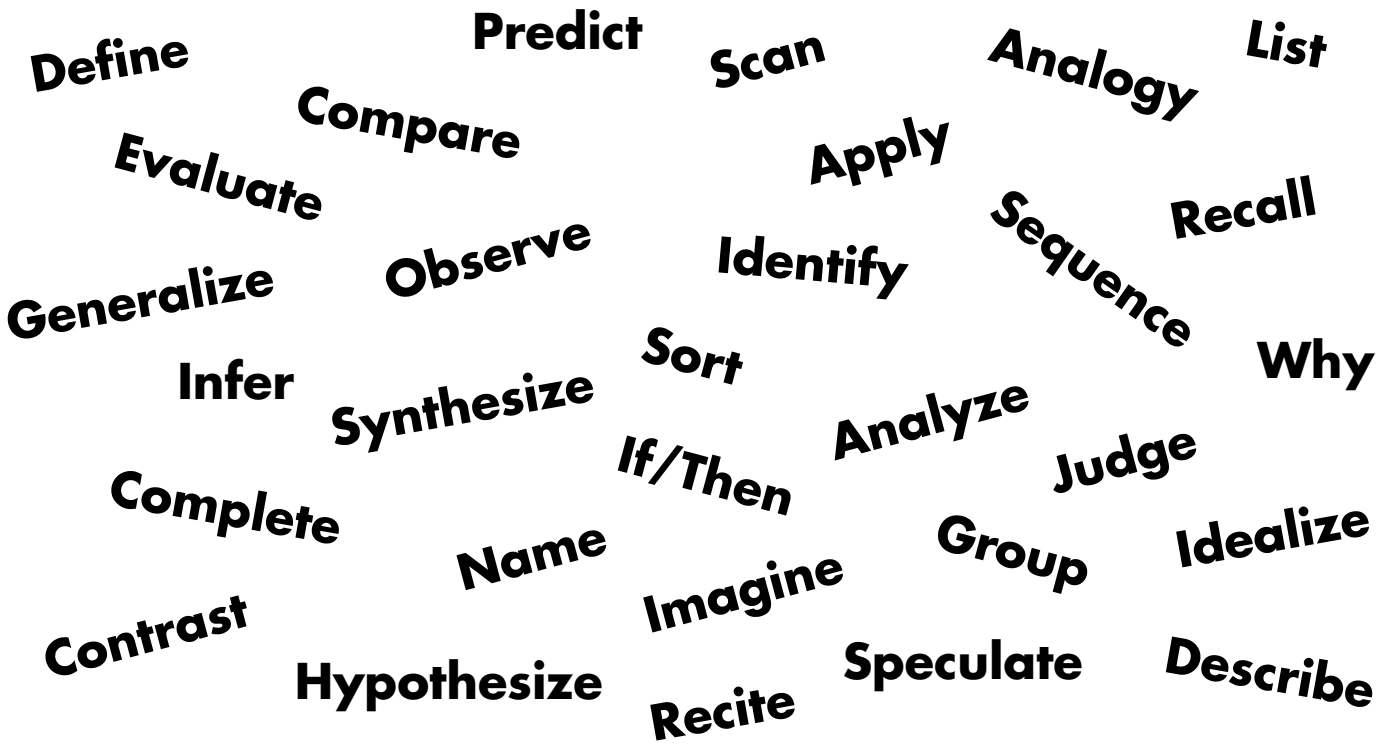
- How would the companies adjust their pricing structure after the break-even point?

- Illustrate your solution with a graph.

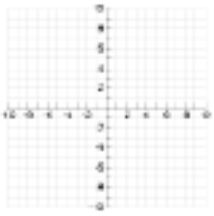
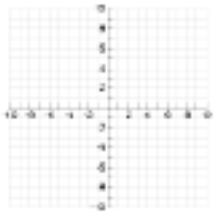
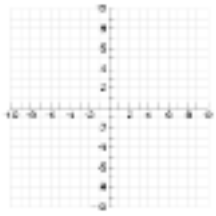
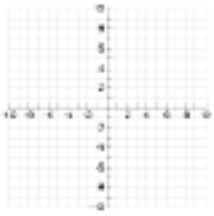
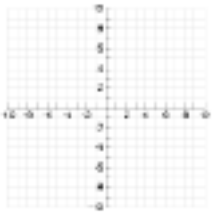
Costa's Levels of Questions

Organize the words below into the following categories:

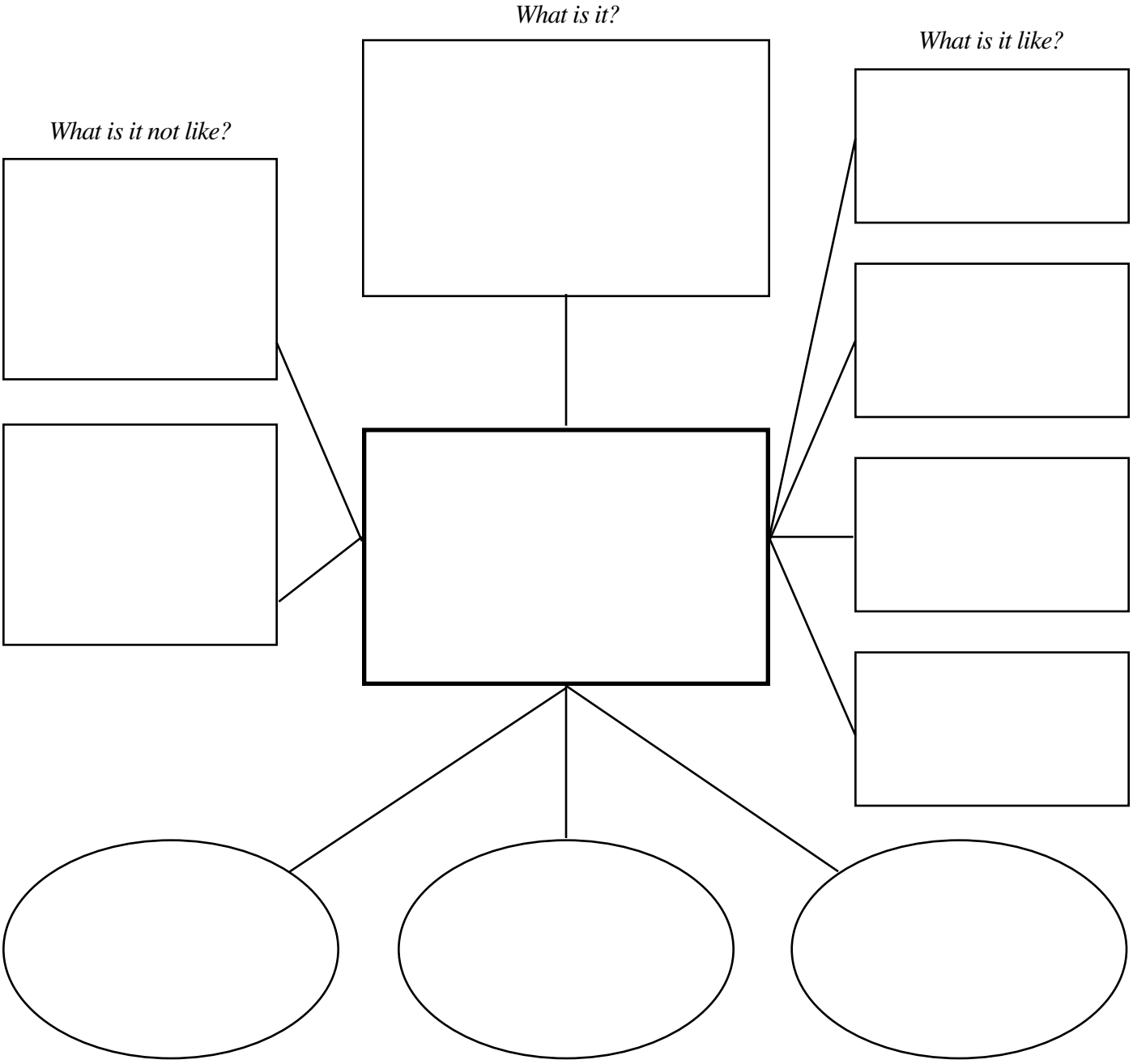
Level One: Input	Level Two: Process	Level Three: Output



Characteristics of Functions

Function	Linear	Quadratic	Cubic	Square Root	Absolute Value
Parent					
Graph					
Domain					
Range					
Symmetric					
Zeroes					

Concept Definition Map



Illustration

Learning Logs

Name: _____

Date(s): _____ Subject: _____

Guiding Questions

Write a paragraph of at least 5 sentences about what happened in class and what you learned. Below are some questions you may want to answer:

- What did you learn?
- What did you find interesting?
- What questions do you have about what you learned?
- What were the main ideas?
- What did you understand best?
- How will you find more information?
- How does the current information relate to what you have already learned in class?
- How might it relate to what we will be doing in the future?

Socratic Seminar Fishbowl

Directions: Choose three participants in the inner circle to observe during the seminar. Take careful notes and pay close attention to the dialogue, individual behaviors, and the group's dynamics.

Participant Name	New Idea	Asked a Question	Referred to Text	Positive Comment	Negative Behavior	Other Notes/ Observations
1.						
2.						
3.						

Mathematical Symbols *Example*

It is necessary to learn the meaning of mathematical symbols that are scattered throughout textbooks and technical writing. You know many of them already. Listed below are some frequently used symbols related to the math you are studying. Try to translate each symbol into a word or word phrase.

Symbol	First Guess	Correct Meaning	Write a Statement and Explain
$\sim$			
$\approx$			
$\cong$			
$m\angle ABC$			
${}_nP_r$			
(x,y)			
$p \leftrightarrow q$			
${}_nC_r$			
$\sum_{n=1}^{3} n^2 + 5$			

While I Was Reading, What Did I Do?

Instructions: Put an √ in the appropriate column.

Strategy (brainstorm strategies below)	**Not very much**	**A little**	**Much of the time**	**All the time**
Drew a picture/illustration				