S C H O O L W I D E

The Write Path II: Mathematics

Teacher Guide

AVID PRESS

THE WRITE PATH II: MATHEMATICS

Teacher Guide

Written by

Jim Donohue

Tim Gill

AVID®

Proven Achievement.
Lifelong Advantage.

Acknowledgements

We would like to thank and acknowledge the following leading math educators and nationally recognized AVID *Write Path* Mathematics trainers for their contribution and collaboration with the Co-Writers/Revisers in developing the framework and creating a vision for the revised AVID *Write Path II: Mathematics* book: Amy Armstrong, Amy Kosler, Anna Lotti, Erich Gott, Kym Butler, Maggie Aleman, Ricardo Gomez, and Sia Lux. In addition, we used the foundation established by the authors and contributors for the previous edition of *The Write Path II: Mathematics,* including Karin Ching, Katherine Dooley, Mary Catherine Swanson (AVID Founder), Rob Gira (AVID Executive Vice President National Programs), Becky Breedlove, and Ivette Sanchez. We thank these educational leaders for their visionary contributions and their commitment to the enrichment of mathematics education. The lessons they contributed, along with their comments and suggestions for the revision of this book, have enabled us to extend the vision for WICR in the mathematics classroom into the twenty-first century. Many took time out of their busy schedules to field test and give feedback on various drafts as they evolved. They have been instrumental in ensuring that the revised materials will meet the needs of mathematics teachers throughout the AVID world. None of these efforts would have been possible without sage advice from Mark Wolfe, AVID's National Director of Curriculum, and the team of professionals at AVID Center. We would also like to extend our heartfelt appreciation to Christy Ball, our editor, and Kathy Johnson, our graphic designer and typesetter on the project. Their support, advice, and encouragement have provided the anchor we needed. We greatly appreciate the efforts of all these dedicated professionals. Thank you!

Jim Donohue and Tim Gill—Co-Writers/Revisers

Table of Contents

HOW TO USE THIS GUIDE

The Write Path II: Mathematics Teacher Guide was designed by and for AVID-trained Math teachers to augment the texts and syllabi utilized in mathematics classrooms in grades 6 through 12. The lessons offered here are exemplars of lessons that are rich in content and pedagogy. The authors and contributors are hopeful that you will "personalize" these lessons and use them as a springboard to create extensions and improvements that support your grade-level content standards and lessons. A Learning Resource Log is provided on the following pages as a place to record your thinking during the Write Path II Educator Training.

The intended audience of this guide is grades 6 through 12 mathematics teachers who have (1) attended AVID Summer Institute and participated in the Mathematics I content strand, and (2) spent a school year implementing the WICR-based teaching methodologies espoused in *The Write Path I: Mathematics* Teachers Guide. The lessons in *The Write Path II: Mathematics* are designed to assist teachers in taking WICR strategies to a deeper, more complex level of implementation.

WICR—Writing, Inquiry, Collaboration, and Reading

The AVID-influenced classroom is distinctive with clear evidence of writing as a tool for learning, inquiry methodologies, collaboration, and reading as a tool for learning (WICR). WICR-infused lessons provide students at all levels with an opportunity to practice the literacy skills they must master in preparation for accessing rigorous course work and post-secondary education access and success.

Writing is basic to thinking, learning, and growth. It allows students to think in complex ways by clarifying and ordering experiences. It contributes to self-knowledge and helps students to become better readers. The *Write Path II: Mathematics'* lessons in note-taking, learning logs, and writing about mathematics will provide students with the opportunity to write and think like a mathematician.

Inquiry immediately engages students with their own thinking processes. It teaches them to think for themselves instead of chasing the "Right Answer." Student ownership for expanded understanding of concepts is the result. *The Write Path II: Mathematics'* lessons introduce students to strategies for crafting higher-level questions using Arthur Costa's model of intellectual functioning. Strategies such as Philosophical Chairs, inquiry-based tutorials, and Socratic Seminars along with skillful questioning will empower students to gain mastery of their own learning.

The AVID philosophy and belief is that students learn best when they are actively manipulating materials by making inferences and then generalizing from those inferences. Collaborative learning groups encourage this kind of thinking. *The Write Path II: Mathematics'* lessons emphasize collaborative group work whenever possible. In small groups, students will ask, explore, and answer questions as they become better listeners, thinkers, speakers, and writers. They will discover ideas and remember them because they will actively engage them. Their teachers will become coaches, carefully guiding them in their learning.

AVID incorporates strategies that help students become more effective readers when used strategically with rich and varied curricula. The reading strategies introduced in *The Write Path II: Mathematics* are designed to make sense of the mathematics text and help students synthesize their understanding. The lessons that are included in *The Write Path II: Mathematics* will help students understand what effective readers do, how to activate prior knowledge, investigate text structure, and utilize text-processing strategies to improve comprehension of their mathematics texts.

The lessons in *The Write Path II: Mathematics* are divided into four themes/units: Writing, Inquiry, Collaboration, and Reading (WICR). Most of the lessons involve more than one theme. Because writing is one of the most important skills, nearly every lesson includes a writing to learn component. In all cases, higher-level thinking and discourse are emphasized.

In addition to WICR strategies, the AVID-influenced classroom is noted for its incorporation of active learning methodologies. In the following section, you will find a summary of some of the more common methodologies that are integrated into the lessons in the *The Write Path Mathematics* books. The authors and contributors have sought to incorporate an active learning methodology that seemed best suited to the particular lesson or concept. Please use these as opportunities to personalize the lesson by substituting the active learning methodology and content of your choice.

Common Active Learning Methodologies

Note: The names of the methodologies may be different in other texts.

Keys to Success when implementing Active Learning Methodologies

- Start small!
- Plan an active learning methodology, try it out, collect feedback, then modify and try it again.
- Start on the first day of class.
- Explain to students why you are doing this and how it will aid them in the learning process.
- Develop classroom routines for transitions.
- Work collaboratively with a colleague while you're implementing active learning methodologies.

Think-Pair-Share

1. Instruct students to think carefully about a specific topic or a question. This may be facilitated by a quick write.
2. Instruct students to find a partner near to them.
3. When you give a signal, one partner shares his/her answer to the question and the reasons that support it, while the other partner listens.
4. The partners exchange roles.
5. The partners prepare to share their answers/responses with the large group.

Think-Pair-Share—Squared

1. Participants listen to a question, concern or scenario.
2. Individuals think and make notes about the questions, concern or scenario.
3. Individuals pair and discuss their responses.
4. Pairs join into groups of four and discuss responses.
5. Foursomes prepare to share their answers/responses with the large group.

KWL

1. Draw three columns on chart paper. Label the columns of the KWL chart; What we Know, What we Want/Need to Know, and What we Learned.
2. Identify a text selection or topic for students to consider during the activity.
3. Ask students to brainstorm and enter information in the columns indicating what they know and want/need to know.
4. Provide students with frequent practice.

Carousel Brainstorming

1. Prepare the same number of wall charts as groups.
2. Assign each group to begin at a specific chart. It may be helpful to assign a different color marker for each group.
3. On the first signal—groups move to assigned charts and generate and record as many ideas as possible for that item.
4. On the second signal—groups rotate clockwise to the next chart, review what previous group wrote, generate and add additional ideas.

5. On the third signal—groups rotate clockwise to the next chart, review what the previous group wrote, generate and add additional ideas … continue until all groups have written on all charts … then,
6. Ask the students to take a "Gallery Walk" of all charts and be seated.

Jigsaw—Home Group/Expert Group

When discussion of new information is desired, but time is limited, use Jigsaw reading/study groupings.

1. Divide students into small groups. The number of sections of the reading or the number of concepts being reviewed or introduced will determine the number and size of the groups.
2. Assign each member of the group a number that corresponds to the section of the text to be read or the concept to be mastered. Each member is responsible for completing one part of the reading or mastering one of the assigned concepts. Encourage students to take notes.
3. Students then leave their "home" groups and form "expert" groups with other students with the same number. Each "expert" group works on its part of the assignment; members assist each other with questions, clarifications, and summaries. In preparation for going back to his or her "home" group as an "expert," each student rehearses and teaches the lesson to the other members.
4. Students return to their "home" groups and share, discuss information, and teach their part of the assignment.
5. Students reassemble as a whole class and share their thoughts and responses.

Jigsaw Sequencing Groups

1. Cut sections of a solution process, reading or proof into individual parts. Each part should have a complete meaning and show a type of transition at the beginning or the end of that section.
2. Form groups of students that correspond to the number of "jigsaw" pieces.
3. Each group member receives a different piece of the text, problem or proof.
4. Each member of the group must then decide where their piece fits in the text, problem or proof.
5. If a student thinks he/she has the first section of the text, problem or proof, the student must give the reasons why without letting the group read the section. He/she tells the group, "I think I have the first piece because…"
6. If the group agrees that it is the first section, the student reads the text, problem or proof aloud to the group and then places it on the table.
7. The group then proceeds to look for the next section following the same rules as above.

Numbered Heads Together

1. Place students in groups of four.
2. Have students in each group number off from one to four.
3. Ask students a question for discussion or review.
4. Have students discuss the question in their groups, making sure that each member of the group can answer the question if called upon.
5. Select a random number corresponding to a number of a group member.
6. Select one or two students to respond to the question. Additional students with the same number can respond to the question by adding new information to the previous response(s).

Fishbowl

1. Set up a small inner circle of students to demonstrate an activity for the class. Have all other students form a larger outer circle around the inner circle (Fishbowl group) of students.
2. The inner circle (Fishbowl) listens carefully to teacher directions and then demonstrates the activity to the rest of the class.
3. As necessary, clarify and correct the activity steps with the Fishbowl group.
4. Debrief with the entire class.

   Note: The Fishbowl can also be used as a type of Socratic Seminar, where the inner circle of students participate in a discussion and the outer circle students listen and take notes. Later, the outer circle students can comment on the discussion, using their notes and then exchange places with the fishbowl students.

Novel Ideas Only

1. Place students in groups and assign groups to list ideas about a given topic. Set a time limit for the task.
2. Have a spokesperson from each group stand and share one "novel" idea from the group's list.
3. Students in each group must listen attentively to ensure that no group repeats information already provided by another group. (Each group spokesperson can only give information not previously mentioned.)
4. As students hear an item shared by another group, they check it off their own group's list
5. Each spokesperson sits down after they have either read or checked off all the items on their list.
6. The activity continues until all "novel" ideas about the topic have been shared and all students are again sitting down.

Novel Ideas—Four Corners

1. Allow students to divide themselves into four groups based on their perceived level of understanding or mastery of a question or concept.
2. Ask the groups to brainstorm all that they know about the question or concept.
3. Ask a representative from the level one group to share all that was on their group's brainstorm list.
4. Proceed in turn with each sequential group allowing them to share new information not previously mentioned.
5. Finish with the group that perceived themselves as having mastered the material.
6. Clarify misconceptions and misstatements.

Inside/Outside Circles or Parallel Lineups

1. Divide students into two equal groups.
2. Place half the group in the inner circle directly facing a member of the second half of the group in an outer circle. (Alternatively, form parallel lines.)
3. Provide a limited amount of time for the partners to quiz each other on vocabulary, review questions or to discuss another teacher-designated topic.
4. Have the outer circle move to the left (or right) two or three partners down. With parallel lineups, have one or two persons at one end of the line walk quickly to the other end of the line, and all other move one or two spaces to face a new partner.
5. Repeat step 3.

Give One/Get One

1. Ask each student to make a list of ideas related to a teacher-generated topic or question on a sheet of paper. Give students two to three minutes to create as long a list as possible.
2. Tell students to draw a line after their final idea.
3. Have students stand with their list in hand and talk, one on one, with as many other students as they can in a period of three to five minutes.
4. Students must give each other student they meet an idea from their list; they must also write down one new idea from each partner's list.
5. At the end of the activity, create a class list of information completed from the individual lists of students.

Talking Chips

1. Have students each create three name cards ("Talking Chips").
2. During discussion groups, have student take out their name cards ("Talking Chips"). Tell them that when they are ready to contribute to the discussion they must place one of their chips in the center of the table. When they do this, all other students at the table must stop talking and listen attentively.
3. When students have used up all of their talking chips, they must wait for others to use theirs up, too, before they can contribute to the discussion again.
4. Once all chips are in the center of the table, they can be redistributed and all participants invited to join in the discussion again.

Take Five

This process is used to gain consensus decision-making. It is an effective way to assess group needs and gather information for problem-solving.

1. Divide the group into smaller groups of four or five students each.
2. Provide quiet time for each student to complete a 5 to 10 minute quickwrite.
3. Provide time for groups to collaborate and brainstorm.
    a. Each student should share his or her writing one at a time.
    b. Groups should look for common themes and record consensus.
    c. Each group should then share their top agreements/priorities with the larger group.
4. The larger group records common themes/priorities.

Parking Lot

1. Provide students with sticky notes on which they can record questions and concerns. Designate a location in the room for students to "post" their questions and concerns.
2. Encourage students to add to the Parking Lot at any time.
3. Check the Parking Lot frequently and address any notes that have been posted.

Consultation Groups

1. Divide the group into small issue groups based on needs or interests.
2. Utilize a Jigsaw structure or other reporting out or sharing strategy for the "experts" to share what they know or have learned.

Whip Around

1. Divide students into small groups of four to five students each
2. Present a question or discussion prompt.
3. Give a time limit, usually two to three minutes.
4. Going around the group sequentially, each student is provided an opportunity to comment on the question or discussion prompt.
5. A student may pass one time, but must comment the next time it is his or her turn.

Popcorn

1. Give students an opportunity to share ideas and comments with the whole group. Students do not have to raise their hand.
2. Standing to share improves thinking, keeps comments short and provides an opportunity to include movement.
3. Record the ideas/comments on chart paper.

Learning Logs—Minute Papers

1. Provide students with the opportunity to synthesize their knowledge and to ask unanswered questions during a few minutes at the end of the class. Writing prompts could include:
   - What was the most important thing you learned today?
   - What important question remains unanswered?
   - Variations of these questions, and the student questions and answers they generate, enhance your students' learning process and provide you with feedback on students' understanding of the subject material.

Concept Mapping

1. Ask students to create visual representations of models, ideas, and the relationships between concepts.
2. Ask them to draw circles containing concepts and lines, with connecting phrases on the lines, between concepts. These can be done individually or in groups,
3. Provide an opportunity for students to share, discuss, and critique the work of their peers.

Note-checking Pairs

At the end of a class segment (after 10 to 15 minutes) ask students find a "Shoulder Partner" to review their notes. The note review activities could include:

- Summarize the three important points.
- Choose the most important idea that will appear on the exam.
- Check the completeness and accuracy of your partners notes.
- Use the notes to solve an example problem.
- Write questions in the left column of their Cornell notes.
- Use the notes to work on a teacher-generated question.

Collect the student work as a formative assessment.

Games

Games such as Buzz counting games, Jeopardy®, matching, mysteries, group competitions, puzzles, charades, Scrabble®, Pictionary®, etc. can be designed to introduce or review specific vocabulary and concepts.

Why Vertical Teams?

Part I: Case Study

The Concept of Mathematics/AVID Vertical Teams

To help counteract the barriers that have prohibited equal access to honors and Advanced Placement (AP) mathematics courses, it is recommended that middle schools and high schools form Mathematics/AVID Vertical Teams. Their challenge should be to address the question:

> *"What would it take to build a mathematics program so strong and inviting that a large percentage of students—perhaps every student—could be prepared to successfully complete challenging mathematics courses, such as calculus, before leaving high school or upon entering college?"*

The Advanced Placement Program Mathematics Vertical Team Toolkit 1998, funded by The College Board and Dana Center, suggests the vision, evidences the need, and recommends the purposeful and necessary work of a mathematics-focused vertical team.

California Mathematics Standards

California Mathematics Content Standards, which expect enrollment in Algebra for all students at the eighth-grade level, are now more closely aligned with the competitive realities of admission to the state's better four-year universities. In California and nationally, barriers exist that inhibit schools in their attempts to successfully meet this new expectation. The gap between the mathematics that has historically been taught to most eighth graders and the newly adopted state standards is tremendous. The shortage of well-prepared mathematics teachers is another limiting factor.

The truth is that students who do not take a minimum of five years of college preparatory mathematics culminating in Advanced Placement Calculus or Advanced Placement Statistics are far less likely to be admitted to the nation's more competitive public and private institutions. The most likely scenario for such a sequence requires that students successfully complete college-preparatory Algebra as eighth graders.

Nevertheless, the legislation of standards by itself cannot change the attitudes and practices of those educators, parents, and students who for years have accepted college-preparatory Algebra at eighth grade as an honors course, with limited access, reserved only for the "best and brightest." It will take the ongoing work of educators committed to equity and access to profoundly impact the systems that have effectively denied access and impeded success for some groups of students far more than others.

The mandate of "Algebra for all" and the shift of that curriculum from the high school level to the middle school will require additional assistance for middle-level mathematics teachers, parents, AVID coordinators and tutors, all of whom will need to support young people from Algebra through Calculus. One means of creating viable support systems is through the formation of Mathematics/AVID Vertical Teams.

AVID: Advancement Via Individual Determination

AVID is a nationally recognized program designed to give students who ordinarily would not be programmed into rigorous, academic, college-preparatory classes the opportunity to take such classes and the support necessary to succeed in them.

The core of this support structure is the AVID coordinator, whose elective AVID course aims to help students develop the skills they will need to succeed in rigorous classes. By coupling high academic expectations with strong, persistent support, ordinary students are enabled to do extraordinary things. Writing to Learn, Inquiry, Collaboration, and Reading to Learn (the WICR methodologies) are emphasized, and students experience college entry tests and writing activities. They learn and practice study skills, they participate in collaborative study groups facilitated by college tutors, and they enrich their secondary years by learning about college and careers. Fears of a system that seems unapproachable are assuaged as their confidence and academic achievement increase. AVID parents become partners in the preparation for college, and teams of educators at the school follow an action plan to fulfill the mission of AVID.

AVID is designed to increase schoolwide learning and performance. The mission of AVID is to ensure that all students, and most especially students in the middle, are capable of completing a college-preparatory path and:

- Will succeed in the most rigorous curriculum;
- Will enter mainstream activities of the school;
- Will increase their enrollment in four-year colleges; and
- Will become educated and responsible participants and leaders in a democratic society.

"The students of the 21st century desperately need educational opportunities. The long-term economic success of the nation and the perpetuation of democracy are going to hinge on building an education system to accommodate and prepare all of our students. A society that defines itself as a democracy is obligated to create and sustain public education for the full, broad sweep of its citizenry."

—Mary Catherine Swanson, AVID Founder

Tomorrow's Jobs and the Need for a Better Educated Populace

"In recent years, the level of educational attainment of the labor force has risen dramatically. The trend toward higher educational attainment is expected to continue. Projected rates of employment growth are faster for occupations requiring higher levels of education or training than for those requiring less. Workers in occupations requiring higher levels of education have higher incomes. Many of the occupations projected to grow most rapidly between 1992 and 2005 are among those with higher earnings."

—*Occupational Outlook Handbook,* US Dept. of Labor's Bureau of Labor Statistics

"Increasingly, over the past 30 years, new jobs have been filled by people with a college degree, including more than 90% of the new jobs created since 1980."

—*Source:* McCarthy & Verney, *Immigration in a Changing Economy: California Expansion,* RAND 1997

The US Department of Labor predicts that, "For 12 of the 20 fastest growing occupations, an associate degree or higher is the most significant level of postsecondary education or training."

—Retrieved February 24, 2008, from <http://www.bls.gov/oco/oco2003.htm>

The trend toward higher educational attainment is expected to continue. Projected rates of employment growth are faster for occupations requiring higher levels of education or training than for those requiring less. Workers in occupations requiring higher levels of education have higher incomes. Many of the occupations projected to grow most rapidly between 2006 and 2016 are among those with higher earnings.

Part II: Advanced Placement Calculus

Brief History

The Advanced Placement Program began over four decades ago to enable students to complete college-level studies while they are still in high school and to obtain college placement or credit on the basis of their performance on rigorous AP® examinations. The AP program is administered by the College Board which contracts with Educational Testing Service (ETS) for technical and operational education services.

There are numerous AP courses and examinations offered in a variety of disciplines including: Calculus AB, Calculus BC, and Statistics.

Most university applications specifically ask for a list of all AP courses taken in high school, and admissions officers use AP coursework as a significant part of their selection process. Students who take AP courses and exams have a distinct advantage in being accepted to a university. Because AP coursework is more rigorous, students with the AP advantage are better prepared for college coursework.

AP Mathematics Prerequisites

Prior to enrolling in an AP Calculus or AP Statistics course, because AP coursework is more rigorous, students are expected to take the following sequence of courses: Algebra I, Geometry, Algebra II/Trigonometry, Precalculus or equivalent integrated courses that cover the same content and have prior college approval. Since these courses are sequential, students must begin the sequence in the eighth grade or attend summer school.

Students enrolled in Advanced Placement courses experience:

- Significant increase in mathematical knowledge
- Increased chances of acceptance by the best universities
- Increased number of possible university majors
- Increased confidence in their ability to handle university-level work
- Cost-saving advantages through placement in advanced standing
- Time-saving advantages through advanced placement
- Priority registration at some institutions

Access for AVID Students to Rigorous Mathematics in High School

The AVID mission and practices have always expected students to enroll in rigorous coursework, but most students at most schools are not yet being accepted into AP Calculus or AP Statistics as high school seniors. Traditionally, a majority of students do not take, or successfully complete, the prerequisite Algebra course in the eighth grade and so they are not eligible for the highest track of mathematics in high school.

In *Class Struggle: What's Wrong (and Right) with America's Best Public High Schools* (1998), Jay Mathews concludes that students will strive for the best if they get the chance. He estimates that at least 25,000 students are told each year that they cannot take the AP courses they want, and another 75,000 students and probably far more have the ability to do well in such courses but do not ask to enroll because no one encourages them to do so.

CREATE (Center for Research, Evaluation and Training in Education) published the following recommendation in an executive summary based on findings from a longitudinal study of the AVID program.

> "Increase the emphasis on algebra in middle-level AVID. Early indicators in this study show that, in the first semester of high school, students who took algebra in middle grades out-performed students who did not. **In fact, algebra was the single most critical predictor of GPA and college preparation credit accumulation in the one term studied thus far."**
>
> —*Longitudinal Research on Middle-Level AVID: Year 2 Report – Executive Summary,* 1998

As part of their ongoing work, vertical team members will want to examine the attitudes, practices, and policies that exist within their communities and effectively limit access to rigorous mathematics. Discussions related to these issues are more powerful when connected to appropriate data. As stated previously, teams may want to collect data to describe the local situation and make comparisons to state and national realities.

Part III: Common Concerns about Placement of AVID Students in Honors-Track Math Courses

Frequently when teachers, counselors, and parents are asked why they hesitate to enroll AVID middle school students in pre-Algebra and Algebra courses, the following concerns surface in the conversation. For real change to occur, vertical team members need to anticipate the typical fears of others, evaluate their own beliefs and practices, and become proactive in changing the attitudes and climate at their schools. If the vertical team is going to increase the number of students succeeding in Algebra through Calculus and Statistics, it must carefully and critically review current local practices which determine student placement.

Some of the most commonly asked questions and concerns from parents, teachers, and students are presented here with suggested responses.

1. **Concern:** AVID students' self-esteem will be damaged if they don't do well in Algebra.

   Response: At the middle school level, both educators and parents are dedicated to educating the whole child. There is great concern for the student's self-esteem. Yet, real self-esteem does not come from getting good grades in mediocre programs. Rather, self-esteem develops as a result of meeting with success in new and challenging work. Students know the difference. Students know when the work they are doing matters. They also know when the system is praising them through inflated grades for work that is substandard or remedial. A grade of C or D in Algebra might translate to the achievement of higher-level standards more than an A or B in a subject that simply revisits material already taught in previous grades.

   AVID builds self-esteem by believing in its students and in their ability to meet high academic standards. AVID stands by its members when the learning becomes difficult and reminds them that their self-determination is the key to their success.

2. **Concern:** Teachers will need to water down their curriculum.

   Response: It is essential that Algebra teachers continue to maintain high standards in their math classes. It is imperative that the essential content as identified through articulation and vertical team consensus be taught.

   Teachers need staff development to hone their own math skills and to develop strategies that work well with a diverse group of students. They need to learn and practice methodologies designed to increase their effectiveness with a diverse student population.

   Teachers need to trust that through the support of tutors in the AVID elective, students will receive opportunities to strengthen their arithmetic skills as well as additional help with their rigorous Algebra curriculum.

3. **Concern:** Unless students have adequate English language skills they cannot be enrolled in advanced math courses.

   Response: Students who have English language difficulties will indeed find Algebra challenging, especially if the mathematics curriculum at their school is based on acquisition of skills through the process of solving word problems. However, when collaborative group work is promoted in the classroom, as it is in the AVID elective, students can overcome their language barrier. Much of the symbolism used in mathematics is universal, and placement in honors level mathematics courses is often the arena where students whose first language is not English can excel.

4. **Concern:** Grades will suffer.

   Response: Research on the AVID program has found that as much as two years may be required for students to develop the skills for academic success as demonstrated by C or better grades in college-preparatory classes. This further substantiates the need for enrollment in honors courses, specifically Algebra, at the middle school level. The research also suggests that when students persist and teachers hold them to high standards, their grades improve in time to ready them for the university.

   Even a poor grade in Algebra at the eighth-grade level assures that the student will be placed in Algebra in the ninth grade, thus enabling the child's continued tracking in a college prep curriculum. AVID provides the infrastructure, the immediate intervention strategies, and the support students need to succeed.

5. **Concern:** Parents will worry about their children's placement.

   Response: It is true that parents will be concerned should their child bring home poor grades and may want to have the counselor move the student to a less rigorous mathematics class. But parents need to remember that what matters most at the middle school level is the track to which their children are gaining access. Parents need to be more concerned about access to rigorous curriculum that will open the doors of higher education to their children than they are to grades. A student who "fails" Algebra will know more than a student who is never given the opportunity to be in the Algebra classroom.

6. **Concern:** Heterogeneous classes hurt GATE students.

   Response: "We have mountains of research evidence indicating that homogeneous grouping doesn't consistently help anyone learn better ... no group of students has been found to benefit consistently from being in a homogeneous group." (Oakes, J. (1985). *Keeping track: How schools structure inequality*. New Haven, CT: Yale University Press.)

Rigor in Mathematics

Mathematics teachers are essential members of the AVID site team, dedicated to equity in the classroom and to providing students with access to academic rigor *(AVID Essential 4)*. The importance of mathematics teachers to the future success of students has been delineated in numerous contemporary research findings. Clifford Adelman states in *Answers in the Toolbox* (1999) that "the academic intensity and quality" of a student's course of study is a far more powerful predictor of bachelor-degree attainment than class rank, grade point average, or test scores. He finds that this impact is "far more pronounced for African-American and Latino students than for any other group."[1]

In 2006, Adelman revisited his *Answers in the Toolbox* and the critical role of mathematics teachers was more clearly defined. In *Answers in a Toolbox Revisited*, Adelman states that the "highest level of mathematics reached in high school continues to be a key marker in precollegiate momentum, with the tipping point of momentum toward a bachelor's degree now firmly above Algebra 2."[2] In addition to college access and success, a rigorous curriculum "predicts greater skill in the workforce and greater wage-earning potential. An extensive study conducted by ETS found that 84 percent of highly paid professionals and 61 percent of "well-paid, white-collar" professionals had taken Algebra 2 or higher-level mathematics courses while only 30 percent of low-to-moderately skilled and low-paid workers had done so."[3] These findings make a strong case for all schools to provide all students, not just those enrolled in "college prep," with a rigorous academic program including preparation for and access to Algebra II and beyond. Sadly, not all schools provide equal access to rigor.

Research findings from *Answers in the Toolbox Revisited* indicate "Latino students … are far less likely to attend high schools offering Trigonometry (let alone Calculus) than white or Asian students, [and that] students from the lowest socioeconomic status (SES) quintile attend high schools that are much less likely to offer any math above Algebra 2 than students in the upper SES quintiles."[2] These finding, among others, make it clear that mathematics teachers are critically important on the AVID site team in ensuring not only equity and access, but that academic support, encouragement, and rigor are a part of every students' school experience. The extent to which opportunities that require high-level cognitive mathematical processing are offered to students at all levels is the charge of the math leaders in our schools and districts. However, these opportunities will not be available to students if a clearly defined scope and sequence of skills and concepts are not identified and vertically aligned.

What is Rigor in Mathematics?

The vital role of rigor in school curricula is unmistakably supported by research. However, an agreed upon definition of what constitutes rigor in mathematics is more elusive. Many professionals in the educational community have embraced the definition put forward in *Teaching What Matters Most: Standards and Strategies for Raising Student Achievement*[4] in which the authors argue that, "rigor is the goal of helping students develop the capacity to understand content that is complex, ambiguous, provocative, and personally or emotionally challenging."

A statement issued by the Institute for Learning captures much of what many educators in mathematics support:

> *"Academic rigor in a thinking curriculum holds that students must be exposed to a rich knowledge core that is organized around the mastery of major concepts. This curriculum should provide students with regular opportunities to pose and solve problems, formulate hypotheses, justify their reasoning, construct explanations, and test their own understanding."*[5]

Several versions of standards released by The National Council of Teachers of Mathematics (NCTM) support these definitions of rigor, which promote mathematical thinking, reasoning, and understanding (NCTM, 2000, 1991, 1989). The NCTM has repeatedly extended a philosophy of students as "active constructors of mathematical knowledge, and teachers are to serve as facilitators of students' learning by providing classroom experiences in which students can engage with rich mathematical tasks, develop connections between mathematical ideas and between different representations of mathematical ideas, and collaboratively construct and communicate their mathematical thinking."[6] Briefly stated, students must be given an opportunity to investigate important and worthwhile mathematics with understanding that goes beyond procedural knowledge. This being said, it becomes the imperative of the AVID site team, and more specifically, that of the mathematics leaders on the site team, to help their schools and colleagues define and integrate rigor throughout their curricula.

---

1. Adelman, C. (1999). "The executive summary." *Answers in the toolbox: Academic intensity, attendance patterns, and bachelor's degree attainment*. Washington, DC: U.S Department of Education. Electronic version. (Retrieved January 13, 2008, from <http://www.ed.gov/pub/Toolbox/Exec.html>).
2. Adelman, C. (2006). *The toolbox revisited: Paths to degree completion from high school through college*. Washington, DC: U.S. Department of Education. Electronic version. Retrieved January 13, 2008, from <http://www.ed.gov/rschstat/research/pubs/toolboxrevisit/index.html>.
3. The American Diplomat Project (ADP) (2004). *Ready or not: Creating a high school diploma that counts*. Washington, DC: Achieve Inc. (Retrieved January 13, 2008, from ERIC Document Reproduction Service No. ED 494 733).
4. Strong, R.W., Silver, H.F. & Perini M.J. (2001). *Teaching what matters most: Standards and strategies for raising student achievement*. Alexandria, VA: Association for Supervision and Curriculum Development.
5. Boston, M. & Wolf, M.K. (2006). *Assessing academic rigor in mathematics instruction: The development of the instructional quality assessment toolkit* (CSE Technical Report 672). Los Angeles, CA: National Center for Research on Evaluation, Standards, and Student Testing (CRESST). (Retrieved January 13, 2008, from ERIC Document Reproduction Service No. ED 492 868).
6. Boston, M. & Wolf, M.K. (2006). *Assessing academic rigor in mathematics instruction: The development of the instructional quality assessment toolkit* (CSE Technical Report 672). Los Angeles, CA: National Center for Research on Evaluation, Standards, and Student Testing (CRESST). (Retrieved January 13, 2008, from ERIC Document Reproduction Service No. ED 492 868).

Learning Resource Log

Lesson Name	Location	Notes/Modification

Learning Resource Log (Continued)

Lesson Name	Location	Notes/Modification

Learning Resource Log (Continued)

Lesson Name	Location	Notes/Modification

Learning Resource Log (Continued)

Lesson Name	Location	Notes/Modification

Unit One: Writing in Mathematics

Introduction to Writing in Mathematics

Writing to Learn in Mathematics

It is critical that the mathematics curricula of the 21st century incorporate opportunities for students to become effective communicators. The National Council of Teachers of Mathematics (NCTM) highlights the importance of communication in their standards.

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- analyze and evaluate the mathematical thinking and strategies of others; and
- use the language of mathematics to express mathematical ideas precisely.

Writing to learn in mathematics will benefit students in a number of ways. The practice promotes clear thinking, helps students make connections between math and other disciplines, and it raises the students' awareness of what is known and not known when problem-solving. Writing to learn also helps students raise questions about new ideas, organize their thinking, and perhaps most importantly, it helps students construct meaning out of complex material.

The practice of writing to learn is also beneficial to teachers because it provides invaluable insight into the students' thinking and level of understanding. Those teachers who add writing to their pedagogy will often find it easier to recognize and diagnose their students' conceptual problems. The time teachers invest in helping students clearly explain their thinking will be recouped in instructional time later on as lessons become more prescriptive in nature. Students too often remark, "I can do it but I can't explain it." What is the value to the student to "do" math if they cannot communicate about it? The number one issue separating successful college students and potential employees is the ability to communicate effectively. It will not be enough for 21st century students to master the concepts of mathematics. They will need to be equipped with the skills to successfully communicate highly technical information.

The syntax, vocabulary, and structure of writing in mathematics are unique and the mastery of them requires direct explicit instruction and ongoing opportunities to practice. While some students will resist writing in math class, they need to understand that writing is the single most powerful tool for thinking and learning.

The contemporary mathematics curricula offers a wide variety of entry points for Writing to Learn activities. Daily note-taking, journals, quickwrites, extended response questions, reports, portfolios, and learning logs are only a sampling of the formal and informal writing opportunities in the mathematics classroom.

Getting Started

Start small. Incorporate Cornell note-taking into the daily routine. Try quickwrites or a journal with a particular unit and utilize a rubric to help students understand what it takes to "write like a mathematician." Spend time modeling technical writing and formal oral communication. Students will experience the use of the language of mathematics, begin to understand the kind of writing that is expected of them, and know that it will support their learning. Encourage students to talk about mathematics and to practice their academic language. Use prompts to guide writing and critical reflection. Scaffold activities that will lead to formalized technical writing.

Becoming Better Communicators

Students take a great risk putting their thinking on paper and potentially exposing their lack of deep understanding of what they are doing and why they are doing it. They must simultaneously wrestle with style and content while making public their ideas. For students who do not understand a math concept or a problem solution, the process may seem overwhelming. They are trying to explain new material that they do not fully understand to someone whose understanding is greater than their own. For students who think they do understand the math, the process seems wasteful. "I know what I am doing; I can show my work symbolically; why do I have to talk about it in words?" It is up to the teacher to help such students realize the benefit that writing provides even when it exposes their areas of limited understanding or confused thinking. The mathematics teacher can emphasize that writing helps students remember what they are learning as they find connections with prior knowledge, and that writing about mathematics will be necessary to communicate with others whether inside or outside the mathematics community.

Rubrics

Many writing activities include holistic scoring guidelines that students can use to shape their writing and to evaluate their own work and the work of other students. Prewriting for many assignments might be prefaced with discussion of the rubric. Once students are familiar with rubrics, asking them to develop rubrics of their own can be a valuable exercise. The more students learn to use a well-developed rubric to guide their writing, the easier it is for a teacher to assess student work, thus encouraging both teacher and students to spend more time writing. A general scoring rubric is provided here as a guide in developing an agreed upon rubric in the classroom.

General Scoring Rubric	
4 Points	• Contains a complete response with a clear, coherent, and unambiguous explanation. • Includes a clear and simple diagram, if appropriate. • Communicates effectively to an identified audience. • Shows an understanding of the question and the mathematical ideas/processes. • Identifies all the important elements of the question. • Includes examples and counterexamples. • Gives strong supportive arguments.
3 Points	• Contains a good response with some, but not all of the characteristics outlined above. • May include an incomplete diagram. • The identified audience may be unclear and the ideas are communicated less effectively. • The understanding of the question and the mathematical ideas/processes is not clear. • Includes examples but counterexamples may not be included or may be unclear. • May include minor errors of execution but not of understanding.
2 Points	• Contains a complete response, but the explanation is muddled. • Presents an incomplete argument. • Includes diagrams that are inappropriate or unclear, or the response fails to provide diagrams when it would be appropriate. • Indicates some understanding of the mathematical ideas/process, but in an unclear way. • Shows clear evidence of understanding some important ideas/processes, while also making one or more fundamental errors.
1 Point	• Omits parts of the question in the response. • The response includes major errors or incorporates inappropriate strategies.

Models

Models, anchor papers, and templates can be powerful teaching tools. Ask students to read the models for ideas. Engage them in critiquing the work. This will help them develop a map for their own writing. The *Problem-Solving and Guide for Solution Write-up* is provided as a model for developing an agreed upon template for the classroom.

Problem-Solving and Guide for a Solution Write-up

SOLVING THE PROBLEM

I. Read the problem carefully again and again.

II. Restate the Problem

- Use your own words.
- Include all parts of the problem including specific information such as lengths of line segments, size of angles, number sets mentioned, etc.
- Include any given figures and draw them to scale.
- Include any given formulas.

III. Search for a Solution

- Identify words and symbols you don't know. Look up their meanings and write them down.
- Think about the problem
- Use a graph, a data table, algebraic reasoning, technology or a combination of these to investigate the problem.

WRITING THE SOLUTION

IV. Describe Your Solution Process

- Describe how you got started.
- What strategies have you used to solve the problem?
- Include any charts, graphs, lists, geometric figures, drawings, manipulatives that you used or created.
- If you only found a partial solution, state what it is.
- If you know your solution is incomplete or wrong, explain how you know.
- If you have a general solution, state what it is and support it with specific examples.
- State whether you think there could be other correct solutions and support your position.

V. Reflect about What You Have Learned

- What mathematics was required to be able to solve the problem?
- What advanced mathematics helped you solve the problem in a more sophisticated way?
- How, if at all, does this problem relate to previous work you have done?
- How and when did collaboration with peers assist you in reaching a solution?
- What "real-world" applications might there be for problems like this?

Individual and Collaborative Writing

Quickwriting

Through quickwriting activities, students can explore ideas without fear of criticism and without the premature editing that can inhibit expression. Writing encourages writing. By reducing anxiety about writing and producing material that can become a foundation for further writing, quickwriting is an excellent tool for prompting the thought and focus central to the entire writing process.

Clustering

Based on the premise that working with the natural rhythms of the brain to create writing produces work that is rich in meaning, clustering is a nonlinear brainstorming process that helps writers discover the ideas and patterns of organization that characterize strong writing. The practice not only promotes creative writing, but it also produces material that is abundant in memories, metaphor, and wholeness, and as such, its application to the academic terrain of analytical writing (or even test review) is equally impressive.

Listing and Grouping

Listing helps students recall what they already know about a topic and discover what they may need to find out about it. Lists alone sometimes suffice as a prewriting activity. There may be other occasions when students can use their lists to generate groupings of items that may be helpful as they consider how to organize the information they will write about.

Guided Reflection

Guided reflection can take the form of class discussion or individual student conferences with the teacher and/or tutors about the writing in which students are engaged.

Choosing a Topic

Some assignments are designed to provide students with exposure to a variety of discourse modes and topics. Students may need some coaching to identify and narrow topics, but experience with defining topics will be good practice for the many times in college when students will be expected to do so.

Choosing an Audience, Purpose, and Format

Activities that require students to write for different audiences and purposes greatly enhance the legitimacy of the writing process. By providing students with flexibility and choice in audience, purpose, and format, writing takes on the dimension of a real world venture, and the decision-making process required by such variety produces energetic revision and refreshing writing. Discussions that address audience, purpose, and form remind students of the importance of the choices they make as writers.

Planning

Students often benefit from help planning the organization of their writing. Strategies like outlining and grouping, as well as individual conferences with other students, tutors, and teachers are helpful. Making available copies of various forms of graphic organizers reinforces the importance of planning writing before drafting.

Individual and Collaborative Drafting

Done individually, drafting produces writing that is ready for response and revision. Done collaboratively, drafting incorporates a variety of points of view forged into writing that is also ready for response and revision. *Note:* Students often need reminders that during the drafting stage of the writing process they should focus on content and logical organization rather than mechanical correctness.

Individual and Response Group Editing

Response to drafts and revision comprise a stage of the writing process frequently abandoned by students working too close to deadlines and teachers unfamiliar with classroom groupings and/or guidance of students as they learn to respond to each other's work. The college preparatory class provides a perfect opportunity to help students integrate this portion of the process into their work with writing.

As frightening as it may be for students, responding to each other's work and soliciting critique of their own work helps them to develop a sense of audience that reaches beyond the teacher alone. By giving and receiving responses, students begin to think of themselves as writers whose work must communicate with others. If students are taught the characteristics of good writing—instruction that often demands the direct involvement of teachers—and are convinced that their commentaries are met with appreciation and often with tangible results in the evaluation stage, they can gain the courage to contribute to the successful writing of others. As students read each other's work, they become better at identifying good and bad writing. They become better writers.

Oral Response Groups

Groups that mix students by strengths and areas needing improvement in writing can provide consistency of commentary as students work through revisions of various writing assignments and prompt unusual unity around the task of writing. Changing group composition by assignment and/or to blend experienced writers with novices can afford students a variety of responses to their writing and increase the camaraderie of an entire class. Students are often self-conscious reading their writing aloud; they are sometimes inclined to dismiss their work or apologize for it with comments like, "This is just a first draft," or "My baby brother kept me up until four this morning, so this isn't very good." Since response groups should be safe places for even the most unpolished writing, students may need reminders that "no apologies" should precede reading their work.

Another tendency of some writers when presented with critique is to become defensive or to try to explain what they intended the essay to say. Students need guidance to relax into the safety of accepting critique as helpful without feeling the need to defend or explain their work. Finally, students often need a great deal of coaching to phrase specific and meaningful commentary. Lacking self-confidence or fearful of hurting feelings, students sometimes slip into excessively general commentary such as, "It's good," or "It just needs a little work," which doesn't give writers much information to shape revision. Have listeners write down comments as they listen. Then they can provide specific oral comments that provide useful information to the writer. Helping students with phrasing, directly addressing some of the causes of unnecessarily general commentary, and modeling effective commentary by joining groups during response sessions can contribute greatly to the honesty and thoroughness of student critique.

Written Response

When oral response groups are not feasible, or as a supplement to oral response group commentary, written critique can provide ideas for revision. Students should be encouraged to frame comments similar to those expressed in response groups directly on each other's papers. Written commentary can be reviewed by individual writers and discussed with other students, tutors, or the teacher as students prepare for revision.

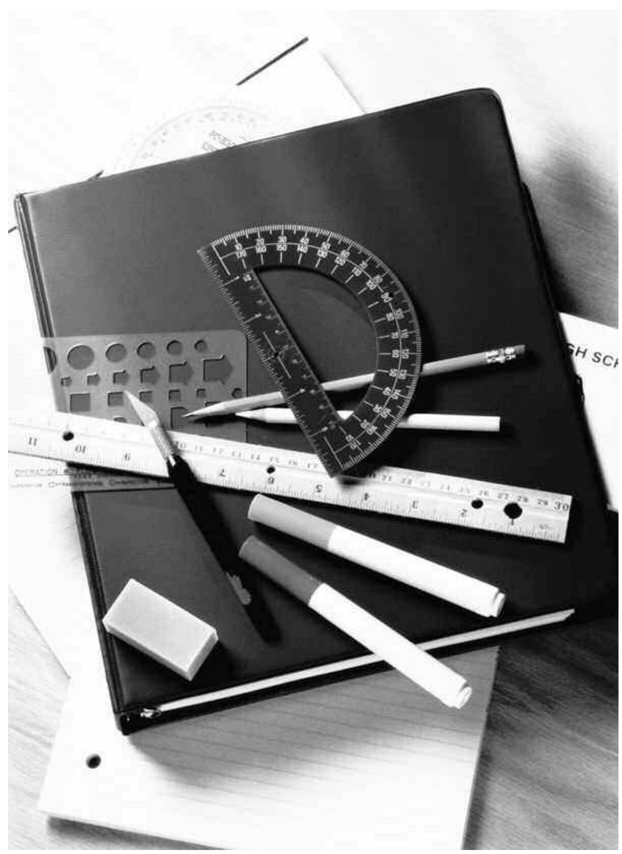
Revision allows students to make use of the comments they receive from response groups and written critique and to redefine what it is that they want to communicate in a piece of writing. For many students, revision is a painful process. Nevertheless, revision is evidence that a writer really cares about a piece of writing. As students revise, they need to revisit the critical areas of audience, purpose, and form. In addition, they need to review the components of writing pertinent to the type of writing they are producing. Revision is a teachable skill. Students benefit from reminders of their power as writers: While critique should be considered seriously, revision need not automatically respond to all suggestions for revision mentioned during critique. Part of the integrity of the writing process is assuming ownership for the decisions made that acknowledge critique but preserve the writer's own vision. Final editing for cosmetics familiarizes students with standard mechanics and guides them toward submitting writing that is free from mechanical distractions.

AVID writing lessons adhere to the following "writing process" sequence:

- Study and discussion of samples
- Pre-writing
- Drafting
- Peer evaluation
- Revision
- Peer evaluation
- Final revision
- Teacher evaluation
- Discussion and revision
- Reflection...

Technical Writing Tips for Mathematics

- Use a word processor and equation editor
- Use complete sentences (begin with a capital letter; use a subject and verb)
- Start with a short summary when writing a report
- Use your textbook as a guide for style and syntax
- Don't start a sentence with a symbol
- Use standard and accepted symbols
- Do not use symbols in a sentence unless they are part of an equation
- Do not insert words in place of symbols in equations
- Leave a double space between a symbol and text
- Generally expressions used as nouns are singular and the verb should agree
- State a theorem and provide an example before proving it
- Use headings to divide your paper into different sections
- Number all tables and write a caption for each table
- Number all figures in a separate numbering system from the tables
- There is no substitution for formal mathematical terminology
- Write variables in *italics* or **bold** face
- Avoid starting a sentence with a number
- Spell out numbers that begin a sentence
- Use care, pronouns can confuse the reader
- Avoid being judgmental, i.e., "It is easy to see that..."
- Place math expressions centered on their own line
- Equations are dependent clauses
- Check calculations with care
- Create an outline
- Double space
- Use active verbs



1.1: Cornell Notes

Topic

- Refining Cornell note-taking in Mathematics

Objectives

Students will:

- Become familiar with refinements to the Cornell note-taking system
- Develop specific note-taking skills
- Develop facility in writing questions that synthesize the notes' content
- Develop skills in writing connections, summaries, reflections, and analyzes of daily notes
- Develop strategies for interacting with the content of their notes
- Understand how to use Cornell Notes to review and study for exams

Timeline

- One 50-minute class period to review and further develop student usage of Cornell Notes

WICR Strategies

Writing to Learn

- Write notes in a two-column format
- Write questions that synthesize the content of the notes
- Write daily summaries of notes

NCTM Standards

Communication

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- analyze and evaluate the mathematical thinking and strategies of others; and
- use the language of mathematics to express mathematical ideas precisely.

Connections

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- recognize and use connections among mathematical ideas;
- understand how mathematical ideas interconnect and build on one another to produce a coherent whole; and
- recognize and apply mathematics in contexts outside of mathematics.

Representation

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- create and use representations to organize, record, and communicate mathematical ideas;
- select, apply, and translate among mathematical representations to solve problems; and
- use representations to model and interpret physical, social, and mathematical phenomena.

Rationale

Cornell Notes is a cornerstone strategy in AVID world. Practitioners of *The Write Path I: Mathematics* Teacher Guide have been introduced to the Cornell note-taking system originated by Walter Pauk at Cornell University. The system is based on research done in the area of memory and learning theory. It is a very valuable system because it takes students through the cycle of learning. It is much more than just a way to record information. Teaching Cornell Notes will take time; however, it will be one of the most valued skills that students will take with them when they enter college. In AVID-inspired classes, note-taking is considered an essential skill and therefore will improve with time and practice. The Cornell note-taking system incorporates what students do with their notes once they have taken them. By using Cornell Notes consistently, students learn to see writing as a tool for learning in mathematics as well as other subject areas.

The Cornell note-taking system is a system that students need to be taught and to review regularly. It is not intended to change how teachers deliver information, but rather how students record and interact with that information.

The refinements to the Cornell note-taking system presented here focus on the specific needs of the mathematics classroom.

Vertical Alignment

- Formal note-taking in mathematics is a critical skill that can be introduced and refined at all levels. In school districts that make note-taking a priority at middle and high school levels, the students' development of skills generated by Cornell Notes are clearly articulated from grade 6 through grade 12 through a vertical team process.

Materials/Preparation

- *Student Handout 1.1a:* "The Cornell Note-taking System"
- *Student Handout 1.1b:* "Tips for Studying with Notes"
- *Student Handout 1.1c:* "Power Math Notes"
- *Student Handout 1.1d:* "Interactive Student Notebooks"
- *Student Handout 1.1e:* "Common Math Abbreviations"
- *Student Handout 1.1f:* "GIST Summary"
- *Student Handout 1.1g:* "Higher-Level Reflections"
- *Student Handout 1.1h:* "Cornell Note-taking Checklist"
- *Student Handout 1.1i:* "STAR Note-taking Strategy"

- *Student Handout 1.1j:* “Taking Cornell Notes—Some Tips”
- *Student Handout 1.1k:* “Two-Column Quiz”
- *Student Handout 1.1l:* “Two-Column Homework”
- *Student Handout/Overhead Transparency 1.1m:* “Student Samples of Cornell Notes”
- Initially, students should be provided with *Student Handouts 1.1a, 1.1c,* and *1.1d* to help them with the structure of Cornell Notes. Once the routine has been established, no special materials are required.
- Review the Active Learning Methodologies (see the *Introduction*).

Instructions

- Ask students to complete a quickwrite explaining how they learned to take notes.
- Use “Popcorn” or another Active Learning Methodology activity for students to share their training in note-taking.
- Ask students if anyone has taught them what to do with their notes once they have taken them.
- Brainstorm with students why taking notes might be a good skill to learn.
- In pairs or groups, ask students to list some effective ways to use notes.
- Explain to students that there are several skills needed to become an effective note-taker, for example:
  — Know what to write down.
  — Be able to listen to what the teacher says and write it down at the same time.
  — Learn how to use abbreviations.
  — Use symbols and/or indentations on the note page to organize notes while writing.
  — Know what to do with notes after taking them (see *Student Handout 1.1b:* “Tips for Studying with Notes”).
- Distribute *Student Handout 1.1a:* “The Cornell Note-taking System,” *1.1c:* “Power Math Notes” or *1.1d:* “Interactive Student Notebooks” to teach students how to set up their paper for Cornell Notes and identify the major elements of the note-taking format.
- Use the selected *Student Handout 1.1a, 1.1c,* or *1.1d* to give a 7–10 minute lecture.
- Ask students to take notes on the right hand side of their paper.
- Ask students to pair-share their notes with a partner and encourage them to add to their notes if they missed any information. Train students to use a different color of ink when they add to their notes so that they can see what they missed.
- Survey the class to see if anyone used abbreviations and share those with the whole class. See *Student Handout 1.1e:* “Common Math Abbreviations” for a list of commonly used abbreviations. Encourage students to begin to create their own list of abbreviations. Asking students to think about how they use abbreviations in “instant messaging” may help them understand this concept.
- Ask students to highlight the main idea(s) and key words.
- Teach students how to use the questioning column. Students should generate questions that can be answered with their notes on the right, and may be possible test or quiz questions.

- Ask students to write one to three questions.
- Ask a few students to share their questions with the whole class. (There should be duplication and/or overlap).
- As students become familiar with Bloom's Taxonomy or Costa's Higher Levels of Questions, you may require that they include the higher-level questions in their notes. Initially students will most likely write Level One or Level Two questions (see section on "Inquiry" for more information).
- Share a well-written reflection to model the characteristics of a "good" reflection. Describe how the reflection provides a "big picture" and ties the main ideas together to reflect learning.
- Distribute and review *Student Handout 1.1f:* "GIST Summary."
- Provide time for students to work in pairs or individually to write their summaries.
- Distribute *Student Handout 1.1g:* "Higher-Level Reflections" and ask students to review and rewrite their reflections at a higher level. Ask students to share their reflections with the whole class. As they do this, take time to point out which parts are the most effective. Teaching students effective summarizing will take time, but ultimately this skill will improve their writing overall. Continue to reinforce the difference between the re-telling of information versus connecting the main ideas to show new learning.
- Distribute *Student Handout 1.1h:* "Cornell Note-taking Checklist" and review each of the rubric's descriptors.
- Ask students to complete a self-assessment of their notes using the rubric and then compare it with an assessment by a partner using the rubric.
- Survey the class to see how students scored. Discuss how taking notes is a skill and the expectation is that students will improve with time.
- Provide students with opportunities to practice using the rubric provided on *Student Handout 1.1h:* "Cornell Note-taking Checklist."
  - — Distribute copies of the rubric
  - — Post copies of the sample math notes or other exemplars.
  - — Divide students into small groups.
  - — Ask groups to use the rubric and reach a consensus score for the each of the sample/exemplar notes.
  - — Lead a class discussion about what characteristics contributed to "good" notes.
- Collect student notes regularly and grade them using the rubric previously introduced.
- Review and problem-solve difficult note-taking situations.
- Emphasize note-taking skills consistently and hold students accountable for all of the steps they need to do on their own outside of class to get the most use of their notes. Use *Student Handout 1.1i:* "STAR Note-taking Strategy" and *Student Handout 1.1j:* "Taking Cornell Notes—Some Tips" to scaffold student note-taking skills.
- Encourage "Habits of Mind" by using Cornell-like structures for quizzes, tests, and homework. See *Student Handout 1.1k:* "Two-Column Quiz" and *Student Handout 1.1l:* "Two-Column Homework."

Higher-Level Questions

Level Two

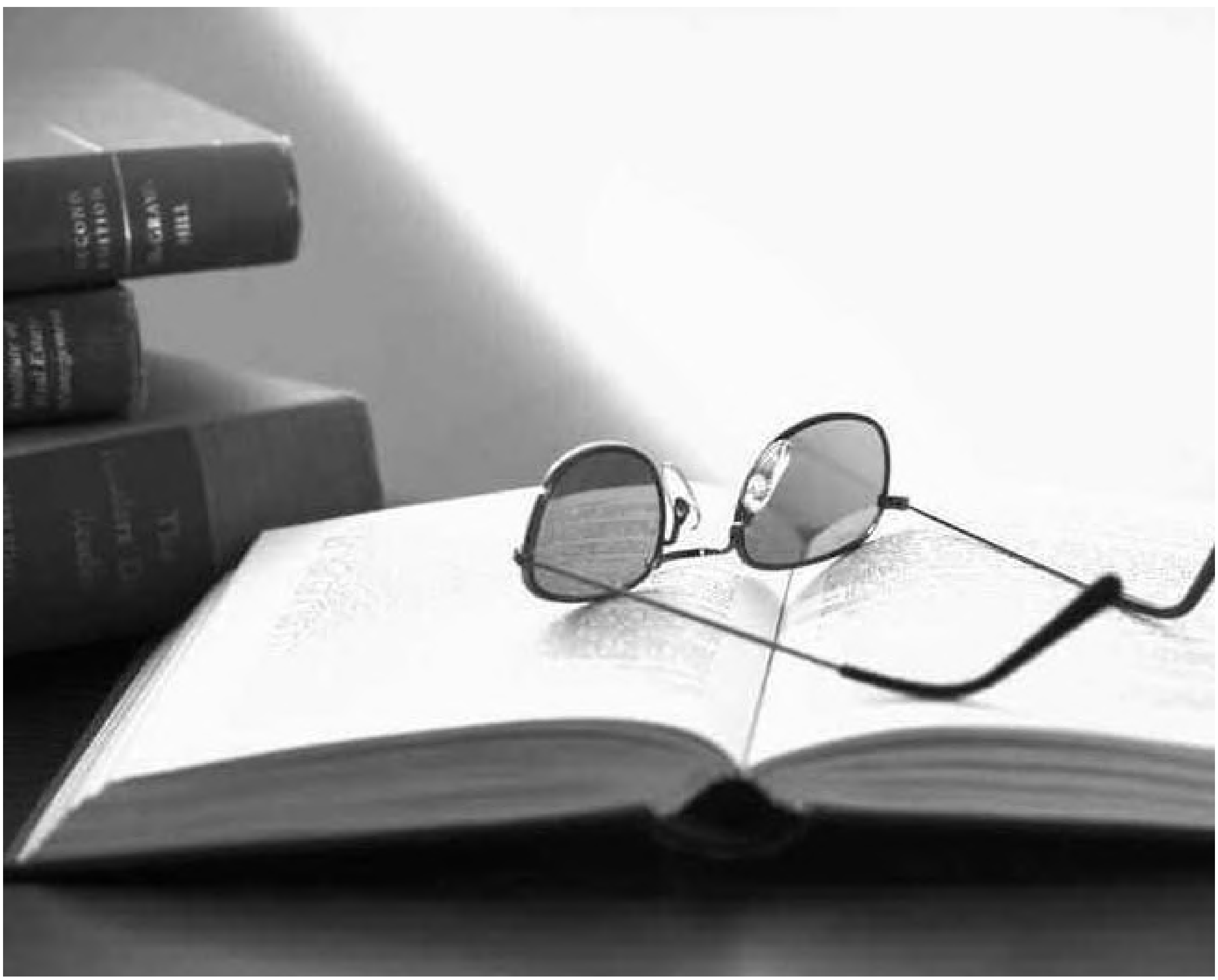
- Compare Cornell note-taking system with traditional notes.

Level Three

- Describe how Cornell note-taking skills might be useful in a practical or "real-world" setting.

Formative Assessment

- Use a Cornell Notes' rubric occasionally to give students feedback on their notes.
- Have students use the rubric to peer check another student's Cornell Notes.



Name: ________________________ Quarter: ____________

Begin Date: ________________________ Period: ____________

The Cornell Note-taking System

What are the advantages?

Three Advantages:

1. It is a method for mastering information, not just recording facts.
2. It is efficient.
3. Each step prepares the way for the next part of the learning process.

What materials are needed?

Materials:

1. Loose-leaf paper or graph paper to be kept in binder.
2. 2½ inch column drawn at left-hand edge of each paper to be used for questions.
3. 3–4 lines left at the bottom of page for connections, summary, reflection, analysis section.

How should notes be recorded?

During class, record notes on the right-hand side of the paper:

1. Record notes in paragraphs, skipping lines to separate information logically.
2. Don't force an outlining system, but do use any obvious numbering.
3. Strive to get main ideas down. Facts, details, and examples are important, but they're meaningful only with concepts.
4. Use abbreviations for extra writing and listening time.
5. Use graphic organizers or pictures when they are helpful.

How should notes be refined?

After class, refine notes:

1. Write questions in the left column about the information on the right.
2. Check or correct incomplete items:
   - Dates, terms, names.
   - Notes that are too brief for recall months later.
3. Read the notes and underline key words and phrases.
4. Read underlined words and write in recall questions in the left-hand column (use key words and very brief phrases that will trigger ideas/facts on the right). These are in addition to the questions.
5. Write a reflective paragraph about the notes at the bottom of the page.
6. If possible, compare notes with a study buddy.

What are the ways to recite notes?

Recite notes three ways:

1. Cover up right side of page. Read the questions. Recite information as fully as possible. Uncover the sheet and verify information frequently. This is the single most powerful learning tool!
2. Reflect on the organization of all the lectures. Overlap notes and read recall cues from the left side. Study the progression of the information. This will stimulate categories, relationships, inferences, personal opinions/experiences. Record all of these insights! REFLECTION = KEY TO MEMORY!!
3. Review by reciting, reflecting, and reading insights.

What are the six steps of this system?

This system in brief:

1. Record lectures in the main column.
2. Within 8 hours, read over notes to fill in gaps and to make notes more legible.
3. Identify main ideas and write recall questions.
4. Recite by covering main column and expanding on recall cues. Then verify.
5. Write a connections, summary, reflection, analysis at the bottom of the page.
6. Review your notes regularly. Short, fast, frequent reviews will produce better understanding and recall.

Tips for Studying with Notes

Study/Review Questions:	Topic: *Tips for Studying with Notes*
How can the format of the cornell notes help with studying for tests?	• *Spread out or hold notes so that right side of page is covered; review ideas and answer study questions from the left-hand column; use right-hand section as an answer key.* • *Engage in an oral quiz with others using study questions from the left-hand column.* • *Cover the right-hand column with blank paper; write out answers to the left-hand study questions and explanations of main ideas.*
What should you write in the summary/reflection section?	• *Write summaries and reflections about the most important material in the summary/reflection section.* • *Write a quiz for others using the notes; exchange and correct.* • *Write anticipated test questions beyond those already in the left-hand column and write answers to the questions.*
How should you use notes for review?	• *Look over notes frequently to keep information and questions still unanswered fresh in mind.* • *Recite information from notes.*
How can you use notes when studying in a group?	• *Exchange notes with others to flesh out information and understanding.* • *Use notes in study groups to provide a common ground of material for reference and review. Rewrite notes if necessary.*

Connections, Summary, Reflection, Analysis:

Power Math Notes

Name: ______________________ Date: ______________________

Title: ______________________

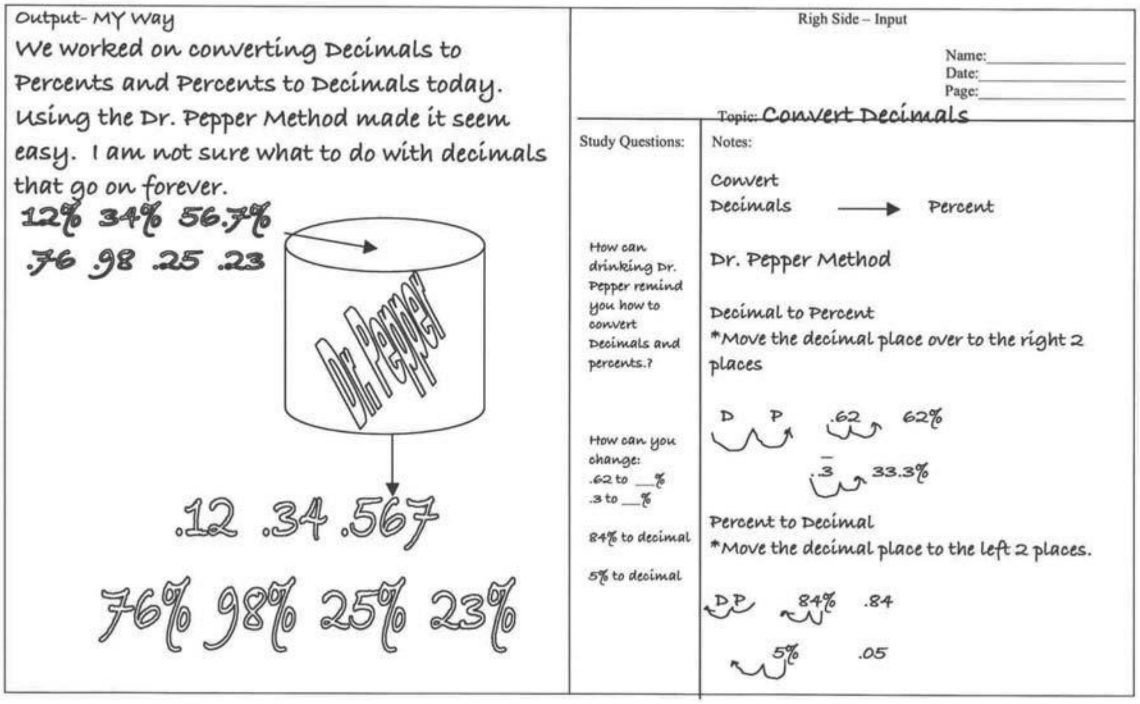
Tool Box:

Connections, Summary, Reflection, Analysis:

Study Questions:

Adapted from the Cornell note system by: James O. Donohue (2003)

Interactive Student Notebooks



What are Interactive Student Notebooks?

Note-taking is an engaging and interactive activity when done thoughtfully. You can become involved with the material by making charts, illustrating concepts, creating graphic organizers, making connections between concepts and the real world, and applying knowledge to new situations. Interactive Notebooks will encourage you to use critical thinking skills to organize and contemplate new ideas. The right side, referred to as the Input side of student notebooks, looks much like normal Cornell Notes. However, the left side, or the Output side, is used for processing new ideas and will be colorful, creative, and engaging.

What are the advantages?

Four Advantages:

1. It is a method for mastering information, not just recording facts.
2. It is a strategy that encourages independent thinking.
3. It is efficient.
4. Each step prepares the way for the next part of the learning process.

What materials do I need?

Materials:

1. Loose-leaf paper to be kept in binder or spiral book.
    - When using a spiral book, leave several pages at the beginning for the table of contents, grade record, etc.

- Color pens/pencils, glue sticks or tape, and scissors

2. Two columns at the top of the page to be used for a "Tool Box" and for the reflection and connections.
3. 2½ inch (6.5 cm) column drawn at left-hand edge of each paper to be used for questions.

Must haves

1. A title page for the notebook
2. Taped or glued-in handouts
3. Title page for each unit containing the title of the unit and some relevant pictures and or symbols
4. A table of contents
5. Number all pages

How do I record the notes?

During class, record notes on the right-hand side of the paper:

1. Record notes in paragraphs, skipping lines to separate information logically.
2. Don't force an outlining system, but do use any obvious numbering.
3. Strive to get main ideas down. Facts, details, and examples are important, but they're meaningful only with concepts.
4. Use abbreviations for extra writing and listening time.
5. Use graphic organizers or pictures when they are helpful.
6. Use only one side of the paper for the Input—reserve the back side for the output of the next day's notes.

How do I refine the notes?

After class, refine notes:

1. Write questions in the left column about the information on the right.
2. Check or correct incomplete items:
   - Dates, terms, names.
   - Notes that are too brief for recall months later.
3. Read the notes and underline key words and phrases.
4. Read underlined words and write in recall cues in the left-hand column (key works and very brief phrases that will trigger ideas/facts on the right). These are in addition to the questions.
5. Write a reflective paragraph about the notes in the upper right reflection and connections box.
6. Complete the left, "Output" side of the notebook. (Back side of previous days notes.) Possible ideas for processing notes include:
   - Brainstorming
   - Concept maps

- Questions
- Process descriptions
- "Wanted" Posters
- Flow charts
- Matrices, clustering, Venn diagrams, T-charts or other graphic organizers
- Cartoons, caricatures
- Graphs
- News' article
- Drawing or illustration

7. If possible, compare notes with a study partner.

What are the ways to recite notes?

Recite notes four ways:

1. Cover up right side of page. Read the questions. Recite information as fully as possible. Uncover the sheet and verify information frequently (single, most powerful learning tool!)
2. Reflect on the organization of all the lectures. Overlap notes and read recall cues from the left side.
3. Study the progression of the information. This will stimulate categories, relationships, inferences, and personal opinions/experiences. Record all of these insights! REFLECTION = KEY TO MEMORY!!
4. Review by reciting, reflecting, and reading insights.

What are the seven steps of this system?

This system in brief:

1. Record lectures in the main column.
2. Within 8 hours, read over notes to fill in gaps and to make notes more legible.
3. Identify main ideas and write questions in left-hand column based on main ideas.
4. Recite by covering main column and expanding on recall cues. Then verify.
5. Write a reflection and connect the information to something you already know in the Reflection and Connections box.
6. Complete the left-hand output page of the notes using critical thinking and creative ways to process the new information.
7. Review your notes regularly. Short, fast, frequent reviews will produce better understanding and recall.

Left Side - Output

Student Directed:

- Interpretations and reactions
- Illustrations
- Cartoons
- Songs
- Poems
- Graphic organizers
- Apply knowledge to a new situation
- Use critical-thinking skills
- Colorful and varied representations of concepts
- Process new ideas in creative ways
- Flow charts
- Math graffiti
- Charts
- Sequence chains
- Advertisements
- Personal responses
- Wrestle with the new information—personalize it
- News articles
- Brainstorming
- Wanted Posters
- T-charts
- Matrices

Students must actively do something with the information before they can internalize it.

Everything on the left side is the student's work where they show how they have processed the information and used critical-thinking skills.

Right Side - Input

Name:______________________________ Date: __________

Title:______________________________

Tool Box:

Connections, Summary, Reflection, Analysis:

Study Questions:

Adapted from the Cornell note system by: James O. Donohue (2003)

Common Math Abbreviations

Name ______________________________ Date ______________ Period _____

Common Short Cuts For Note-taking—Abbreviations/Acronyms

For	4	Factorial	!
To	2	Difference/change	Δ
With	w	Therefore	$\therefore$
Without	w/o	Perpendicular	$\perp$
Within	w/i	Mean	μ
And	& or +	Pi	π
Minus	–	Theta – used for angles	θ
Equal/same	=	Sigma – standard deviation	σ
Not equal	$\neq$	Infinity	∞
School	sch	Union	$\cup$
No/not ever	$\varnothing$	Intersection	$\cap$
Part	prt	Then – implies	$\rightarrow$
Point	pt	Empty set	$\varnothing$
Be	b	Sum/summation	Σ
Between	b/w	Similar	~
Reference	ref	Approximately equal	$\approx$
Symbols	$> < \geq \leq$	Congruent	$\cong$
If and only if	IFF, $\longleftrightarrow$	Parallel	$\parallel$

Additional Suggestions

- Make names and titles into acronyms after writing them the first time.
- Write first few syllables of long words and complete the word when reviewing notes.
  - -- coll Collect
  - -- comm Communicate
- Write words deleting vowels until notes are reviewed.
  - -- spk Speak
  - -- commnct Communicate
  - -- commnty Community

Think of some of your own short cuts.

1. ______________________
2. ______________________
3. ______________________
4. ______________________
5. ______________________
6. ______________________
7. ______________________
8. ______________________
9. ______________________
10. ______________________

GIST Summary

Include the following in a GIST summary:

A. Explain what you are summarizing.
B. Describe the concept you are learning about.
C. Highlight or list five key phrases/words that encompass what the notes are about.
D. Use your five key phrases/words to write three to five complete sentences summarizing your notes.
E. Check your summary to be sure the details support the topic and the concept in your notes.

Topic: ____

____

Concept: ____

____

Highlight or list five key phrases/words:

1. ____
2. ____
3. ____
4. ____
5. ____

Write three to five sentences using the key phrases/words:

Higher-Level Reflections

Just as it is important to bring higher-level questions to the tutorial, it is equally important to write a higher-level reflection at the conclusion of the tutorial.

Costa's Levels of Questioning

Level 1	Level 2	Level 3
❑ complete	❑ compare	❑ evaluate
❑ define	❑ contrast	❑ generalize
❑ describe	❑ classify	❑ imagine
❑ identify	❑ sort	❑ judge
❑ list	❑ distinguish	❑ predict
❑ observe	❑ explain (why?)	❑ speculate
❑ recite	❑ infer	❑ if/then
❑ select	❑ analyze	❑ hypothesize
		❑ forecast

Student Samples

Level 1 Reflection

Today I learned that the perimeter of a polygon is the sum of the lengths of all its sides. Since a rectangle has 4 sides, and the opposite sides of a rectangle have the same length, a rectangle with sides 5 cm and 8 cm would have a perimeter of 26 cm. When I write my answer to a perimeter problem, I need to remember to indicate the specific units I'm using. *(Describe)*

Level 2 Reflection

The perimeter of a polygon is the sum of the lengths of all its sides while the area of a figure measures the size of the enclosed region of the figure. Area is expressed as square units whereas perimeter is not. For example, the perimeter of a figure would be centimeters while the area would be described as square centimeters. If a polygon has sides that measure 5 cm and 8 cm, the perimeter (5+5+8+8) would be 26 cm while the area of the polygon (5 x 8) would be 40 square cm. *(Compare and Contrast)*

Level 3 Reflection

The perimeter of a polygon is the sum of the lengths of all its sides while the area of a figure measures the size of the enclosed region of the figure. Area is expressed as square units whereas perimeter is not. For example, the perimeter of a figure would be centimeters while the area would be described as square centimeters. If a polygon has sides that measure 5 cm and 8 cm, the perimeter (5+5+8+8) would be 26 cm while the area of the polygon (5 x 8) would be 40 square cm. In my own life, I needed to know the perimeter of my poster paper for my science project when I was making a special border for it. My father asked me to help him calculate the area of our kitchen floor at home when he needed to find out how many tiles to buy. *(Evaluate/Generalize)*

Cornell Note-taking Checklist

Name____ Period ____________

Do your notes have the following characteristics?

1. Consistent Cornell physical format, notes dated and titled, readable — 3 pts
2. Use of abbreviations, key words/phrases, underlining, starring — 1 pt
3. Main ideas are easily seen; correct sequencing of information — 1 pt
4. Questions are completed on left hand side; Level 2 and 3 questions — 3 pts
5. An accurate, complete reflection follows the notes — 2 pts

Characteristics **Date**					
1. Consistent Cornell physical format, notes dated and titled, readable					
2. Use of abbreviations, key words/phrases, underlining, starring					
3. Main ideas are easily seen; correct sequencing of information					
4. Questions are completed on left hand side; Level 2 and 3 questions					
5. An accurate, complete reflection follows the notes					
Total Points					

Rubric

Consistent Cornell physical format, notes dated and titled, readable

3. Lines drawn to delineate areas for the tool box, reflection/connections, study questions and notes. The notes are titled. Notes are adequate length.
2. Minor problem with format
1. No date or no title; short
0. Fails to use Cornell note-taking format or date and title are missing or notes are inadequate in length

Use of abbreviations, key words/phrases, underlining, starring

1. Techniques used throughout
0. Too much verbiage

Main ideas are easily seen; correct sequencing of information

1. Information is complete and in correct order
0. Notes confusing

Questions are completed on left hand side; Level 2 and 3 questions

3. A substantive number of higher order thinking questions are noted in the left margin which are answered in the notes to the right
2. Level 1 questions are many; level 2 and 3 questions minimal
1. Level 1 questions only
0. No questions in the left hand margin

An accurate, complete reflection follows the notes

2. Detailed reflection covers the main topics of the notes
1. Reflection is generic or incomplete
0. Reflection missing

Name: ______________________________ Quarter: ____________

Begin Date: ____________________________ Period: ____________

STAR Note-taking Strategy

S = Set Up Paper

1. Put your name, period, class, and date in upper right-hand corner.
2. Give your notes a title.
3. Draw lines to delineate areas for the tool box, reflection/connections, study questions and notes.

T = Take Notes

1. PARAPHRASE the text or lecturer in the right-hand column.
2. Listen to decide which parts of the information are most important. Notice if the lecturer seems to stray from the topic.
3. Use whatever it takes to cue your own memory system. For example, use capital printing, underlining, arrows, or even pictures.
4. Don't get hung up on spelling. If you know what you mean, that is what counts. If you use this information later for another assignment or an essay, check for proper spelling then.
5. Use abbreviations that work for you. Develop your own shorthand.

A = After Class

1. Within five minutes of class, or as soon as humanly possible, edit your notes. Reread them looking for places to make additions, deletions, or clarifications.
2. Work with a partner to review your notes whenever possible.
3. Use a highlighter or underlining to emphasize important points.
4. Note any points that need to be clarified with the lecturer in the next session.
5. Finally, fill in the left-hand column with questions, icons, symbols, pictures, and memory keys.

R = Review Notes

1. Review notes regularly, after class, at least once a week.
2. Cover the right-hand column with blank paper. Read aloud or rewrite the right-hand column by using the cues in the left-hand column.
3. Paraphrase the answers.
4. Reflect by summarizing the notes, relating the subject to yourself, or relating the subject to personal experiences.

Taking Cornell Notes—Some Tips

Level 2: sort, infer, analyze, sequence, organize, solve, explain, compare, contrast, classify, isolate, characterize, make analogies.

Level 3: conclude, criticize, reorganize, justify, judge, estimate, predict, speculate, make a model, extrapolate, apply a principle, interpret, hypothesize, if/then

Name: ______________________

Class: ______________________

Period: ________ Date: ______________________

Topic *Note-taking Strategies*

Study/Review Questions	
How can you use the speaker's style to identify important points?	Become familiar with the speaker's style. Listen for important points that might be emphasized when the speaker: · Pauses or slows down · Repeats a point · Modulates the volume of her/his voice · Uses introductory phrases (e.g., "The four main points are" or "Note the relationship") · Writes on the board · Gestures or uses visual aids Write only the important ideas such as name, examples, terms, definitions, effects, evaluations, cross references: make it brief, but clear. Use abbreviations for familiar words.
How can you keep-up with the speaker?	**Speaker says:** "An altitude of a triangle is the perpendicular segment from a vertex to the opposite side or the line that contains the opposite side. An altitude can lie inside, on, or outside the triangle." **Notes say:** Altitude of Δ is $\perp$ from vertex to opp side or line contain opp side. · Can be inside, on or outside Δ
How should you use your notes to review?	Use notes to review: · Develop study questions and identify the main ideas. · Fill in details for clarity. · Look up and add to the definitions of new words/terminology. · Identify information that is unclear and/or questions that need to be answered; write and mark them so they can be easily found; get answers to the questions from other students and/or the speaker. · Add symbols to highlight important ideas and key words. · Delete irrelevant information. · Review the overall organization of the material: add symbols to make the organization clear or rewrite for clarity as needed. · Write a reflection about the significant ideas.

Connections, Summary, Reflection, Analysis

Three important note-taking strategies are reviewed in the notes. Identifying important points and main ideas, using abbreviations to paraphrase information provided during the class and the use of notes for review are outlined. The important cues in identifying main points and the use of questions to help with review are particularly helpful strategies as is the writing of summaries.

Two-Column Quiz

Topic: ______________________

Name: ______________________

Class: ______________________

Period: ________ Date: ______________________

Questions	**Show all your work and box the solution**

Connections, Summary, Reflection, Analysis:

Two-Column Homework

Topic: ______________________________

Name: ______________________________

Class: ______________________________

Period: ________ Date: ______________________________

Questions	Show all your work and box your solution

Connections, Summary, Reflection, Analysis:

Student Sample 1

Title: 5.1B

Tool Box;

Academic Goal:

Intercept form Quadratic function- $y=a(x-p)(x-q)$

If $x>0$ ______________

If $x<0$ ______________

Axis of symmetry $x=$ ____________

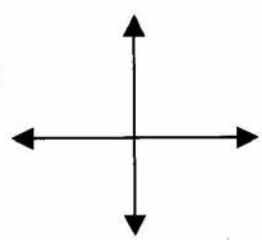
Study Questions:

Notes:

Example: Name the x-intercepts, axis of symmetry and vertex for: $y=\frac{2}{3}(x-7)(x+1)$

1. Find the x-intercepts. Let ____________ and solve using the ______________________.
2. Name the axis of symmetry and x-coordinate of the vertex. The axis of symmetry is ____________ between the intercepts. So, take the ______________ of the intercepts.
3. To find the y-coordinate of the vertex. Substitute the x-coordinate of the vertex into the equation and solve for y.
4. Plot the vertex and the intercepts and sketch the graph.

Example: Graph $y=-2(x-1)(x+1)$



Connections, Summary, Reflection, Analysis:

Student Sample 2

Name ______________________

Date ______________________

Title: Proving Lines are Perpendicular

Tool Box:

Outcome:

Learn and apply the concept of the negative reciprocal

Parallel lines – the slopes are equal.

Perpendicular Lines – the product of the slopes is negative one.

Connections, Summary, Reflection, Analysis: If the product of the slopes of lines is -1 then the lines are perpendicular. If the slopes are the same then the lines are parallel. Finding the slope of a line was review. The negative reciprocal was new. I am not sure about finding other solutions for other points on the line perpendicular to a given line and going through a point. There must be more than one.

Study Questions:

Flip the fraction – Reciprocal

$\frac{1}{2} \rightarrow \frac{2}{1}$ $\quad \frac{-2}{3} \rightarrow \frac{3}{-2}$

Can you give an example of negative reciprocals?

Negative Reciprocal $\frac{7}{2} \rightarrow \frac{-2}{7}$ only one has to be negative!!

$\frac{-6}{7} \rightarrow \frac{7}{6}$ If you multiply you get -1

$4 \rightarrow \frac{-1}{4}$

A(-2,-3) B(4, 5) C(-1,3) D(4, -1)

Are AB and CD perpendicular?

Is AB perpendicular to CD

A(-2, -3) B(4, 5) C(-1, 3) D(4, -1)

Slope AB $\frac{5+3}{4+2} = \frac{8}{6}$ Slope CD $\frac{-1+-3}{4+1} = \frac{-4}{5}$

Since $\frac{4}{3}$ *and* $\frac{-4}{5}$ are not negative reciprocals, the lines are not perpendicular. If the product of their slopes had been -1 then they would have been perpendicular.

Given E(0,5) F(5,3) can you find a point G such that it is on a line that is perpendicular to EF and goes through H(2,10)?

E(0,5) F(5,3) Find point G on a perpendicular line going through H(2,10).

Slope EF $\frac{3-5}{5-0} = \frac{-2}{5}$ slope perpendicular line $= \frac{5}{2} = \frac{10-y}{2-x}$

One choice (0,5)

Adapted from the Cornell note system by: James O. Donohue (2003)

Student Sample 3

Name ______________________

Date ______________________

Title: Box and Whisker Plot

Tool Box:

Outcome:
Understand and apply Box-and-Whisker Plots
Mean – Average
Median – Middle Number (median middle of street)
Mode – Most frequently occurring number
Quartiles – Arrange number set in order – divide into 4 equal parts
Box-and-Whisker Plot – Graphic Representation of Quartiles.

Connections, Summary, Reflection, Analysis:

Box-and_Whisker Plots can be used to analyze grades and to compare the free throw percentages between the eastern and western conferences of the WNBA. I am still unsure how the upper quartile of 78.5 was determined since you can not get a half point in basketball. This reminds of earlier in the year when we studied mean, median and mode.

Study Questions:

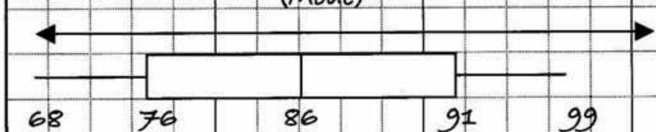
Given the data: 68, 72, 76, 81, 84, 86, 86, 89, 91, 95, 99 can you construct a Box-and-Whisker Plot?

Start by organizing the numbers in order from smallest to largest.

Example:

68, 72, 76, 81, 84, 86, 86, 89, 91, 95, 99

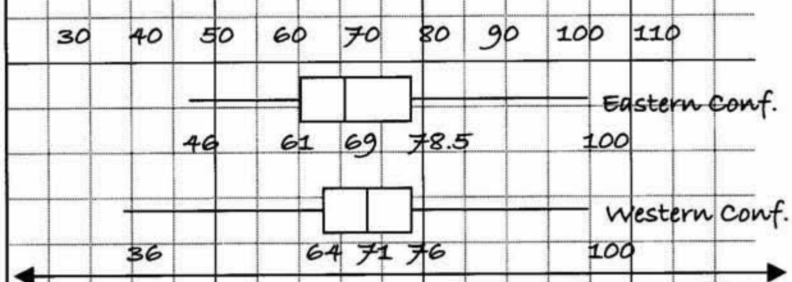
Lower Quartile — Median (Mode) — Upper Quartile



When would you use this kind of plot?

Example:

WNBA Free Throw Percentages



Adapted from the Cornell note system by: James O. Donohue (2003)

Student Sample 4 *(1 of 2)*

Review – Volume of Right Circular Cones

Tool Box:

Outcome

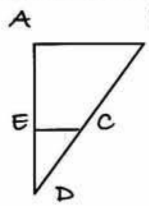
- Apply knowledge of cones
- Integrate knowledge of similar triangles.

Homework

- Finish the table of values, construct a graph and write a formal report interpreting the data.

Vocabulary/Formulas

- Circular Cone – A solid that has a circular base and a vertex that is not in the same plane as the base.
- Altitude or height – The perpendicular distance between the vertex and the base of the cone.



$\Delta ABD \sim \Delta ECD$

$$\frac{ED}{AD} = \frac{EC}{AB}$$

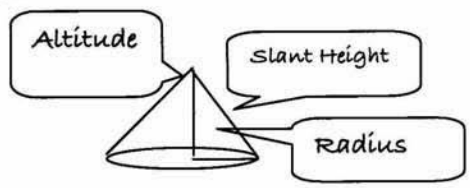
Connections, Summary, Reflection, Analysis:

The work was done in groups. Review work included finding the volume of cones given the radius and height. The use of similar triangle in cones was new information and added a step to the process. This could have been a stumbling block, but with the help of other groups it was easily overcome. It would be interesting to see if there are cone shaped water towers and to know how large they are. If there is an easier way to set up a relationship on the calculator it would also generate the needed graph.

Study Questions:

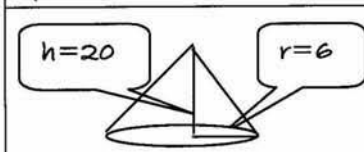
Brainstorm with a partner and write all that know about cones.

$$v = \frac{1}{3}\pi r^2 h$$



Work with a partner to draw/label a cone w/ r=6 cm and h=20 cm – show how to find the volume in terms of π

How can you find the volume of a cone r=6 and h=20?



$$V = \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3}\pi(6cm)^2(20cm)$$

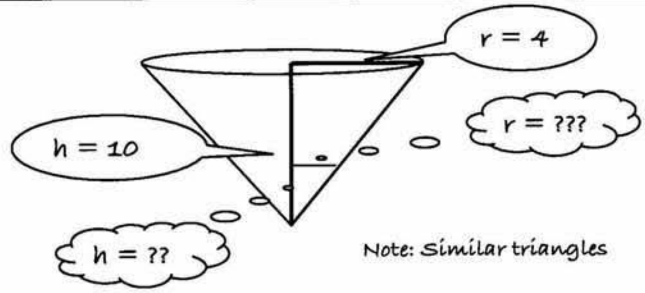
$$V + \frac{1}{3}\pi 720cm^3$$

$$V = 240\pi cm^3$$

Student Sample 4 *(2 of 2)*

How can you use similar triangles to find he height of the water? (r=4 h=10 fill rate 2 cubic meters per minute.)

Assume that the water tower is being filled at a rate of 2 cubic meters per minute. Work with a partner or two to construct a table in your notes that will help analyze the amount of water in the tower and the height of the water at the end of each of the first 10 minutes.



$r_1 = 4$

$h_1 = 10$

$\frac{4}{r} = \frac{10}{h}$

$4h = 10r$

$r = .4h$

$V = \frac{1}{3}\pi r^2 h$

$h = \frac{3V}{\pi r^2}$

$h = \frac{3V}{\pi(.4h)^2}$

$h = \frac{3V}{\pi .16h^2}$

$h^3 = \frac{3V}{.16\pi}$

$h = \sqrt[3]{\frac{3V}{.16\pi}}$

How can we be sure that the way we found the height is correct?

How fast would the fill rate have to be to fill the cone in 10 minutes?

Time - minutes	Volume – cubic meters	(*See below for the derivation of the height*) Height of water = $h = \sqrt[3]{\frac{3V}{.16\pi}}$
0	0	**0 m**
1	2	**2.285 m**
2	4	
3	6	
4	8	
5	10	
6	12	
7	14	
8	16	
9	18	
10	20	

1.2 Math Bookmark

Topic

- Developing a study tool

Objectives

Students will:

- Personalize a math bookmark
- Add key vocabulary and formulas to their bookmark
- Use the bookmark to set-up and utilize Cornell Notes
- Develop the math bookmark as a study aid

Timeline

- One 50-minute class period for students to create and begin entries on a personal math bookmark

WICR Strategies

- Writing to Learn
  — Write definitions, question starters, and study suggestions

NCTM Standards

Communication

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- analyze and evaluate the mathematical thinking and strategies of others; and
- use the language of mathematics to express mathematical ideas precisely.

Rationale

Constructing a *"Math Bookmark"* with reminders about levels or questions, a Cornell Notes' rubric, and study suggestions will help students review and personalize study and learning skills. It will also provide a daily reminder of these skills. Vocabulary bookmarks can also be created at the beginning of a new unit or chapter.

Vertical Alignment

- "Math Bookmark" can be done at any level and many students will choose to take their bookmarks along with key vocabulary and formulas forward to their next math class. The bookmarks will become important reference tools summarizing the concepts from each class.

Materials/Preparation

- *Student Handout 1.2a:* “Bookmark Sample”
- *Student Handout 1.2b:* “Bookmark Template”
- Color markers and pens
- Before class, duplicate *Student Handout 1.2b:* “Bookmark Template” on card stock; then cut-to-size and three-hole punch the cards following the handout example.
- Review the Active Learning Methodologies (see the *Introduction*).

Instructions

- Review the importance and value of setting-up Cornell Notes with a two and one-half inch (6.5 cm) left margin for questions.
- Discuss the advantages of having a ready reference for important vocabulary, formulas, question starters, and a rubric for self-assessment of Cornell Notes.
- Divide students into small groups.
- Distribute and review *Student Handout 1.2a:* “Bookmark Sample.”
- Distribute the precut and hole-punched *Student Handout 1.2b:* “Bookmark Template.”
- Provide students with time to design and personalize their bookmarks.
- Recognize exemplar bookmarks with a “Gallery Tour” or other sharing activity.
- Remind students to use their bookmarks as an aid in setting-up their Cornell Notes, when they are writing questions, or doing a self-assessment of their notes. Encourage them to add to their bookmarks when they learn a new vocabulary word or formula.
- Allow the use of the bookmarks from time-to-time during formative assessments.

Higher-Level Questions

Level Two

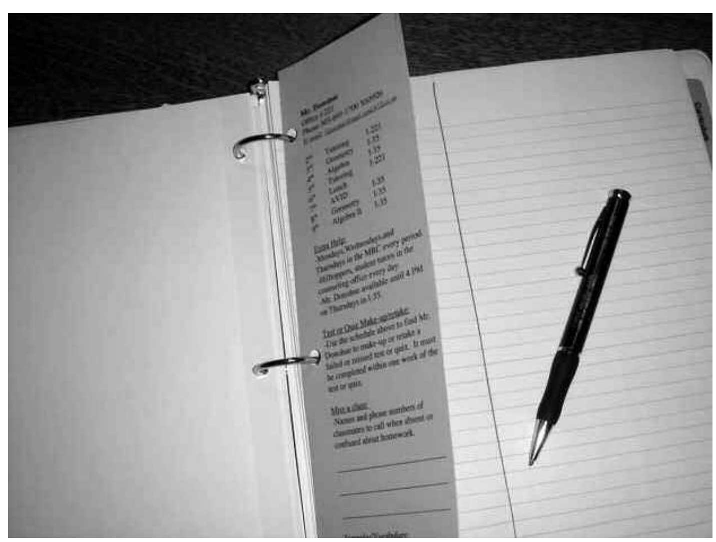
- What are the characteristics of a “good” study aid?

Level Three

- How can you decide which vocabulary words or formulas to add to the limited space on your bookmark?

Formative Assessment

- Review the question starters, rubric, and study aids that students include in their bookmarks.
- Review the vocabulary words, formulas, and other additions made to the bookmark during the course.



Bookmark Sample

Do your notes have these characteristics?

1. Consistent Cornell physical format, notes dated & titled, readable. — 3 pts.
2. Use of abbreviations, key words/phrases, underlining, starring — 1 pt.
3. Main idea are easily seen; correct sequence of information. — 1 pt.
4. Questions are completed on left-hand side: Level 2 and 3 questions. — 1 pt.
5. An accurate, complete reflection follows the notes. — 3 pts.

CHARACTERISTICS	PTS.
1. Consistent Cornell format	
2. Use of abbreviations, key words etc.	
3. Main ideas are easily seen?	
4. Questions are completed on left-hand side: Level 2 & 3	
5. An accurate, complete reflection	
Total Pts.	

RUBRIC

Consistent Cornell format:

3 pts — Vertical line 2.5" (6.5 cm) from the left hand margin heading is complete with name, date, subject. The notes are titled. Notes are adequate length.
2 pts — Minor problem with format
1 pt — No date or no title short
0 pts — Fails to use Cornell note-taking format or date and title are missing or notes are inadequate in length

Use of abbreviations, key words/phrases, underlining,

1 pt — Techniques used throughout
0 pts Too much verbiage

Main ideas are easily seen: *Sequencing of information.*

1 pt — Information is complete and in correct order
0 pts — Notes confusing

Questions are completed on left-hand side: *Level 2 & 3.*

3 pts — A substantive number of higher-order thinking questions are noted in the left margin which are answered in the notes to the right.
2 pts — Level 1 questions are many; level 2 and 3 questions minimal.
1 pt — Level 1 questions only
0 pts — No questions in the left-hand margin.

An accurate, complete reflection follows the notes

2 pts — Detailed reflection covers the main topics
1 pt — Reflection is generic or incomplete
0 pts — Reflection missing

Suggestions for terms to use and to be avoided.

Level 1 — questioning (needs to be avoided): defining, identifying, naming, label, listing, observing, reciting
Level 2 — questioning: analyzing, comparing, contrasting, grouping, inferring, sequencing, synthesizing
Level 3 — questioning: applying a principle, hypothesizing, imagining, judging, predicting, speculating.

Start—Name, Date, Warm-up, Terms, and Homework Assignment
Taking notes
After Class—review your daily notes.
Reflect—Write questions you have for class.

AVID Proven Achievement. Lifelong Advantage.

Do your notes have these characteristics?

1. Consistent Cornell physical format, notes dated & titled, readable. — 3 pts.
2. Use of abbreviations, key words/phrases, underlining, starring — 1 pt.
3. Main idea are easily seen; correct sequence of information. — 1 pt.
4. Questions are completed on left-hand side: Level 2 and 3 questions. — 1 pt.
5. An accurate, complete reflection follows the notes. — 3 pts.

CHARACTERISTICS	PTS.
1. Consistent Cornell format	
2. Use of abbreviations, key words etc.	
3. Main ideas are easily seen?	
4. Questions are completed on left-hand side: Level 2 & 3	
5. An accurate, complete reflection	
Total Pts.	

RUBRIC

Consistent Cornell format:

3 pts — Vertical line 2.5" (6.5 cm) from the left hand margin heading is complete with name, date, subject. The notes are titled. Notes are adequate length.
2 pts — Minor problem with format
1 pt — No date or no title short
0 pts — Fails to use Cornell note-taking format or date and title are missing or notes are inadequate in length

Use of abbreviations, key words/phrases, underlining,

1 pt — Techniques used throughout
0 pts Too much verbiage

Main ideas are easily seen: *Sequencing of information.*

1 pt — Information is complete and in correct order
0 pts — Notes confusing

Questions are completed on left-hand side: *Level 2 & 3.*

3 pts — A substantive number of higher-order thinking questions are noted in the left margin which are answered in the notes to the right.
2 pts — Level 1 questions are many; level 2 and 3 questions minimal.
1 pt — Level 1 questions only
0 pts — No questions in the left-hand margin.

An accurate, complete reflection follows the notes

2 pts — Detailed reflection covers the main topics
1 pt — Reflection is generic or incomplete
0 pts — Reflection missing

Suggestions for terms to use and to be avoided.

Level 1 — questioning (needs to be avoided): defining, identifying, naming, label, listing, observing, reciting
Level 2 — questioning: analyzing, comparing, contrasting, grouping, inferring, sequencing, synthesizing
Level 3 — questioning: applying a principle, hypothesizing, imagining, judging, predicting, speculating.

Start—Name, Date, Warm-up, Terms, and Homework Assignment
Taking notes
After Class—review your daily notes.
Reflect—Write questions you have for class.

AVID Proven Achievement. Lifelong Advantage.

Bookmark Template

1.3 Nonlinguistic Representations

Topic

- Representing information in imagery modes

Objectives

Students will:

- Select appropriate nonlinguistic models
- Demonstrate an understanding of various models for representing information in a nonlinguistic modality
- Use nonlinguistic patterns as a study and review strategy

Timeline

- One 50-minute class period for students to practice representing information in a nonlinguistic mode

WICR Strategies

- Writing to Learn
  - — Write a reflection of information provided in a nonlinguistic mode
  - — Use the development of nonlinguistic patterns as a prewriting strategy
  - — Use nonlinguistic representation as a part of daily note-taking strategies
- Inquiry
  - — Investigate new information in linguistic and nonlinguistic modes
- Collaboration
  - — Work in collaborative groups to investigate new information in linguistic and nonlinguistic modes
- Reading to Learn
  - — Read and transform information provided in a linear mode to a nonlinguistic mode
  - — Use signal words to identify text patterns and structure

NCTM Standards

Communication

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- analyze and evaluate the mathematical thinking and strategies of others; and
- use the language of mathematics to express mathematical ideas precisely.

Connections

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- recognize and use connections among mathematical ideas;
- understand how mathematical ideas interconnect and build on one another to produce a coherent whole; and
- recognize and apply mathematics in contexts outside of mathematics.

Representation

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- create and use representations to organize, record, and communicate mathematical ideas;
- select, apply, and translate among mathematical representations to solve problems; and
- use representations to model and interpret physical, social, and mathematical phenomena.

Rationale

Visual learning strategies, often referred to as nonlinguistic representations, are widely understood to improve student performance. Piaget first introduced the importance of developing schemata to enhance memory. Robert Marzano and others explain that "...knowledge is stored in two forms—a linguistic form and an imagery form... [and] the more we use both systems of representation—linguistic and nonlinguistic—the better we are able to think about and recall knowledge" (Marzano, R., Pickering, D. & Pollock, J. [2001]. *Classroom instruction that works: Research-based strategies for increasing student achievement.* Alexandria, VA: Association for Supervision and Curriculum Development).

Graphic organizers are the most common way to introduce nonlinguistic processing into the classroom. While they often incorporate linguistic components, they offer students with wide and varied alternative strategies for representing information. David Hyerle identifies, "six common patterns into which most information can be organized: descriptive patterns, time-sequence patterns, process/cause-effect patterns, episode patterns, generalization/principle patterns and concept pattern" (Hyerle, D. [1996]. *Visual tools for constructing knowledge.* Alexandria, VA: Association for Supervision and Curriculum Development).

The following patterns can be utilized to enhance student understanding of mathematical content:

Descriptive patterns can be used to represent facts and does not need to be in any particular order.

Time-sequence patterns organize information into sequences in which order matters.

Process patterns organize information/steps in to a network that leads to a specific outcome.

Episode patterns represent information about a specific event.

Generalization/principle patterns organize information into generalizations with supporting examples. Inductive and deductive arguments lend themselves well to this pattern.

Concept patterns organize information around a word, phrase or concept.

In addition to formalized nonlinguistic patterns, physical models (manipulatives), mental pictures, making illustrations, and engaging in kinesthetic activities all enhance student achievement and should be included in any discussion of nonlinguistic strategies.

Vertical Alignment

- Nonlinguistic representations should be introduced at an early age and refined as students mature.

Materials/Preparation

- *Student Handout/Overhead Transparency 1.3a:* "Nonlinguistic Models"
- *Student Handout 1.3b:* "Guiding Questions"
- *Student Handout 1.3c:* "Signal Words for Identifying Text Patterns/Structure"
- Math text
- Cornell Notes

Instructions

- Provide students with several good examples of completed nonlinguistic representations.
- Model the construction of a nonlinguistic representation.
- Guide students through the completion of a nonlinguistic representation.
- Assign a selection from the students' textbook or other mathematics text.
- Divide students into small groups.
- Distribute all three student handouts. Ask students to use *Student Handout 1.3a:* "Nonlinguistic Models" to brainstorm ideas for representing information in a nonlinguistic modality.
- *Note:* Do not provide copies of the nonlinguistic representative and simply have students "fill in" information. Rather, have students work collaboratively in developing their patterns for representing information.
- Provide students time to work individually and in their collaborative groups to organize information utilizing a selected model.
- Encourage students to clarify relationships between concepts.
- Provide many and varied opportunities for students to practice representing information in a nonlinguistic mode.
- Encourage students to use nonlinguistic patterns in their notes, to review information, to plan writing, and as a text-processing strategy.
- In addition to formal nonlinguistic patterns, model how and then ask students to represent information with physical models (manipulatives), mental pictures, illustrations, and by engaging in kinesthetic activities.

Higher-Level Questions

Level Two

- Why do nonlinguistic representations contribute to improved understanding of mathematic texts and concepts?

Level Three

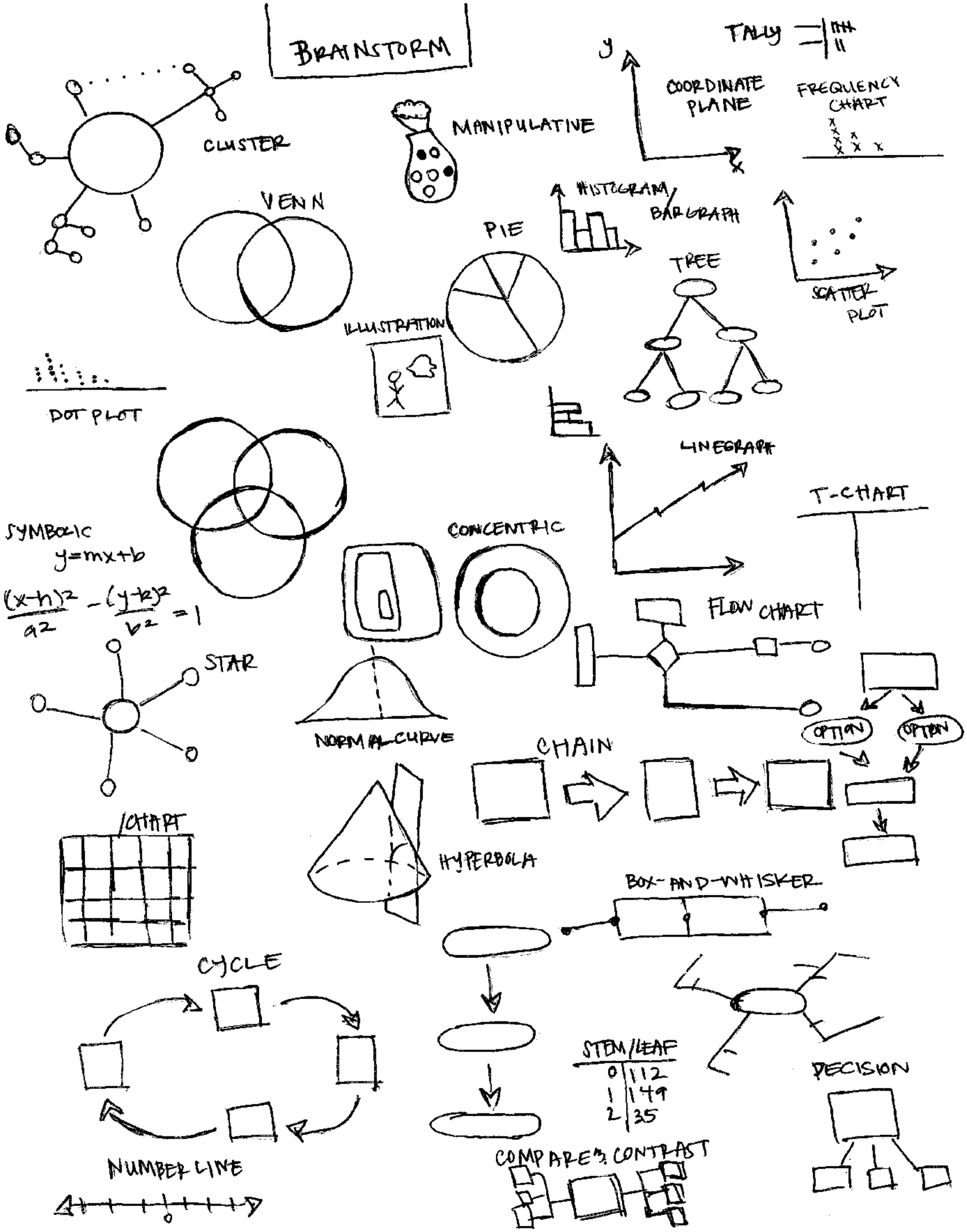
- How can the best nonlinguistic representation be determined?

Formative Assessment

- Did students select a pattern that best represents the text or content?
- Did students utilize *Student Handout 1.3b:* "Guiding Questions" and *1.3c:* "Signal Words" to assist them in determining an appropriate nonlinguistic model?
- Did student represent abstract ideas in a more concrete way?
- Is the information more comprehensible?

AVID
Proven Achievement. Lifelong Advantage.

Nonlinguistic Models



Guiding Questions

1. **Cause/Effect**
   - What is it that happens?
   - What causes it to happen?
   - What is the effect?
   - What are the important elements or factors that cause this effect?
   - How do these factors or elements interrelate?
   - Will this result always happen from these causes? Why or why not?
   - How would the result change if the elements or factors were different?
   - What is the cause/effect process the author is describing?
   - Why did a cause/effect structure emerge?

2. **Compare/Contrast**
   - What is being compared and contrasted?
   - What categories of characteristics or attributes are used to compare and contrast these things?
   - How are the things alike or similar?
   - How are the things not alike or different?
   - What are the most important qualities or attributes that make them different?
   - In terms of the qualities that are most important, are these things more alike or more different?
   - What can we conclude about these things or items?
   - What is the author comparing/contrasting?
   - Why is the author comparing/contrasting these things?
   - When did the comparison/contrast structure emerge?

3. **Problem/Solution**
   - What is the problem?
   - Who has the problem?
   - What is causing the problem?
   - What are the effects of the problem?
   - Who is trying to solve the problem?
   - What solutions are recommended or attempted?
   - What results from these solutions?
   - Is the problem solved? Do any new problems develop because of the solutions?

4. **Sequence/Chronological Order**
   - What is being described in sequence?
   - Why did a chronological order pattern merge?
   - What are the major steps in this sequence?
   - Why is the sequence important?

5. **Description/Definition**
   - What is being described?
   - What are its critical attributes?
   - What are the characters, places, and objects in the passage?
   - Why is this description important?
   - What is the concept?
   - To what category does it belong?
   - What are its critical characteristics/attributes?
   - How does it work?
   - What does it do?
   - What are its functions?
   - What are examples of it?
   - What are examples of things that share some but not all of its characteristics/attributes?

Signal Words for Identifying Text Patterns/Structure

Cause/Effect	Compare/Contrast	Problem/Solution	Sequence/ Chronological Order	Description/ Definition
because	however	problem	on (date)	for instance
since	but	the question is	not long after	to begin with
therefore	as well as	a solution	now	also
consequently	on the other hand	one answer is	as	in fact
as a result of	not only . . . but also	one reason for the problem	before	for example
this led to	either . . . or		after	in addition
so that	same as		when	characteristics of
nevertheless	in contrast		first	such as
accordingly	while		second	to illustrate
if . . . then	although		next	most important
thus	more than		then	another
subsequently	less than		last	furthermore
because of	unless		finally	first
in order to	similarly		initially	second
may be due to	yet		preceding	to begin with
effects of	likewise		following	
for this reason	on the contrary			
	different from			
	similar to			
	as opposed to			
	instead of			
	compared with			

Adapted from Rachel Billmeyer and Mary Lee Barton, *Teaching Reading in the Content Areas,* McREL, and Susan Davis Lenski, Mary Ann Wham and Jerry L. Johns, *Reading and Learning Strategies for Middle and High School Students,* Kendall/Hun.

1.4: Writing Prompts—Five Ws

Topic

- Writing for a variety of audiences for an specified purpose

Objectives

Students will:

- Construct well-written writing prompts
- Practice technical writing skills when responding to a teacher- or student-generated writing prompt

Timeline

- One 50-minute class period to explore constructing and decoding writing prompts

WICR Strategies

- Writing to Learn
  - — Use technical writing skills while constructing a writing prompt
  - — Use technical writing skills while responding to a writing prompt
- Collaboration
  - — Work in collaborative groups to construct a writing prompt
  - — Work in collaborative groups to respond to a writing prompt and edit written work
- Reading to Learn
  - — Read and construct meaning from a problem from a text in order to rewrite it in the Five W format

NCTM Standards

Problem Solving

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- build new mathematical knowledge through problem solving;
- solve problems that arise in mathematics and in other contexts;
- apply and adapt a variety of appropriate strategies to solve problems; and
- monitor and reflect on the process of mathematical problem solving.

Communication

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- analyze and evaluate the mathematical thinking and strategies of others; and
- use the language of mathematics to express mathematical ideas precisely.

Connections

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- recognize and use connections among mathematical ideas;
- understand how mathematical ideas interconnect and build on one another to produce a coherent whole; and
- recognize and apply mathematics in contexts outside of mathematics.

Representation

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- create and use representations to organize, record, and communicate mathematical ideas;
- select, apply, and translate among mathematical representations to solve problems; and
- use representations to model and interpret physical, social, and mathematical phenomena.

Rationale

A well-written prompt will provide the best opportunity for well-developed student writing. *Affective/Attitudinal, Mathematical Content,* or *Process* prompts generate the three most common guided writing activities. Using a template to write the specifications of a prompt can greatly improve the quality of the prompt and the resulting quality of the students' writings. A common template utilizes the Five Ws format:

- **Who** is writing?
- **Who** will be reading?
- **What** is the format?
- **What** is the subject or point of the writing?
- **What** reminders are incorporated?

Opportunities for students to write for different audiences and purposes greatly enhance the legitimacy of the writing process. By providing students with flexibility and choice in audience, purpose, and format, writing takes on the dimension of a real world venture, and the decision-making process required by such variety produces energetic revision and refreshing writing. Discussions that address audience, purpose, and form remind students of the importance of the choices they make as writers.

Providing practice with a variety of forms such as letters, memos, reports, emails, text messages, etc. will enable students to practice their technical writing skills across various genres in many novel situations. It will encourage students to be creative and imaginative in their responses. In addition, providing writing tips in the reminder section will enable students to practice specific writing skills and this will enable focused formative writing assessments.

The "*Writing Prompts—Five Ws"* activity provides a framework for using and refining individual technical writing skills as well as building high-level questioning skills through the design and analysis of an experiment. Students working collaboratively are given opportunities in *"Unit 1: Writing to Learn in Mathematics"* to practice and provide evidence of mastering technical vocabulary, technical writing, and key communication skills.

Vertical Alignment

- The specific content may vary with the level of instruction; however, the concept of incorporating formal writing in the mathematics classroom can be introduced at any level. Students' work at higher levels will be more complex and will demonstrate a higher degree of mastery of technical writing skills specific to mathematics.

Materials/Preparation

- Student textbook or selection of content-specific problems
- *Teacher Reference Sheet/Overhead Transparency 1.4a:* "Five Ws *Example*"
- *Student Handout 1.4b:* "Five Ws for Writing Prompts"
- Review the "Technical Writing Tips for Mathematics" (see the *Introduction to Writing in Mathematics*).

Instructions

- Explain the various aspects of role, (who is writing) audience, format (letter, memo, report, email, text message...), the topic, and strong verbs that students will be considering before writing.
- Display *Teacher Reference Sheet/Overhead Transparency 1.4a:* "Five Ws *Example*" on the overhead or document camera.
- Discuss characteristics of each element with the class.
- Brainstorm with the class to develop a Five Ws prompt.
- Include some "Technical Writing Skills" suggestions in the Reminders part of the Five Ws template. These can be found in the "Technical Writing Tips for Mathematics."
- Distribute *Student Handout 1.4b:* "Five Ws."
- Use the agreed-upon "Five Ws" prompt as a guide to complete a class example. Use brainstorming, graphic organizers, and other planning tools to prepare for writing.
- Assign students to small groups or pairs and have them use the class "Five Ws" as a guide for their writing.
- Use "Popcorn" or another group-sharing activity to share each group's completed assignment with the class.
- Teach students strategies for "decoding the prompts."
  - — How to determine elements of the "Five Ws"
  - — Incorporate the prompt into the lead
  - — Use "Think-Alouds" as they organize responses
  - — Practice brainstorming
  - — Identify and teach specific technical writing skills
  - — Confer with peers
  - — Practice use of graphic organizers
  - — All writing is rough draft—use the writing process
  - — Practice self-assessment using a rubric (What does it take to get a 4?)

Higher-Level Questions

Level One

- What is the solution to the problem in the "Five Ws" example?
- What are the rules for incorporating a formula into your writing?

Level Two

- What are the skills needed to explain math concepts to a "math-challenged" person?
- What are some practical examples of when technical writing skills may be needed?

Formative Assessment

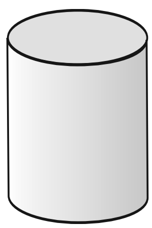
- Did students work collaboratively to construct a well-written prompt?
- Did students include technical writing skills reminders?
- Does their writing reflect the use and mastery of technical writing skills?

Five Ws *Example*

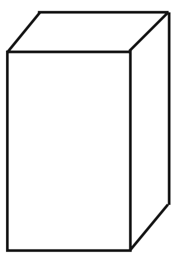
What is the "Frame/Questions?"

Members of the marketing department are arguing about the shape of the new flavored drink container. They argue that the two suggested containers have equivalent volume and that the rectangular container will show-up better on the shelf in the store.

A. Cylinder with radius 2.8cm and a height of 12cm.



B. A rectangular prism with the dimensions of 7cm x 3.5cm x 12cm.



Determine if they are correct and make a recommendation.

Who is writing?

Math consultant for a flavored drink company.

Who is reading?

Members of the marketing department.

What is the format of the writing?

Write a formal one-half page recommendation. Explain in detail how you arrived at your recommendation; use words, numbers, equations and illustrations.

What are the reminders?

- Equations are on their own line separated by a space above and a space below.
- Define all variables and use italics when writing them.
- Label all diagrams.

Five Ws for Writing Prompts

What is the "Frame/Questions?"

Who is writing?

Who is reading?

What is the format of the writing?

What are the reminders?

1.5: What is Math Like?

Topic

- Writing to Learn

Objectives

Students will:

- Become acquainted with "Writing to Learn" as a topic in mathematics
- Practice writing skills
- Provide a sample of their writing skills
- Be formally introduced to the teacher and learn about one another

Timeline

- One 50-minute class period at the beginning of the school year to finish first draft, plus homework time to edit

WICR Strategies

- Writing to Learn
  — Write math similes
- Reading to Learn
  — Read a portion of writing in a small group

NCTM Standards

Communication

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- organize and consolidate their mathematical thinking through communication; and
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others.

Rationale

Writing to learn is a key skill in acquiring deeper understanding in mathematics. Through continuing practice, self-assessment, and guidance, students are given the opportunity to improve their facility and fluency of the precision language of mathematics. Vocabulary and language usage can only be developed through regular practice. *"What is Math Like"* provides students with a bridge between the kind of writing they have done in their English and Humanities classes and the writing they will be doing in mathematics.

Reference:

Gibson, Helen. (1994). "Math is like a used car": Metaphors reveal attitudes toward mathematics. In D. Buerk (Ed.), *Empowering students by promoting active learning in mathematics: Teachers speak to teachers* (pp. 7–12). Reston, VA: National Council of Teachers of Mathematics.

Vertical Alignment

- Providing students with structured writing opportunities regarding their feelings and attitudes about mathematics is a great idea at any level. Activities such as “What is Math Like?” are an effective way for teachers to learn about their students at the beginning of the school year.

Materials/Preparation

- *Student Handout 1.5a:* “What is Math Like?”
- Pen, highlighters, and paper

Instructions

- Distribute *Student Handout 1.5a:* “What is Math Like?”
- Provide students with five minutes to list all the words or phrases they would use to describe math to one of their friends in the first column.
- Provide students with five minutes to list all the feelings they have when doing mathematics in or out of school in the second column.
- Provide students with five minutes to list all the things that math is “like” for them in the third column.
- Now ask students to circle the word or phrase in the third column that best describes what math is like for them.
- Ask students to complete the bottom half of the student handout by responding to the writing prompt, “For me, math is most like...” Provide students with 15–20 minutes to write the first draft of a well-written paragraph.
- Ask students to select one or more sentences that they are most proud of and highlight them.
- Use “Reading into the Center” or another group share-out activity to give students an opportunity to share their favorite parts of their writing.
- Assign homework time for students to edit their first draft.

Higher-Level Questions

Level Two

- What are some other connections that you might use to enrich your simile?

Level Three

- What are some other prewriting activities that you could use to help plan a writing activity?
- Predict how your attitudes towards math will influence your performance in math class this year.

Formative Assessment

- Assess the length and fluency of the students’ writing.
- Evaluate the complexity of the student sentences.
- Were most students’ descriptions positive or negative?
- Did most students feel comfortable sharing their writing?
- Did most students improve their first draft through the editing process?

What is Math Like?

List the words you would use to describe Math to one of your friends.	List the feelings you have when doing math in or out of school.	List the things (nouns) that describe what math is like for you.

Write a complete paragraph responding to the following prompt:

For me, math is most like ...

1.6: The "Dominant Hand" Exploration

Topic

- Exploration of our "Dominant Hand" through experimentation and data analysis

Objectives

Students will:

- Explore higher-level questions
- Work collaboratively to develop an experiment designed to answer higher-level questions
- Write a description of an experiment
- Conduct an experiment
- Collect and analyze data
- Display data in a graphical form
- Review the value of WICR and higher-level questions as a foundation for lessons

Timeline

- One 50-minute class period to prepare and conduct the experiment and to collect data, plus homework time to complete the activity
- One 50-minute class period to analyze and display data

WICR Strategies

- Writing to Learn
  — Write a description of an experiment
  — Construct a graphical display
  — Write a reflection about the process and findings of an experiment
- Inquiry
  — Develop higher-level questions
  — Design and conduct an experiment to answer a higher-level question
  — Discuss and support choices for graphical display of data
- Collaboration
  — Work together with peers to develop experimental questions
  — Work together to design, write, and conduct an experiment
  — Work together to analyze data and develop a graphical display
  — Work together to evaluate different types of graphical displays
- Reading to Learn
  — Read a description of an experiment
  — Analyze data

NCTM Standards

Focal Point Grade 7

Number and Operations and Algebra and Geometry: Developing an understanding of and applying proportionality, including similarity

Focal Point Grade 8

Data Analysis and Number and Operations and Algebra: Analyzing and summarizing data sets

Data Analysis and Probability

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them;
- select and use appropriate statistical methods to analyze data; and
- develop and evaluate inferences and predictions that are based on data.

Rationale

This activity provides a framework for using and refining individual technical writing skills as well as building high-level questioning skills through the design and analysis of an experiment. Students working collaboratively are given opportunities in *"The 'Dominant Hand' Exploration"* to practice and provide evidence of mastering technical vocabulary, technical writing, and key communication skills.

Vertical Alignment

- Designing, conducting, and writing about mathematical experiments can be experienced by students at any level in grades 6–12. The complexity in the level of expectation in both writing and the use of academic language should increase as students move through mathematics courses in middle and high school.

Materials/Preparation

- Flip chart
- Graph paper
- Clock with a sweeping second hand
- Color pens or pencils
- Markers
- Overhead projector
- "Technical Writing Tips for Mathematics" (see the *Introduction to Writing in Mathematics*)

Instructions

Day One

- Demonstrate an experiment on an overhead or document projector that answers the Level One question: “If you write the letter x on a line repeatedly as fast as you can for ten seconds, how much faster is your dominant hand than your non-dominant hand?”
- Identify a class volunteer to demonstrate.
- Record his/her results in a data table as shown below.

X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X					Dominant Hand
X	X	X	X	X	X	X	X														Non-Dominant Hand

- Following the demonstration, use “Popcorn” or another group share-out activity to collect and record possible experimental questions that could be posed and answered using the tools utilized in the demonstration. Record these questions on a flip chart.
- After reviewing Level Two and Level Three questions, come to class consensus to determine the level of each of the questions. Highlight or star the higher level questions.
- Divide the class into pairs.
- Ask each team to select a higher-level question from the class list and design an experiment to help answer the question selected. Advise students that they will be conducting the experiment and as such, must design an experiment that is possible within the constraints of the class period and the materials available. Advise the teams that they will be displaying their data in the graphical way that best aids them in analyzing the data and drawing conclusions.
- Use “Think, Pair, Share,” “Popcorn” or another group share-out strategy to review alternative graphical displays, e.g. Bar Graphs, Circle Graphs, Box and Whiskers, Stem and Leaf Plots, Scatter Plots, Histograms, etc.
- Review the “Technical Writing Tips for Mathematics” that will be assessed in this activity.
- Ask each team to discuss and agree upon an experiment and a data analysis tool. Give them time to write a description of the design. Their collaboration must demonstrate the teams’ best “Technical Writing” skills so that anybody reading the description would be able to replicate the experiment. (A variation could be that another team uses their description to conduct the experiment.)
- Give teams time to conduct the experiment and collect data.
- Students may continue their data analysis as a homework activity.

Day Two

- Provide teams with graph paper and color markers or pencils. Ask teams to display data graphically and write a short summary of what they did and what conclusions, if any, they can draw. Encourage them to include additional questions that have come to mind. Emphasize the importance of creativity in the visual display.
- Use a “Gallery Tour” or other group-share activity to get student feedback.

- Ask students to write their questions/comments on sticky notes for each display.
- Review the aspects of WICR.
- Emphasize the use of higher-level questions and inquiry.
- Use the "Four Corners" or another Socratic discourse strategy to discuss the advantages of different types of data.

Higher-Level Questions

Level Two

- If time and expense were unlimited, what kind of experiment could you design?
- What are the uncontrolled variables in your experiment?
- What does this activity remind you of?
- What are some other methods that you could use to display your data?

Level Three

- Predict how the results of your experiment would change if the majority of students in the class were left-handed.
- What is the best way to display the data?
- How can you control for some of the uncontrolled variables?

Formative Assessment

- Did all of your students participate in the whole-group brainstorming activities?
- Did students meet your expectations with their written descriptions of their experiment?
- Do your students have a deep understanding of Costa's Levels of Questions?
- Did the students' graphical representations of their data meet your expectations?
- Did the student summaries of the experiment adequately answer the research questions?
- Did all of your students fully participate in the Socratic discourse regarding the dominant hand experiments?

1.7: Three-Column Proofs

Topic

- Providing justification for the steps taken in solving linear equations

Objectives

Students will:

- Work with a partner to solve multiple-step linear equations
- Write descriptions of the steps involved in solving a multiple-step linear equation
- Provide the mathematical property or reasoning associated with each step in solving a multiple-step linear equation

Timeline

- One 50–60-minute class period for students to solve linear equations using the three-column proof format

WICR Strategies

- Writing to Learn
  - — Write descriptions of the steps involved in solving a multiple-step linear equation
  - — Practice writing the academic language of mathematics
- Inquiry
  - — Provide justification for the steps involved in solving multi-step linear equations
- Collaboration
  - — Work with a partner to complete the activity

NCTM Standards

Focal Point Grade 7

Number and Operations and Algebra and Geometry: Developing an understanding of operations on all rational numbers and solving linear equations.

Communication

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others; and
- use the language of mathematics to express mathematical ideas precisely.

Rationale

Students often perform the tasks involved with solving linear equations without a solid understanding of why they are doing these steps. In the primary grades of elementary school, students first encounter the concept of solving linear equations when asked to "fill in the box" with a number that makes the number sentence true. This task is generally dependent on the students' knowledge of "Fact Families." In the upper grades of elementary school, students begin to encounter one-step linear equations and are generally taught to perform the opposite operation to solve the equation. Discussion of solving linear equations is routinely confined to the procedure for solving the equation, and justifying the solution by "checking your answer." "*Three-Column Proofs"* is an activity that takes students on a journey much deeper into solving linear equations. Students are asked not only to describe their solution steps in Academic English, they are asked to state a mathematical property or reason that justifies each step.

Vertical Alignment

Justifying mathematical thinking is essential for developing deep levels of understanding at all levels of mathematics instruction. "Three-Column Proofs" is an activity that students can engage when they first begin to solve multiple-step linear equations. The activity can be repeated whenever students encounter new types of equations to solve (absolute value, square root, quadratic, rational, etc.) throughout their mathematics education. This activity also lays the foundation for the proof writing skills they will need in their study of Euclidean Geometry. Geometry teachers can utilize *Student Handout 1.7a:* "Three-Column Proofs" by having students write the "Given" and "Prove" statements in the upper left hand cell of the table.

Note: In *Student Handout 1.7b:* "Three-Column Proof *Example*," the Subtraction Property of Equality and the Division Property of Equality are listed as the properties for two of the linear equation solution steps. Teachers may wish to use the Additive Inverse and Multiplicative Inverse Properties as well.

Materials/Preparation

- *Student Handout 1.7a:* "Three-Column Proofs"
- *Student Handout 1.7b:* "Three-Column Proofs *Example*"
- A set of multiple-step linear equation problems
- Review the Active Learning Methodologies (see the *Introduction*).

Instructions

- Place students into partner groups.
- Distribute one *Student Handout 1.7a:* "Three-Column Proofs" to each pair of students, or ask students to divide a piece of binder paper into three columns. You may also distribute *Student Handout 1.7b:* "Three-Column Proofs *Example*," if desired.
- Model a three-column proof. You can also ask students to generate a list of properties and reasons for various steps in solving linear equations.
- Assign one of the linear equations from the day's assignment to each group to solve using the three-column proof format.
- When students have completed their proof, you can choose one of the Active Learning Methodologies to share-out the students' work, or you may choose to collect the work as a formative assessment.
- Ask students to use their "Three-Column Proofs" as an outline for writing a one-paragraph narrative proof for their solution.

Higher-Level Questions

Level Two

- Compare the sequence of your solution steps to another group's sequence of solution steps. If possible, show how your equation can be solved using another groups sequence of solution steps.

Level Three

- Is your three-column equation proof mathematically beautiful? Discuss what it means for mathematics to be beautiful.

Formative Assessment

- Are your students using target vocabulary words in their descriptions of the steps involved in solving linear equations?
- At what level of understanding are your students performing in relation to the mathematical properties involved in solving linear equations? What can you do as a teacher to help your students acquire a deeper understanding?

Three-Column Proofs

Steps	Property or Reason	Verbal Description

Three-Column Proofs *Example*

Steps	Property or Reason	Verbal Description
Solve for x: $3(x + 3) – 5 = -2x – 11$		
$3x + 9 – 5 = -2x – 11$	Distributive Property	Use the distributive property to clear the parenthesis in the equation.
$3x + 4 = -2x – 11$	Addition of Integers	Simplify the equation by combining the constant terms on the left side of the equation.
$5x + 4 = -11$	Addition Property of Equality	Add $2x$ to both sides of the equation to gather the variable terms on the left side.
$5x = -15$	Subtraction Property of Equality	Subtract 4 from both sides of the equation to gather the constant terms on the right side.
$x = -3$	Division Property of Equality	Divide both sides of the equation by 5 to find the value of x.

1.8: Test Corrections

Topic

- Developing and agreeing upon a protocol for test corrections

Objectives

Students will:

- Participate in the development of a test corrections protocol
- Utilize the agreed upon protocol to master material on formal assessments

Timeline

- One 50-minute class period to develop and practice using a protocol for test corrections

WICR Strategies

- Writing to Learn
  - — Incorporate writing to learn to demonstrate mastery of material on formal assessments
- Collaboration
  - — Work collaboratively to develop and use a test correction protocol
- Reading to Learn
  - — Read and assess peer writing

NCTM Standards

Communication

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- analyze and evaluate the mathematical thinking and strategies of others; and
- use the language of mathematics to express mathematical ideas precisely.

Representation

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- create and use representations to organize, record, and communicate mathematical ideas; and
- select, apply, and translate among mathematical representations to solve problems.

Rationale

Incorporating writing in test corrections will promote clear thinking, help students make connections, and help students become aware of what is known and not known. Writing will encourage students to raise questions about new ideas, organize thinking, and help construct meaning. Most importantly, it will inform instruction by providing an invaluable insight into the thinking and level of understanding of student writers. Teachers who add writing to their pedagogy will often find it easier to recognize and diagnose students' conceptual problems. The time teachers invest in helping their students clearly explain their thinking will result in saved instructional time later on as lessons become more prescriptive in nature.

Vertical Alignment

- Incorporating writing as a methodology for students to clarify thinking and complete test corrections can be implemented at any level. At the higher grades, the degree of complexity will increase.

Materials/Preparation

- *Student Handout 1.8a:* "Test Corrections"

Instructions

- Discuss the role and importance of writing in clarifying and organizing thinking. The use of the Socratic Seminar structures may assist in the promotion of the discussion and the subsequent ownership of a test correction protocol.
- Discuss the value of test corrections in reviewing and demonstrating mastery of material.
- Distribute *Student Handout 1.8a:* "Test Corrections."
- Discuss ways to incorporate the themes from the sample form in designing an agreed upon protocol for test corrections for the class.
- Reach class consensus.
- Formalize the class agreements and distribute the agreed upon form and protocol.
- Use and refine the form and protocol as needed.
- Provide students an opportunity to do peer assessments of test corrections prior to submission.
- Provide opportunities for student groups to submit group test corrections using the agreed upon protocol.

Higher-Level Questions

Level Two

- How can writing contribute to clarifying and organizing thinking?

Level Three

- What are alternative ways to demonstrate mastery of material?

Formative Assessment

- Assess student discourse skills.
- Assess student methods for reaching consensus.
- Assess student use of the agreed upon protocol.

Test Corrections

Test ______________________ Date ____________ Name ______________________

Instructions: Write all of your responses in complete sentences. **This form is due within one week of the test and neatness counts!**

Identify the Error: 1. Restate the problem. 2. Explain in **complete sentences** what it was about the problem that caused you difficulties.	**Demonstrate Understanding:** 1. Show the solution. (Show all steps and all work.) 2. Show how you checked your solution.	**Test Understanding:** 1. Write a similar problem. 2. Explain in **complete sentences** what you understand now that you did not understand before.	**Make Connections:** 1. Show the solution to the new problem. 2. Explain in **complete sentences** how this problem is related to the mathematics you have done in the past and are currently doing in class.
Problem #__________			

Unit Two: Inquiry in Mathematics

Introduction to Inquiry in Mathematics

AVID is based on inquiry, not lecture, because it is the process of posing and answering questions that teaches students to think. Many activities, such as tutorial and Cornell note-taking are built around asking questions and enable students to clarify, analyze, and synthesize material. Learning how to ask the right questions is a crucial skill, because many students have difficulty clarifying thoughts and asking the right questions to get the information and help that they need. Tutors and teachers are trained to ask questions that move students to successively higher levels of thinking.

Inquiry engages students with their own thinking processes. It teaches students to think for themselves instead of chasing the "right answer." The result is student ownership of the learning process and a better understanding of concepts and values. When Socrates encourages Crito in the Platonic dialogue to "examine the question together" and attempts to persuade him, he captures the essence of inquiry as an instructional method. In the AVID-inspired classroom, questioning takes many different forms: skilled questioning and writing questions (most often in collaborative learning groups), Socratic Seminars, Quickwrites, discussions, critical thinking activities, and open-mindedness activities.

Inquiry within the Math Tutorial

In the math tutorial, students engage in all levels of critical thinking, from recall to evaluation. Students pursue understanding with mutual respect and civility and are mindful of each other's dignity. They are willing to be persuaded by arguments/evidence more powerful than their own and to change their minds in light of fresh insights.

To begin tutorial, students bring questions for discussion. Guided by a teacher/tutor, students exchange responses and collaborate in search of understanding. By returning to notes and texts, students often gain a deeper understanding of the answers to the questions raised. This collaboration rests on the belief that the group can arrive together at some understanding that would not be arrived at independently.

There are several questioning strategies teachers/tutors can use to lead their groups. Two highly recommended methods outlined below are based on work in cognitive functions by Benjamin Bloom and Arthur Costa, respectively.

Using the revised Bloom's hierarchy of cognitive skills, teachers, tutors, and students can ask questions that follow along a continuum:

Remembering - Retrieving, recognizing, and recalling relevant knowledge from long-term memory.

Understanding - Constructing meaning from oral, written, and graphic messages through interpreting, exemplifying, classifying, summarizing, inferring, comparing, and explaining.

Applying - Carrying out or using a procedure through executing, or implementing.

Analysis - Breaking material into constituent parts, determining how the parts relate to one another and to an overall structure or purpose through differentiating, organizing, and attributing.

Evaluating - Making judgments based on criteria and standards through checking and critiquing.

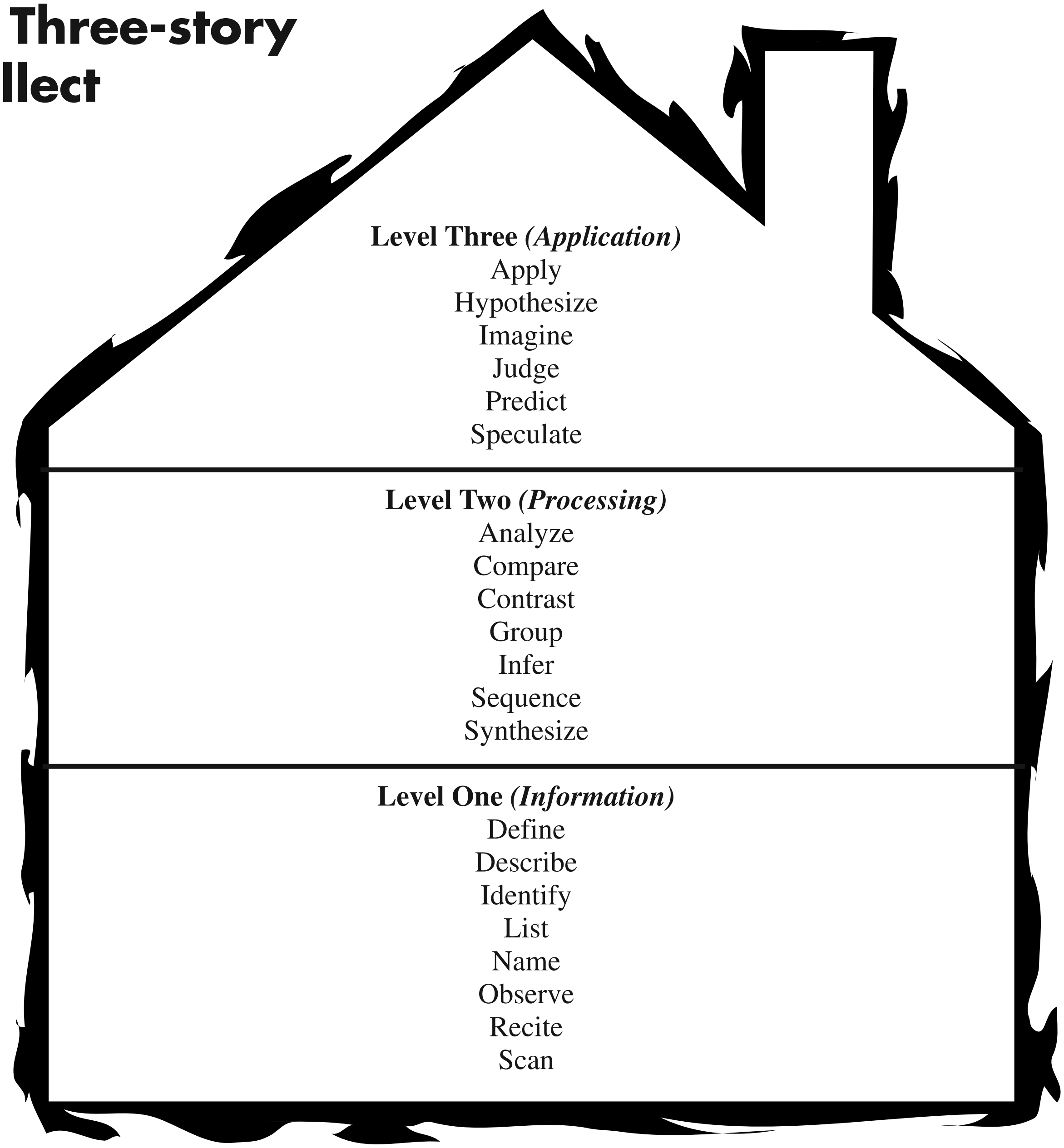
Creating - Putting elements together to form a coherent or functional whole; reorganizing elements into a new pattern or structure through generating, planning, or producing.

(Anderson & Krathwohl, 2001, pp. 67–68)

Using Costa's Model of Intellectual Functioning in Three Levels, students ask three levels of questions:

1. **Level One** questions focus on gathering and recalling information.
2. **Level Two** questions focus on making sense of gathered information.
3. **Level Three** questions focus on applying and evaluating information.

The Three-story Intellect



Costa's Levels of Questions

Level One	
defining	What is the definition of the slope of a line?
describing	What do all isosceles triangles have in common?
identifying	Identify the trig ratios of an acute angle in a right triangle.
naming	Name 5 different quadrilaterals.
listing	Make a table of ordered pairs that satisfy the function, $y = 3x + 1$.
observing	In a triangle, the longest side is opposite of which angle?
reciting	State the quadratic formula.
Level Two	
analyzing	In the proportion, $\frac{y}{5} = \frac{5}{x}$ what happens to the values of y as x increases?
comparing	How can you decide which graphic representation (line graph, box-and-whisker plot, stem leaf, etc.) to use with a given set of data?
contrasting	How are permutations and combinations different?
grouping	Group the following polygons according to a common characteristic: square, equilateral triangle, rectangle, scalene, triangle, regular pentagon, isosceles triangle, isosceles right triangle, trapezoid, rhombus, parallelogram
inferring	Given the first five members of a sequence below, find the sequencing rule that generates them and then find the next two members of the sequence: -5, -3, -1, 1, 3, ...
synthesizing	Consider the function of the form $f(t) = a(b)^t$ describe the function by means of a data table, its graph and an application. Choose any a and b you like.

Costa's Levels of Questions

Level Three	
applying a principle	Describe a use for the Commutative Property for real numbers.
hypothesizing	Under what conditions for x and y would $\frac{x}{y} > 1$? or $\left\|\frac{x}{y}\right\| < 1$?
imagining	What must be true about a set of data in which the median is larger than the mean?
judging	If the money and interest rates in two accounts are the same, will my principal earn more money in an account compounded quarterly or in a simple interest account? Why?
predicting	What happens to the value of a fraction if the numerator stays the same and the denominator keeps increasing?
speculating	In a normal distribution curve, can a data value ever be more than 3 standard deviations to the right of the mean? If you think the answer is yes, about how frequently would you expect that to happen?

Socratic Seminar

Socratic Seminars are teacher- or student-led dialogues regarding specific texts that encourage students to think for themselves. These seminars develop habits of thoughtfulness and analysis through close and collaborative questioning of the meaning of a text, a math problem or set of data, a work of art or music, or a presentation. Participants demonstrate careful thinking and self-expression. They search for and weigh evidence and explore differing views. The teacher or the leader of the seminar does not guide participants to a specific goal or conclusion but leads them to discover their own truth or interpretation of text. The physical arrangement of the classroom is vital to the success of Socratic Seminars. All students should be seated in desks or tables arranged in a rectangle or a circle to encourage eye contact. This also encourages equality, sharing, and face-to-face interactions within the group.

Source: *AVID Implementing and Managing the AVID Program for High Schools* 2006 Revision pp. 130–133.

2.1: The Evolution of a Great Question: Costa's Triples

Topic

- Writing higher-level questions in mathematics

Objectives

Students will:

- Become more familiar with "Costa's Levels of Questions"
- Practice writing Level Two and Three math questions

Timeline

- 10–20 minutes to explore the evolution of a question

WICR Strategies

- Writing to Learn
  - — Write Level Two and Level Three questions
- Inquiry
  - — Investigate the characteristics of Level Two and Level Three questions
  - — Predict the outcome for extending the problem from squares to cubes
- Collaboration
  - — Work in collaborative learning triads to evolve a question from Level One to Level Three

NCTM Standards

Communication

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- analyze and evaluate the mathematical thinking and strategies of others; and
- use the language of mathematics to express mathematical ideas precisely.

Rationale

In the *Write Path I: Mathematics'* activity, "Costa's Levels of Questions," students explored the cognitive demand of a question by sorting state assessment items into Costa's Three Levels of Questions. In *"The Evolution of a Great Question: Costa's Triples,"* this idea is expanded upon by asking students to work together to evolve Level One questions into Level Two and Three questions. As AVID students move through middle and high school, the expectation for their ability to write better and more complex questions rises. This expectation is realized in the AVID elective class, in Cornell Notes, in the AVID tutorial, and hopefully, in mathematics courses schoolwide.

Vertical Alignment

- Practice in writing higher levels of questions can begin in the upper grades in elementary school and should occur every year through grade 12 and beyond. As students move through the grades, the expectation for their abilities to write better and more complex questions should increase.

Materials/Preparation

- *Student Handout 2.1a:* "Costa's Levels of Questions"
- Index cards or sticky notes

Instructions

- Place students into groups of three.
- Distribute one index card to each student.
- Ask each student to write a Level One math question pertaining to a particular math topic.
- Each student then passes the card to the student on their right, who rewrites the question as a Level Two question.
- Finally, the card is passed to the right once again and this student rewrites the question as a Level Three question.
- The cards are passed back to the original student and the results are read aloud to the entire group.

Example

- *Level One Question:* State the "Pythagorean Theorem."
- *Level Two Question:* Given 3 sides of a triangle, can you determine if it is a right triangle?
- *Level Three Question:* Why is a triangle with sides 3, 4, and 5 called a "Pythagorean triple?" Can you find 3 more "triples?"

Higher-Level Questions

Level Two

- Why are Level Two and Three questions more difficult to ask?
- Why are Level Two and Three questions more difficult to answer?

Level Three

- If you don't know the answer to a Level Two or Three question, where can you look to find more information about the question?

Formative Assessment

- Have a fellow teacher sit in on your class and record the number of times you ask a Level One, Two, or Three question.
- Have a fellow teacher sit in on your class and record the number of Level One, Two, or Three questions asked by your students.

COSTA'S LEVELS OF QUESTIONS

Level One	Level Two	Level Three
Define	Analyze	Apply
Describe	Compare	Hypothesize
Identify	Contrast	Imagine
List	Group	Judge
Name	Infer	Predict
Observe	Sequence	Speculate
Recite	Synthesize	
Scan		

COSTA'S LEVELS OF QUESTIONS

Level One	Level Two	Level Three
Define	Analyze	Apply
Describe	Compare	Hypothesize
Identify	Contrast	Imagine
List	Group	Judge
Name	Infer	Predict
Observe	Sequence	Speculate
Recite	Synthesize	
Scan		

COSTA'S LEVELS OF QUESTIONS

Level One	Level Two	Level Three
Define	Analyze	Apply
Describe	Compare	Hypothesize
Identify	Contrast	Imagine
List	Group	Judge
Name	Infer	Predict
Observe	Sequence	Speculate
Recite	Synthesize	
Scan		

2.2 Jigsaw Questions

Topic

- Writing higher-level questions

Objectives

Students will:

- Demonstrate an understanding of Costa's Levels of Questions
- Write higher-level questions given a model
- Propose higher-level test questions

Timeline

- One 50-minute class period to practice writing and understanding higher-level questions

WICR Strategies

- Writing to Learn
  - — Write higher-level questions
  - — Write solution keys for higher-level questions
- Collaboration
  - — Work in collaborative groups to ensure understanding of Costa's Levels of Questions

NCTM Standards

Communication

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- analyze and evaluate the mathematical thinking and strategies of others; and
- use the language of mathematics to express mathematical ideas precisely.

Rationale

This activity provides a means to assess students on their understanding of different types of questions, as well as their ability to write questions for their Cornell Notes and use higher-level questions in mathematical discourse. The *"Jigsaw Questions"* activity can be used as a review for upcoming exams while reinforcing inquiry models. The *"Jigsaw Questions"* lesson is designed to provide practice writing different levels of questions.

Vertical Alignment

- This activity can be done at any level beginning at a very young age. Prior to completing this activity, students should be provided an introduction to both Costa's Levels of Questions and Cornell Notes.

Materials/Preparation

- *Student Handout 2.2a:* "Jigsaw Questions"
- Teachers may revise *Student Handout 2.2a* as new concepts are studied.
- Review the Active Learning Methodologies (see the *Introduction*).

Instructions

- Divide students into small groups.
- Distribute *Student Handout 2.2a:* "Jigsaw Questions"
- Provide time for students to collaborate to complete the missing questions on the student handout.
- Utilize an Active Learning Methodology activity like a "Gallery Tour" or "Popcorn" for groups to share their work.
- Ask students to identify questions to include on the next test or quiz.
- Give students time to prepare solution keys for the higher level questions that they have generated.

Higher-Level Questions

Level Two

- How are the higher-level questions different and/or the same as lower-level questions?

Level Three

- Are there any questions that cannot be rewritten as higher-level questions?
- What is the value of studying by using higher-level questions when many of the test questions may be Level One questions?

Formative Assessment

- Assess students' rewritten questions for higher-level thinking.
- Review questions that students submit for consideration for use in a formal exam.
- Assess student solution keys.

Jigsaw Questions

Instructions: Write corresponding higher- and lower-level questions for each of the following:

LEVEL ONE (complete, count, match, name, define, observe, recite, describe, list, identify, recall)	**LEVEL TWO** (analyze, categorize, explain, classify, compare, contract, infer, organize, sequence)	**LEVEL THREE** (imagine, plan, judge, predict, extrapolate, invent, speculate, generalize)
Simplify: $3x + 4y - 2 + 2x = 2y - 5x + 17$		
What is the definition of a trapezoid?		
What is the equation for finding the surface area of a cylinder?		
What is the sum of the external angles of a pentagon?		
Evaluate the expression: $3x^2 + 4$ if $x = 5$		
	How are rational and irrational numbers the same or different?	
	How does the formula for a parabola change as it shifts right and left or up and down?	
	Arrange the following numbers in order from smallest to largest: $^1/_2$, .7, $\sqrt{5}$, $\lvert -3 \rvert$	
		If the volume of a cylinder increases three fold and the height stays the same, what happens to the radius of the larger cylinder?
		What will the city's population be in 2050 if we continue to grow at 10% per year?

2.3: Test Preparation: Why, Why, Why, Why?

Topic

- Inquiry-based activity for multiple-choice standardized test preparation

Objectives

Students will:

- Determine why the correct response to a multiple-choice math question is correct and why the distracters are incorrect
- Answer a set of five multiple-choice questions similar to the original question
- Write their own multiple-choice item similar to the original questions

Timeline

- 15–30 minutes to find the correct response to a mathematical problem and understand how the incorrect answers function as distracters

WICR Strategies

- Writing to Learn
  - — Write a justification of why the correct response is correct and why the distracters are incorrect for a multiple-choice standardized test question
- Inquiry
  - — Investigate the common mistakes students make in answering multiple-choice questions
- Collaboration
  - — Work in collaborative learning groups to complete the activity

NCTM Standards

Communication

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- analyze and evaluate the mathematical thinking and strategies of others; and
- use the language of mathematics to express mathematical ideas precisely.

Rationale

In today's high-stakes testing, standards-based education environment, a significant amount of instructional time is spent on preparing students for state and national tests. *"The Four Why's?"* is an activity that maximizes the impact of the time spent on test prep by asking students to think deeply about the choices given for a multiple-choice question. For this activity, the teacher chooses one sample test item that is written for a standard emphasized by the state test. In part one of the activity, students work in collaborative groups to find the correct response. In part two, students are asked to defend their selection by justifying the correct answer. They are also asked to determine the common mistake a student would make to arrive at each of the distracters given in the problem. In part three, the students practice a set of five questions that are very similar to the original problem. The last part of the activity asks the students to write their own problem and share it with a partner. The purpose of each step is to foster deep understanding of the mathematics of the question. This type of test prep is very different than giving students worksheets of practice problems. If done on a regular basis, *"The Four Why's?"* can be an excellent alternative during dedicated test preparation time.

(This activity is adapted from Dennis Parker's *Strategic Schooling Model,* 2008)

Vertical Alignment

- "The Four Why's?" can be asked of students at all levels in grades 6–12. The level of sophistication and their ability to justify correct responses and identify the common mistakes that lead to distracters should increase over time.

Materials/Preparation:

- Cornell note-paper
- One sample multiple-choice, state test question, and five similar questions
- Review the Active Learning Methodologies (see the *Introduction*).
- *Student Handout 2.3a:* "Test Preparation: Why, Why, Why, Why?"

Instructions

- Arrange the class into collaborative learning groups of two to four students.
- Distribute or have students create their own Cornell note-paper.
- Distribute or project the sample multiple-choice test item.
- Distribute *Student Handout 2.3a:* "Test Preparation: Why, Why, Why, Why?" and discuss expectations for the activity.
- Ask your students to write the problem down in their Cornell Notes. In their groups, ask the students to find the correct response.
- When each group has an answer, elicit responses from all of the groups. When consensus is reached on the correct response, ask each student to write a justification of why the correct response is correct in their Cornell Notes.
- Use "Think, Pair, Share" or another Active Learning Methodology to share-out a few of the student descriptions. *Tip:* Keep it moving here, we still have a lot to do with this question!
- In their groups, ask the students to determine the common mistake a student would make to arrive at each of the incorrect choices (distracters). Students should record there thinking in their Cornell Notes.
- Distribute or project the five similar practice questions. When everyone has had a chance to write the questions and answers in their Cornell Notes, quickly review the correct answer for each question.
- Ask each student to create their own version of the multiple-choice test question, complete with correct

response and common mistake distracters.

- Have students trade their Cornell Notes with a partner and answer each others' question.
- Have students write a thoughtful reflection at the end of the activity in their Cornell Notes.

Higher-Level Questions

Level Two

- Determine the level of the state test question investigated for this activity. If it is a Level One question, rewrite it as a Level Two question.

Level Three

- If possible, rewrite the state question as a Level Three by changing the question from a specific example to a more generalized form.

Formative Assessment

- As your students are identifying common mistakes made on state test questions, monitor their conversations. Do your students admit that they make similar mistakes? Is there agreement on what the common mistakes are?
- Are your students remembering not to make the common mistakes you discussed once the activity is completed?

Test Preparation: Why, Why, Why, Why?

Example Response

Sample 7th Grade State Assessment Item	A set of headphones that is priced \$30 is part of a 40% off sale. What is the sale price of the headphones? A. \$12.00 B. \$18.00 C. \$22.50 D. \$26.00
What is the correct choice?	The correct answer is B.
Why is the correct choice correct?	If the headphones are on sale for 40% off, then the sale price is 60% of the original price. Since 60% of \$30 is \$18, choice B is correct.
What mistake would be made to arrive at the incorrect choices?	**Choice A:** If you find 40% of \$30, you get \$12. This is how much you save, not the sale price of the headphones. **Choice C:** If you misread 40% as one-fourth, you will incorrectly calculate the savings as \$7.50 and the sale price as \$22.50. **Choice D:** If you misread 40% off as \$4 off, you will incorrectly calculate the sale price as \$26.
Practice Problems	1. A sweater that is regularly priced \$25 is on sale for 20% off. What is the sale price of the sweater? A. \$5.00 B. \$12.50 C. \$20.00 D. \$23.00 *(Students would copy and answer four more questions similar to the original question here.)*
Write a problem just like the ones above.	A pair of shoes that is regularly priced \$30 is on sale for 30% off. What is the sale price of the pair of shoes? A. \$9.00 B. \$20.00 C. \$21.00 D. \$27.00
Connections, Summary, Reflection, Analysis	Today I learned that when you are finding the price of an item on sale, you have to make sure that you subtract how much you save from the original price. Another way to find the sale price is to subtract the percent off from 100%, then multiply by the original price.

2.4: Inquiry Cube

Topic

- Inquiry-based activity involving the slopes of perpendicular lines

Objectives

Students will:

- Ask each other questions to determine what should be on the blank side of the "Inquiry Cube"
- Practice inquiry as a study method
- Access prior knowledge of slope and perpendicular lines to solve the puzzle

Timeline

- One 50–60-minute class period to find the missing side of the "Inquiry Cube"

WICR Strategies

- Inquiry
  — Investigate a problem through the Inquiry method by only asking questions
- Collaboration
  — Work in collaborative learning groups to complete the activity

NCTM Standards

Focal Point Grade 8

Algebra: Analyzing and representing linear functions and solving linear equations and systems of linear equations

Algebra

Instructional programs from pre-kindergarten through grade 12 should enable all students to explore relationships between symbolic expressions and graphs of lines, paying particular attention to the meaning of intercept and slope.

Communication

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- analyze and evaluate the mathematical thinking and strategies of others; and
- use the language of mathematics to express mathematical ideas precisely.

Reasoning and Proof

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- recognizing reasoning and proof as fundamental aspects of mathematics; and
- make and investigate mathematical conjectures.

Rationale

The AVID Math classroom should provide a variety of opportunities for students to experience the Inquiry method of teaching and learning. The *"Inquiry Cube"* is a fun way to engage students in the Inquiry method while making some important connections between math concepts. As students learn the Substitution, Elimination, and Graphing methods of solving Systems of Linear Equations, they do not always make connections between the solution of the system and the intersections of the graphs of the linear equations. They do not always understand that the solution to the system will make both equations true. Most students will memorize the rule for the products of slopes of perpendicular lines, but fail to apply this knowledge when it is required to solve a problem. The *"Inquiry Cube"* is a collaborative activity that asks students to draw on the collective knowledge of the group. By only asking each other questions, they discover what should be on the bottom of the cube. Students are given little direction so they must rely on each other to find the solution. This type of activity will make the target math concept concrete for students, giving them the ability to recognize perpendicular lines and to write the equations for perpendicular lines.

Vertical Alignment

- The "Inquiry Cube" activity enables students to practice inquiry and draw on previous knowledge to complete the square. It can be used at any grade level. As students progress through grades 6–12, the cognitive demand on the student in determining the relationship between the opposite sides of the cube should increase.

Materials/Preparation

- *Student Handout 2.4a:* "Inquiry Cube"
- Cornell note-paper
- Prepare the cubes before class or have students cut out the net and tape the cubes together.
- Prepare some guiding questions to ask groups that are not making progress on the problem. Example: "What form of a line might help you to recognize certain characteristics of the line?"

Instructions

- Arrange the class into collaborative groups of two to four students.
- Distribute one *Student Handout 2.4a:* "Inquiry Cubes" to each group, or the cubes you prepared before class.
- Distribute Cornell note-paper or ask students to create their own.
- Each group should record their ideas, questions, and thoughts that arise while completing the activity.
- Assign someone in the group to record (on Cornell note-paper) the questions, ideas, and thoughts that came up as they worked on the activity. This will help them in summarizing their thinking at the end of the activity.
- Explain that the task is to find what should be on the missing side of the "Inquiry Cube."
- Circulate as students are working to answer clarifying questions or to ask guiding questions.
- If students fail to make initial progress you can prompt them with a question such as, "Do you notice anything about the opposite faces of the cube?"

- When students have a solution, ask them to write a reflection in their Cornell Notes. Do not expect summaries to be written algebraically. Once students have summarized their findings in words, you may want to help them make the connection of how they naturally used inquiry to complete this task.

Higher-Level Questions

Level Two

- Compare and contrast the difference between parallel lines and perpendicular lines.

Level Three

- What other mathematical concepts could you use to create your own inquiry cube?

Formative Assessment

- Assess the students' summaries for understanding of the relationships of the slopes of perpendicular lines.
- Use the Level Three question above and make students or groups responsible for creating their cubes. At what level of complexity are your student-created "Inquiry Cubes?"

Inquiry Cube Answer

The blank side of the cube should have the number 3 in the top right corner (representing the x-coordinate) and the number 4 in the bottom left corner (representing the y-coordinate) where the lines on the opposite faces of the cube intersect.

The equation is $x + 4y = 19$. Once students recognize that it has to be a line perpendicular to the line on the opposite face, they will know that the product of the slopes must be -1. Given the slope and the point of intersection, the equation can be found and written in the form $ax + by = c$.

Sample Inquiry Cube

4

$4x - y = 8$

3

2

$2x + y = 7$

3

1

$2x - y = 0$

2

2

$x - 2y = -4$

3

2

$x + 2y = 5$

1

2.5: Philosophical Chairs - An Investigation of the Volume of Cones

Topic

- The volume of cones

Objectives

Students will:

- Investigate the characteristics of cones
- Develop inquiry skills
- Develop oral and written language skills

Timeline

- One 50-minute class period to complete the "Philosophical Chairs" activity for cone volume and the writing assignment
- One 50-minute class period to complete the extension activity

WICR Strategies

- Writing to Learn
  - — Write and support a prediction
  - — Complete a self-evaluation of participation in the class discourse.
  - — Write about newly acquired understanding
- Inquiry
  - — Investigate the characteristics of cones
  - — Develop a working hypothesis and support it
  - — Change position based on another's argument
- Collaboration
  - — Work as a class to develop a deeper understanding of the characteristics of cones
- Reading to Learn
  - — Read and respond to a written prompt

NCTM Standards

Focal Point Grade 7

Measurement and Geometry and Algebra: Developing an understanding of and using formulas to determine surface areas and volumes of three-dimensional shapes

Focal Point Grade 8

Geometry and Measurement: Analyzing two- and three-dimensional space and figures by using distance and angle

Geometry

Instructional programs from pre-kindergarten through grade 12 should enable all students to analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.

Measurement

Instructional programs from pre-kindergarten through grade 12 should enable all students to apply appropriate techniques, tools, and formulas to determine measurements.

Reasoning and Proof

Instructional programs from pre-kindergarten through grade 12 should enable all students to make and investigate mathematical conjectures.

Rationale

Oral discourse, inquiry learning, and writing to learn are key skills in acquiring a deeper understanding of mathematical concepts. Through daily practice, self-assessment, and guidance, students are given the opportunity to improve facility and fluency and to develop skills in the precise use of the language of mathematics. Vocabulary and language usage can only be improved through regular practice. This activity provides a framework for using and refining oral language and individual technical writing skills as well as building high-level questioning skills. Students working individually and collaboratively are given opportunities in *"The Investigation of the Volume of Cones"* to provide evidence of mastering mathematical content, the use of technical vocabulary, technical writing, and key communication skills.

Vertical Alignment

- "Philosophical Chairs" is a student discussion format that actively engages students from the upper grades of elementary school through high school and beyond. As students progress through courses utilizing AVID strategies, expectations for their abilities to perform at higher levels of discourse will increase. The mathematics that is discussed in "Philosophical Chairs" can range from the most effective strategy for solving a problem in upper elementary school to which proof of the Pythagorean Theorem is the most elegant in a high school Geometry class. Whatever the level, students love the format and should be given ample opportunities in all grades to participate in "Philosophical Chairs" discussions.

Materials/Preparation

- A circle with a radius of approximately 8.5 cm divided into eight to ten sectors. Label each radius.
- Tape
- Scissors
- Graph paper

- Colored markers
- Rulers
- An object to use as a "Talking Stick"
- M & Ms, Skittles, Jelly Beans or other small candies
- *Teacher Reference Sheet 2.5a:* "Using Philosophical Chairs"
- *Student Handout 2.5b:* "Volume of Cones Template"
- *Student Handout 2.5c:* "Philosophical Chairs: Rules of Engagement"
- *Student Handout 2.5d:* "Philosophical Chairs Report"
- *Student Handout 2.5e:* "Philosophical Chairs Written Evaluation Sheet"
- *Student Handout 2.5f:* "Philosophical Chairs Reflection"

Instructions

- Divide students into pairs.
- Distribute *Student Handout 2.5b:* "Volume of Cones Template" and scissors to each pair of students.
- Ask students to cut the template out and cut along one radii of the circle so that by overlapping a cone can be formed. *See Figure 1 below.*

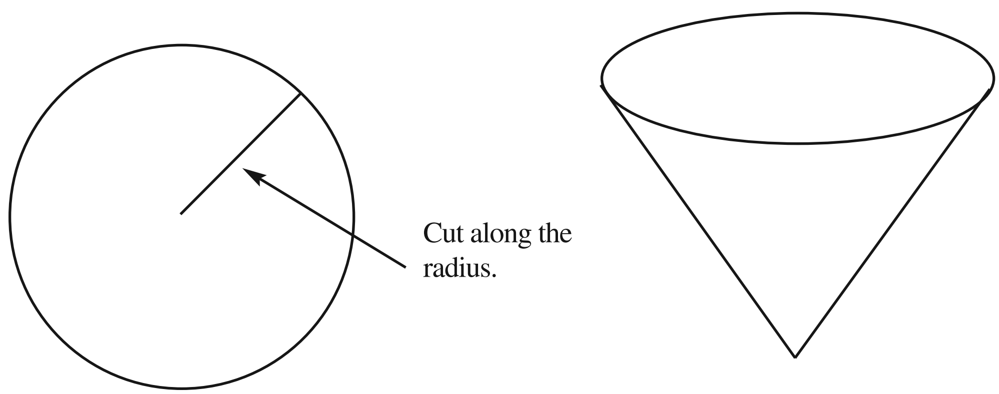


Figure 1

- Ask students to predict what amount of overlap will result in maximum volume. (Labeling each sector will facilitate this discussion.)
- Assign each pair a different overlap amount.
- Ask each pair to form their cone and tape the overlapped edge.
- Pose the Level One question: "Which cone will hold more candies?"
- Ask students to complete a "Two-Minute Quickwrite" predicting the solution and supporting their position.
- Arrange the chairs in the room into four areas representing the possible choices for maximum volume. Will the maximum volume be found by overlapping with sector A, sector B, sector C, or within sectors D–I?
- Ask students to move to the chairs that support their predictions.
- Spend time reviewing and explaining *Student Handout 2.5c:* "Rules of Engagement" for Philosophical Chairs.

- Provide students with an object for use as a "Talking Stick." This will help students honor the commitment to one person talking at a time. See *Student Handout 2.5c:* "Rules of Engagement" for more clarification.
- Encourage students to have an open mind and be willing to be swayed by a good argument.
- When a student's mind is changed, he/she should physically move to the new position. Students may change their mind and move more than one time.
- After giving students sufficient time to discuss their positions, offer to conduct an experiment to find the definitive solution.
- Ask each pair to count the number of candies that their cone will hold. *Alternately, teams could use measurements or the given information (the radius of the template is approximately 8.5 cm) to calculate the volume of their cones.*
- Collect the data in a table of values.
- Ask students to design and construct a graphical method for displaying the data.
- Use a gallery tour or other class sharing activity to share data displays.
- Ask students to draw conclusions based on what they have seen and discussed.
- Complete the recommended post-Philosophical chair writing assignment (*Student Handout 2.5d, 2.5e,* or *2.5f*) or another writing assignment that will help students process their newly acquired understanding.
- An extension of the problem involves developing tables of values and scatter plots that provide opportunities to investigate the volume relationships with a fixed slant height and developing generalizations that could apply to all cones.

Higher-Level Questions

Level Two

- What are "real world" applications of these relationships?

Level Three

- What are the general rules for geometric shapes and maximum volume?
- Is there a general rule that applies to all cones?

Formative Assessment

- How well did students support their arguments in their quickwrite?
- Were students able to follow the "Rules of Engagement" for the Philosophical Chair activity?
- Were students willing to change their position based on another student's argument?
- How well did students explain what they had learned?

Using Philosophical Chairs

Introduction

Philosophical Chairs is a format for classroom discussion and an activity that can be used in any content area. While this activity uses a format similar to debate, it is dialogue that we value. The benefits of this discussion activity include the development of students' abilities to give careful attention to other students' comments and to engage in dialogue with one another to gain a greater understanding of the topic presented.

Like Socratic Seminar, Philosophical Chairs exemplifies the use of WICR strategies in lesson planning. Inquiry and collaboration are inherent in Philosophical Chairs, and writing and reading are easily incorporated into a plan that results in the integration of the four components of WICR. Additionally, this activity makes a great prewriting activity as it allows students to gain and develop a variety of ideas about a topic.

Philosophical Chairs differs from Socratic Seminar in that it is not dependent on a text, but the reading of some text before engaging in the activity can only enhance the process. Philosophical Chairs focuses on a central statement or topic that is controversial.

Because the basic format for Philosophical Chairs remains the same from grade level to grade level, no explicit differentiations are included here. You will differentiate from grade level to grade level by choosing central statements or topics with increased complexity and by decreasing the level of teacher involvement in the process. In the middle school years, the teacher will almost always provide the topic and facilitate the discussion. By the junior and senior years in high school, students should be responsible for developing the central statement and for facilitating the discussions. Included in this unit are three activity sheets that may be used as part of the Philosophical Chairs activity. They provide varying degrees of structure. For middle level, you may want to provide more structure to the reflection after the activity. As students become more practiced at Philosophical Chairs and/or are in high school, you may use the activity sheets that are less structured.

Step-by-step guidelines for Philosophical Chairs and additional ideas for successful implementation of this activity in your classroom follow.

Guidelines for Philosophical Chairs

Classroom Setup

Chairs/desks are set up facing each other with about half facing one way and half facing the opposite way.

Directions

1. A statement is presented to the students. This statement might be based on a reading, a problem, or might be a stand-alone statement. Either way, the statement should be one that will divide the class into those who agree with the statement and those who disagree with the statement. Be sure that the statement is written on the board for reference during the activity. (*Note:* Allowing for a group of students who are undecided is addressed later in these guidelines.)
2. Those who agree with the central statement sit on one side and those who disagree sit on the other side.
3. A mediator—who will remain neutral and call on sides to speak—is positioned between the two sides. (The teacher usually fills this role in the beginning or middle school years. Eventually, students should take on this role.) In addition to facilitating the discussion, the mediator may at times paraphrase the arguments made by each side for clarification. It is important that the mediator always remains neutral.
4. The mediator recognizes someone from the side of the classroom that agrees with the central statement to begin the discussion with an argument in favor of the position stated. Next, the mediator will recognize someone from the other side to respond to the argument. This continues throughout the activity, and part of the job of the mediator is to ensure participation by as many students as possible and to keep just a few students from dominating the discussion. The mediator may also put a time limit on how long each side addresses the issue on each turn.
5. In addition to speaking in the discussion, students may express their opinions by moving from one side to other. Anyone may change seats at any time. Changing seats does not necessarily mean that a person's mind is changed, but rather that the argument made is compelling enough to sway their opinions. Students may move back and forth throughout the discussion.
6. The discussion and movement go on for a designated period of time—usually one class period. The mediator may bring the discussion to a close at any time. Each side may be given an opportunity to make a final statement on the issue. If time allows, each participant states his/her final opinion and may also tell which arguments he/she found most convincing.
7. *Optional:* Ask a few students to observe the process and take notes instead of participating. These students will debrief their observations to the class at the end of the activity. You may have students who were absent or unprepared to participate fulfill this role.

Evaluation

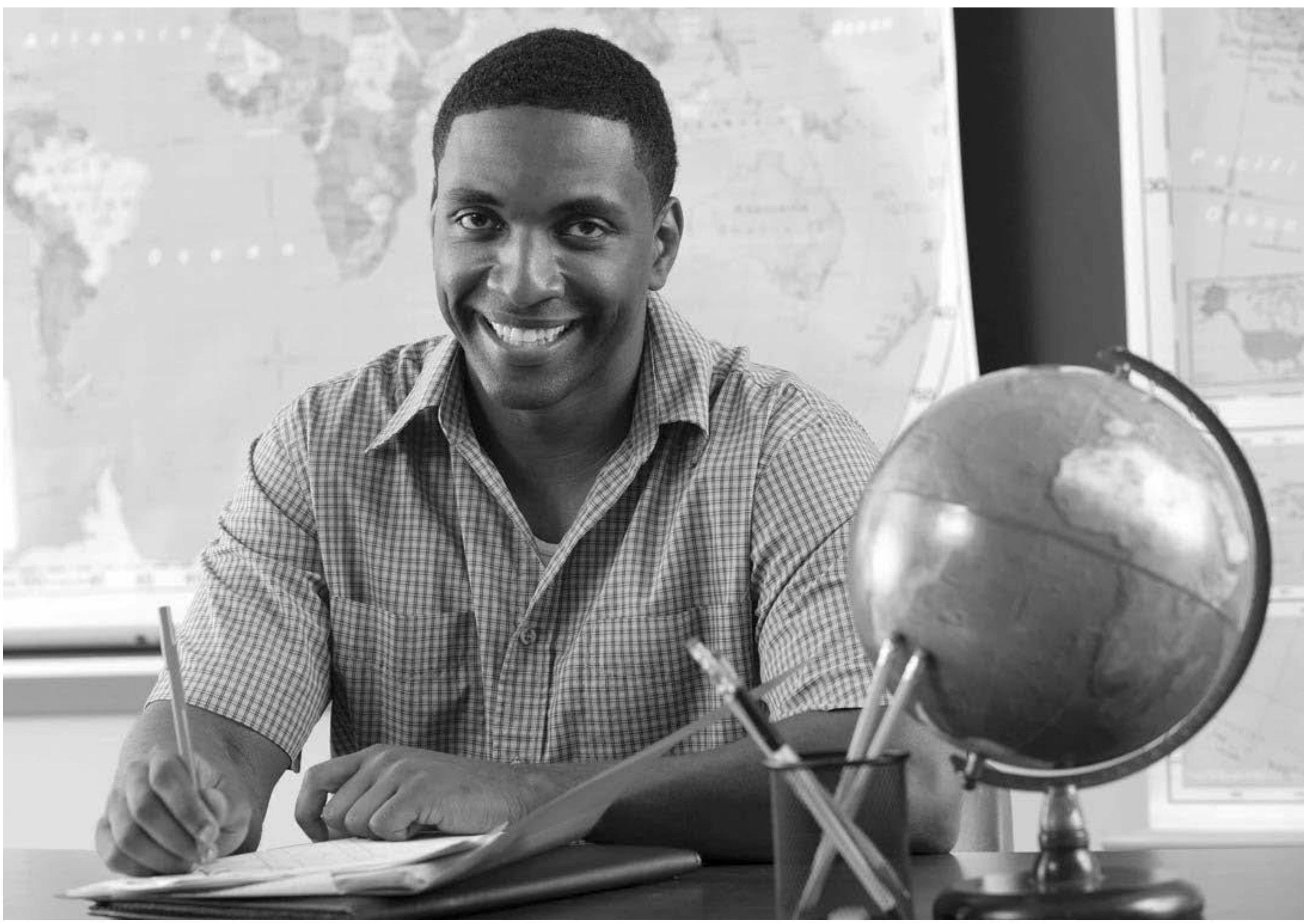
Leave time at the end of the period for students to reflect on the activity. Use one of the activities included in this unit. Students may begin the reflection in class and finish it for homework.

Modifications

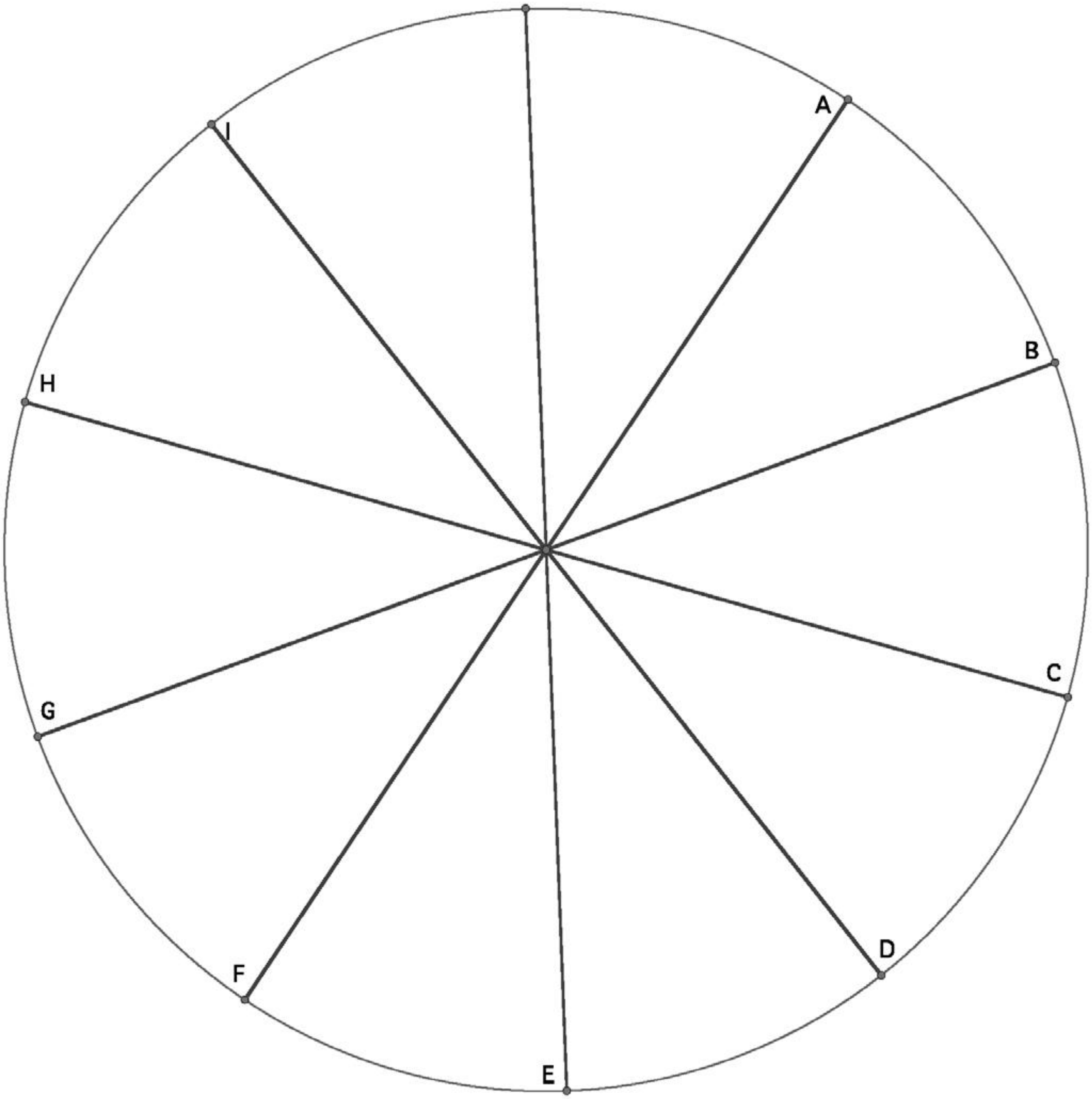
It is recommended that you begin this activity with just two sides. If students have difficulty choosing a side to begin, encourage them to sit on the side that they agree with the most even if they do not completely agree. Once students are accustomed to this format, you may choose to add this additional component: You may add a third section of seats with a few chairs for students who are undecided. This section is placed between the two opposing sides.

During the discussion, you may allow students from the undecided section to participate or you may require that they take a position before participating. Students may move from the sides that agree or disagree with the statement to the undecided section if they wish. Before you end the discussion, require that all students still seating in the undecided zone move to one side or the other depending on which they believe made the most compelling arguments.

When students have mastered the basic structures and strategies of Philosophical Chairs, encourage them to form groups based on their position and discuss possible persuasive arguments. When they have agreed upon a persuasive argument, they should select and send an ambassador or ambassadors to another group to make their argument. Each group should then attempt to convince the ambassador(s) that have been sent to “defect” to their group and point of view.



Volume of Cones Template



Philosophical Chairs: Rules of Engagement

1. Be sure you understand the central statement or topic before the discussion begins. Decide which section you will sit in.
2. Listen carefully when others speak and seek to understand their arguments even if you don't agree.
3. Wait for the mediator to recognize you before you speak; only one person speaks at a time.
4. You must first summarize briefly the previous speaker's argument before you make your response.
5. If you have spoken for your side, you must wait until three other people on your side speak before you speak again.
6. Be sure that when you speak, you address the ideas, not the person stating them.
7. Keep an open mind and move to the other side or the undecided section if you feel that someone made a good argument or your opinion is swayed.
8. Support the mediator by maintaining order and helping the discussion to progress.

Philosophical Chairs Report

- **Central Statement/Topic**

- **My Original Position**

- **How many times did I change my seat? ____________**

- **My Ending Position**

- **How open-minded was I as I listened to other people talk?**
  - ❏ Mostly open-minded
  - ❏ Partially open-minded
  - ❏ Not very open-minded

- **Use the space below to explain why your position did or did not change and the reasons for your thinking.**

Philosophical Chairs
Written Evaluation Sheet

Directions: Answer each of the following questions about today's Philosophical Chairs activity in a few sentences.

1. What was the most frustrating part of today's discussion?

2. What was the most successful part?

3. What statements led you to change your seat or to remain sitting in your original position?

4. What conclusions can you draw about how you form your beliefs based on today's discussion?

5. What would you change about your participation in today's activity? Do you wish you had said something that you did not? Did you think about changing seats but didn't? Explain.

Philosophical Chairs Reflection

Directions: Provide a written reflection of the philosophical discussion you heard in class. Be sure to include the following points in your reflection:

- the statement that was discussed;
- the arguments *for* the statement;
- the arguments *against* the statement;
- your position and the reasons for this position; and
- whether or not you changed your mind during the discussion, which arguments swayed your thinking, and why.

2.6: On Demand Socratic Seminar

Topic

- Engaging students in Socratic dialogue during a lesson or activity.

Objectives

Students will:

- Become more familiar with Socratic Seminar as a formal discussion format
- Engage in Socratic dialogue when issues arise in the flow of a lesson or activity

Timeline

- 10–20 minutes to conduct an "On Demand Socratic Seminar"

WICR Strategies

- Inquiry
  — Explore new concepts and ideas through inquiry.
- Collaboration
  — Work respectfully as a class to investigate new mathematical thinking.

NCTM Standards

Reasoning and Proof

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- recognize reasoning and proof as fundamental aspects of mathematics;
- make and investigate mathematical conjectures; and
- develop and evaluate mathematical arguments and proofs;

Communication

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- analyze and evaluate the mathematical thinking and strategies of others; and
- use the language of mathematics to express mathematical ideas precisely.

Connections

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- recognize and use connections among mathematical ideas; and
- understand how mathematical ideas interconnect and build on one another to produce a coherent whole.

Rationale

In *The Write Path I: Mathematics,* you learned how to engage your students in inquiry-based discussion through Socratic Seminar. The activity in *Write Path I* gave you the formal structure of a Socratic Seminar including many resources to assist you in planning and implementing high-quality Socratic Seminars with your students (See Activity 2.3: “Socratic Seminar” in unit two of *The Write Path I: Mathematics*). In *The Write Path II: Mathematics* we can move into a Socratic dialogue with our students whenever a “Socratic” moment arises in class. The issue could be the various techniques for solving Quadratic Equations, the validity of an argument given for a proof in Geometry, or the concept of absolute value. Whatever the issue is, moving our students in and out of an *“On Demand Socratic Seminar”* is an effective, time-efficient, and thoughtful way for our students to discuss issues of mathematics that arise during a lesson or activity.

Vertical Alignment

- Students should engage in thoughtful discussions involving mathematics at all levels in grades 6–12. As students become more familiar with the structure of Socratic Seminar, they should be able to focus more and more on the mathematics being discussed and less on the rules and norms of the format.

Materials/Preparation

- *Student Handout 2.6a:* “Academic Language Scripts”
- Cornell note-paper
- Individual white boards
- Textbooks or other relevant reference material

Instructions

Note: This activity is given as a strategy to apply to any math lesson or activity. The two activities that follow, 2.7: “The Difference of Two Squares,” and 2.8: “Networks,” are both examples of math activities that lend themselves to an “On Demand Socratic Seminar.” Please see the instructions for these two activities for suggestions on, if, and when you should move your class into an “On Demand Socratic Seminar.”

When you determine that the time is right for an On Demand Socratic Seminar:

- Ask your students to quickly move into a circle. If you have been conducting Socratic Seminars, they should be used to moving into the circle. You may choose to utilize the Fishbowl format for your discussion (see the Active Learning Methodologies in the *Introduction* to the book).
- Ask your students to bring their notes, worksheets, student handouts, calculators, or anything else that they need to discuss the mathematics at hand.
- Ask your students to prepare a piece of paper for Cornell Notes.
- In the left-hand column, ask your students to write down the question to be discussed. You may wish to have a brief pre-discussion with your students to reach consensus on what the question is.
- Conduct the Socratic Seminar adhering to the general principals you learned in *Write Path I.*
- Distribute *Student Handout 2.6a:* “Academic Language Scripts.” Encourage students to use academic language during their discourse.

- Remember that the idea is to get into a discussion and get back out again. You want to get your students back to the lesson or activity as soon as possible.
- When you feel the discussion has reached its conclusion, ask your students to answer the original question in the right-hand column of their Cornell Notes.

Higher-Level Questions

Level Two

- What level was the question that was discussed during our "On Demand Socratic Seminar?"
- How could we have asked a higher-level question?

Level Three

- How can you apply the results of our discussion to the lesson or activity we were working on?

Formative Assessment

- How well did your students adhere to the guidelines for a Socratic Seminar?
- How was the lesson or activity enriched by engaging in the "On Demand Socratic Seminar?"
- What levels of questions did your students ask each other during the discussion?

Academic Language Scripts

Asking for Clarification

- Could you repeat that?
- Could you give me an example of that?
- I have a question about that.
- Could you please explain what _________ means?
- Would you mind repeating that?
- I'm not sure I understood that. Could you please give us another example?
- Would you mind going over the instructions for us again?
- So, do you mean ...?

Requesting Assistance

- Could you please help me understand ...?
- I'm having trouble with this. Would you mind helping me ...?
- Could you please show me how to do this ... write this ... draw this ... pronounce this ... solve this?

Interrupting

- Excuse me, but ... (I don't understand.)
- Sorry for interrupting, but ... (I missed what you said.)
- May I interrupt for a moment?
- May I add something here?

Expressing an Opinion

- I think/believe/predict/imagine that ...
- In my opinion ...
- It seems to me that ...
- Not everyone will agree with me, but ...

Responding

- I agree with what ______ said because ...
- You're right about that.
- That's an interesting idea.
- I thought about that also.
- I hadn't thought of that before.

Disagreeing

- I don't really agree with you because...
- I see it another way.
- My idea is slightly different from yours.
- I have a different answer than you.

Soliciting a Response

- Do you agree?
- What do you think?
- We haven't heard from you yet.
- What did you understand from that answer?

Offering a Suggestion

- Maybe you/we could ...
- Here's something we/you might try.
- What if you/we ...?

Reporting

- __________told me that...
- __________explained to me that ...
- __________pointed out that ...
- __________mentioned that ...
- __________emphasized that ...
- __________shared with me that ...
- __________brought to my attention that ...
- __________pointed out something (interesting, intriguing, surprising).
- I found out from __________ that ...
- I learned from __________ that ...
- I heard from __________ that ...
- I discovered from __________ that ...

Reference: AVID The Write Path Academic Language Scripts, *English Language Learners,* 2006, pp 9–10.

2.7: The Difference of Two Squares

Topic

- Factoring the difference of two squares

Objectives

Students will:

- Create area models representing the difference of two squares
- Pictorially prove the algebraic concept of factoring trinomials in the form of the difference of two squares

Timeline

- One or two 60-minute class periods to create area models for squares and investigate their differences

WICR Strategies

- Inquiry
  - — Investigate geometrical representation of algebraic expressions
  - — Predict the outcome for extending the problem from squares to cubes
- Collaboration
  - — Work in collaborative learning groups to investigate the problem
- Reading to Learn
  - — Read and follow directions in a guided discovery activity
  - — Practice reading algebraic expressions

NCTM Standards

Algebra

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- represent and analyze mathematical situations and structures using algebraic symbols; and
- use mathematical models to represent and understand quantitative relationships.

Rationale

The traditional procedure for factoring the difference of two squares is one of many procedures students encounter during their study of first year Algebra. For many of our students, choosing the appropriate procedure for a mathematical task is often difficult. The *"The Difference of Two Squares"* activity gives students a pathway for connecting the procedure for factoring this special case trinomial to the concept of area of rectangles, which they have been well acquainted with since elementary school. By creating area models for squares and finding the difference in the areas of these squares, students connect the physical model of area to the abstract procedure for factoring trinomials of the form.

Vertical Alignment

- Students can explore many aspects of arithmetic using the area model for multiplication well before the study of first year Algebra. The difference of two squares in Algebra can be explored in earlier grades by using the area model with whole numbers.

  Example: $49 - 36 = (7 + 6)(7 - 6)$

Materials/Preparation

- *Student Handout 2.7a:* "The Difference of Two Squares"
- 5 pages of grid paper per group
- Scissors

Instructions

- Arrange the class into collaborative groups of two to four students.
- Distribute *Student Handout 2.7a:* "The Difference of Two Squares," grid paper, and scissors.
- Ask students to read the directions and respond to clarifying questions regarding the task.
- Remind students that this activity is similar to a model tutorial in the AVID elective.
- As an option, you may have the group prepare and present an oral presentation.
- Circulate as students are working to answer and ask questions. Your role should be to lead the discovery of the formula for factoring the difference of two squares.
- If your students are having difficulty seeing the relationship between the activity and formula we are trying to discover you can move into an "On Demand Socratic Seminar" (see Activity 2.6). Be prepared with some questions for your students to help them get the discussion started.
- Once students have summarized their findings using combinations of words, pictures and symbols, you can help them make the connection to the algebraic notation most often encountered in textbooks.

Higher-Level Questions

Level Two

- Compare and contrast the difference in the area of the different squares. Regardless of the size of the squares you found the same outcome, why?

Level Three

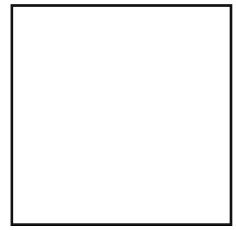
- Predict what would happen if you started off with a cube instead of a square. What do you think your findings would be if you were dealing with volume versus area?

Formative Assessment

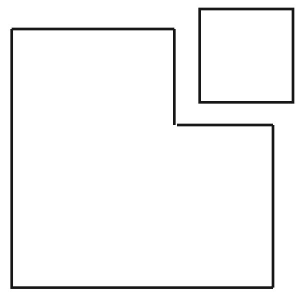
- Were your students able to follow the directions?
- Did your students discover any patterns as they investigated the difference in their squares?
- Were your students able to explain their thinking on this activity? Ask your students to do a quickwrite at the end of the investigation to explain what they discovered.

The Difference of Two Squares

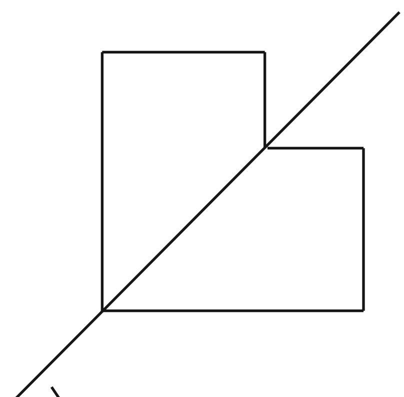
1. Each person in your group needs to have a square. No two squares can be the same size. The smallest square must have an area larger than one square unit.



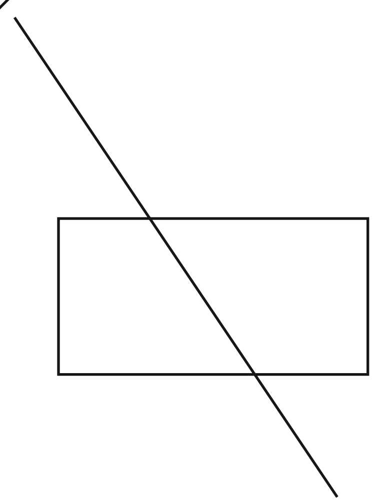
2. Each person needs to cut out a smaller square from the corner of his/her larger square. These smaller squares can be the same or different sizes. Put this square aside for a moment.



3. Imagine that the smaller square is still there. On what is left of the larger square, draw a diagonal line through the corners that would have bisected the smaller square. Cut along that line.



4. Rearrange the two irregular quadrilaterals you have just created into a rectangle. If this seems difficult, have someone in your group who is spatially talented assist you.



5. Fill in the chart below:

Group Member Names (Initials)	Large Square Side Length	Large Square Area	Small Square Side Length	Small Square Area	Difference of the Area of the Two Squares	Rectangle Length	Rectangle Width	Rectangle Area

6. Now summarize your findings. Use words, pictorial symbols, and/or algebraic notation to explain how this activity proves that your findings are true.

2.8: Networks

Topic

- Explore the minimum distance needed to connect three points by applying the distance formula

Objectives

Students will:

- Work in collaborative groups to minimize the straight line distance needed to connect three points
- Apply the distance formula to investigate a Graph Theory problem

Timeline

- One 50–60-minute class period to explore networks

WICR Strategies

- Writing to Learn
  — Write a reflection about the process and the findings of the activity
- Inquiry
  — Investigate extensions to this problem
- Collaboration
  — Work in collaborative groups to find the location of the junction that guarantees the minimal connecting distance
- Reading to Learn
  — Follow the instructions on the student handout

NCTM Standards

Focal Point Grade 8

Geometry and Measurement: Analyzing two- and three-dimensional space and figures by using distance and angle

Geometry

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships; and
- specify locations and describe spatial relationships using coordinate geometry and other representational systems.

Communication

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- analyze and evaluate the mathematical thinking and strategies of others; and
- use the language of mathematics to express mathematical ideas precisely.

Materials/Preparation

- *Student Handout 2.8a:* "Networks"
- Graph paper
- Straightedges or rulers
- Compasses
- Protractors
- Calculators
- Chart grid paper
- *Optional:* Geometer's Sketchpad

Rationale

In a standards-based educational environment where teachers are under increasing pressure to adhere to curriculum pacing schedules, it is difficult to find the time to investigate truly interesting mathematics. Teachers will often say, "I can't afford to waste time exploring a math problem if it isn't directly related to my standards." This position is more than understandable in today's high-stakes testing environment. For teachers to take the time to step out of their pacing schedule to investigate one math problem, it has to satisfy two requirements: first, it has to relate to the grade or course level standards and second, it has to be a really interesting math problem. *"Networks"* satisfies both of these requirements. Most states have the formula for finding the distance between two points somewhere in the middle grades standards, often connecting the formula to the Pythagorean Theorem. Any good problem should promote rich and engaging discussion and the *"Networks"* problem offers at least two opportunities to engage students in an *"On Demand Socratic Seminar"* (see Activity 2.6 in the *Inquiry* section).

Vertical Alignment

Another characteristic of a good math problem is that it offers multiple entry points to the problem. Sixth and 7th grade students who have just learned the formula for finding the distance between two points can apply their new knowledge to this problem, limiting their choices to whole number values for the x and y coordinates; Geometry students can perform Torricelli's Construction to find the Steiner Point; and AP Calculus students can consider the problem in three-dimensional space rather than in the two-dimensional coordinate plane. At any level, "Networks" provides ample opportunity for students to investigate and discuss some very interesting mathematics.

Instructions

- Arrange the class into collaborative learning groups of two to four students.
- Distribute graph paper, straightedges, etc.
- Provide the following background on this problem to the class:

  In 1989, a consortium of telephone companies completed a trans-pacific connection of the islands of Japan, Guam, and Hawaii with fiber optic cable. The project, TPC-3, required laying the cable on the floor of the ocean connecting the islands with the least amount of cable, and thus, the least cost.
- Distribute *Student Handout 2.8a:* "Networks" to each student.
- Model use of the distance formula by finding AB, BC, and CA. Discuss as a class that one way to connect the three points is to connect A to B, then B to C.
- Ask the groups to quickly find the sum of the two shortest distances. This is called the *"Minimal Spanning Tree"* (MST).
- Pose the following questions:

  — "Is this the minimum distance to connect the three points?"

  — "Does anyone have any ideas on how we might connect the points to find a shorter distance?"
- *(Optional)* This would be an excellent opportunity for an "On Demand Socratic Seminar" (see Activity 2.6 in the *Inquiry* section).
- You may have to add the prompt, "What if we add a third point?"
- Ask a student to choose one point inside triangle ABC. Label this point D.
- New Vocabulary: Point D is called a *Junction Point.* When we connect Points A, B, and C to the *Junction Point,* we have created a *Network*.
- Inform the student groups that their task is to find the location of the Junction Point that will result in the shortest distance for the Network.
- Each student in the group should choose a different location for point D. Be prepared with extra handouts for students who want to try more than one location for point D.
- Once students have had sufficient time to investigate the problem, ask them to list the coordinates of their Junction Points and the distances covered by their Networks on chart paper.
- What is the shortest network? Students may identify integer coordinates for the Junction Point D. Encourage them to try rational coordinates to find a shorter network. This is another excellent opportunity for an "On Demand Socratic Seminar." Ask students to discuss, "How can they be certain that they found the shortest network?"
- *(Optional)* If you want your students to investigate this problem, ask them to search the Internet for "Toricelli's Construction." This geometric construction will find the *Steiner Point,* which will be the junction point for the shortest network.
- *(Optional)* See instructions on the next page to find the Steiner Point for any three points in the coordinate plane.

- *(Optional)* If you are proficient with "Geometer's Sketchpad," you can estimate the location of the Steiner Point by asking the program to calculate the distance of the network while you move the junction point.
- Ask your students to write a reflection about the process their group used in investigating this problem and any discoveries they made.

Higher-Level Questions

Level Two

- Predict the shape of the shortest connection (Steiner tree) between 4 points in a square or a rectangular array.
- Explore the special characteristics of the Steiner Point.

Level Three

- In what case would the length of the Steiner network actually be longer than the length of the minimum spanning tree?
- What would the shortest network look like for a 4-point array?

Formative Assessment

- Monitor student use of the distance formula. Are your students able to perform the calculation accurately? Do your students recognize whether the distance calculations are reasonable?
- Monitor student use of the new Graph Theory vocabulary they have encountered in this problem. Are they correctly using the terms "network," "junction point," and "minimal spanning tree" as they investigate the problem?

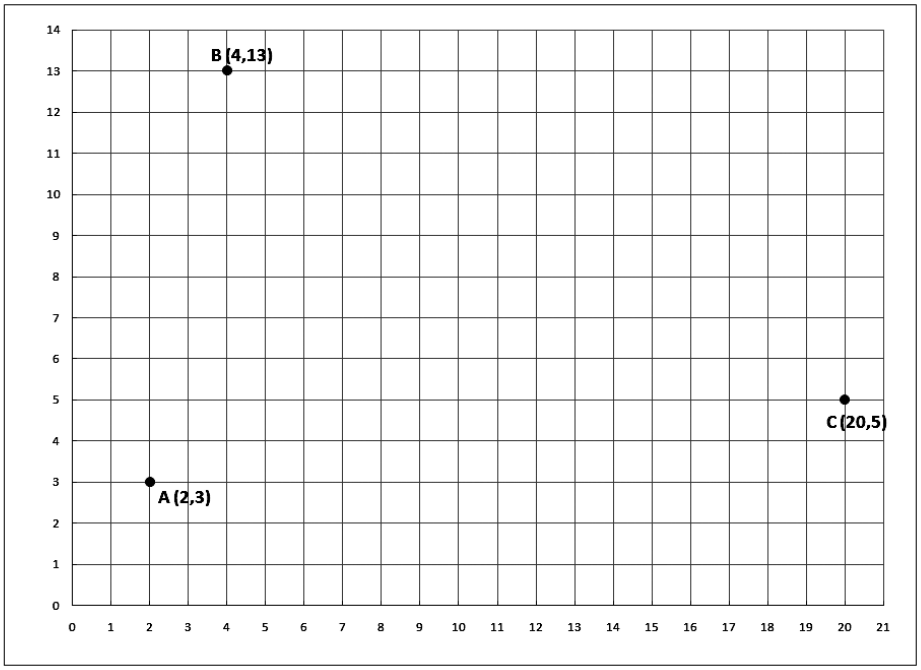
Torricelli's Construction

Finding the Steiner Point (the shortest network to connect 3 points).

- Using a straight edge, draw any acute triangle. Label the vertices, ABC.
- Pick any side, let's say AB, construct equilateral triangle ABD in the exterior of triangle ABC.
- Locate the circumcenter of triangle ABD by constructing the perpendicular bisectors of AB, BD, and DA.
- Circumscribe triangle ABD.
- Draw line segment AD and label the point of intersection of AD and the circle. This is the "Steiner Point." Label it S.
- In a second color, draw line segments SA, SB, and SC. This is the "Steiner Network."

Networks

Directions: Find the shortest distance necessary to connect the points A, B, and C by adding a junction point, D. Each member of your group should pick a different location for point D and find the sum of the distances DA, DB, and DC. Draw your shortest network on the coordinate plane below:



My point D (_____, _____)

DA = _________________ + DB __________________ + DC = _________________

Group Results:

Point								
Distance								

Unit Three: Collaboration in *Mathematics*

Introduction to Collaboration in Mathematics

Within an AVID-inspired classroom, groups are referred to as collaborative. The purpose of collaborative learning is to bring all students together to take responsibility for their own learning. In small groups, they ask, explore, and answer questions, become better listeners, thinkers, readers and writers, and discover ideas and remember them because they are actively involved. The teacher/tutor becomes a coach, carefully guiding students in their learning. Research shows that students learn best when they are actively manipulating materials through making inferences and then generalizing from those inferences. Collaborative groups encourage this kind of thinking.

Collaborative Learning Groups	***Traditional Learning Groups***
Positive interdependence	No interdependence
Individual accountability	No individual accountability
Heterogeneous	Homogeneous
Shared leadership	One appointed leader
Shared responsibility for one another	Responsibility only for self
Social skills necessary for task completion	Social skills ignored
Teacher/tutor observes and intervenes	Teacher ignores group functioning
Groups process their effectiveness	No group processing

Activities that encourage collaboration include: tutorials, jigsaws, group projects, read-arounds, and others.

Preparing for Collaborative Learning Groups

In collaborative learning groups, students experience the process of learning, the "how" as well as the "what" of learning. In order to achieve this, the teacher/tutor must carefully guide the group, thereby encouraging the members to share their ideas and explore and respect the ideas of others. The groups must constantly probe and define and redefine until the expression of ideas is precise and clear. The group task may have individual students share completed assignments or notes, as well as work together to brainstorm and problem solve.

Selection of Groups

In collaborative learning, there is no fixed way to group students. Depending upon the class and the assignment, the teacher may use teacher-determined, self-selected, spatial, or randomly selected groups.

Preparing Students

Students need to be prepared to work in groups, and, indeed, in the beginning, may shy away from group work because they are reluctant to share their work. Group work should begin with experiences that are non-threatening, gradually increasing in task demands and duration. Introduce collaborative group work by discussing group etiquette, stereotyping, and group dynamics with the students before they begin work. Spend time reviewing what a productive group "looks like" and "sounds like" and introduce students to *Student Handout 2.6a:* "Academic Language Scripts." Below are a few reasons why students should collaborate:

1. No one knows everything.
2. Teachers expect analysis, synthesis, and evaluation of subject matter, which is the stuff of collaborative groups.
3. Students will move further faster and remember more.
4. Learning with other people is more fun than studying alone.

Reflecting on Collaborative Groups

Since learning to collaborate in groups is an ongoing process, after completing a group activity the students should write about and discuss what went well in their groups and what they need to improve for the next time.

Avoiding Mayhem

1. Provide the students with careful instructions and simple directions before they move into groups.
2. Establish a specific route for moving into groups.
3. Have students move their desks close together to prevent loud talking and to create a group atmosphere conducive to sharing ideas.
4. Establish a reasonable time limit. Allowing too much time for an activity can cause the groups to deteriorate. It is better for the groups to have too little time than too much. Remember, it takes time and practice for students to learn to work effectively in collaborative learning groups.

3.1: Math Tutorials

Topic

- Working in collaborative tutorials

Objectives

Students will:

- Work collaboratively to explore rigorous tutorial questions
- Demonstrate collaborative skills

Timeline

- One 50-minute class period to model and practice math tutorials

WICR Strategies

- Writing to Learn
  - Write Cornell Notes
  - Write a connection, summary, reflection, and analysis
- Inquiry
  - Investigate tutorial questions
- Collaboration
  - Work in collaborative tutorial groups
- Reading to Learn
  - Read tutorial questions
  - Use a math text as a tutorial resource

NCTM Standards

Communication

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- analyze and evaluate the mathematical thinking and strategies of others; and
- use the language of mathematics to express mathematical ideas precisely.

Connections

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- recognize and use connections among mathematical ideas;
- understand how mathematical ideas interconnect and build on one another to produce a coherent whole; and

- recognize and apply mathematics in contexts outside of mathematics.

Rationale

Collaborative learning groups are the cornerstone of building successful tutorials. Students work together while taking responsibility for their learning, as well as the group's learning. By working in small groups, students have the opportunity to ask higher level questions and work together to explore and answer questions. Collaborative tutorial groups strengthen and enhance the students' listening, thinking, speaking, and writing skills. The collaborative process provides an opportunity to discover new ideas and take ownership of their learning because the students are actively involved.

Vertical Alignment

- Student collaboration for tutorials can be introduced at an early grade and refined as students mature.

Materials/Preparation

- *Student Handout 3.1a:* "Tutorial Learning Process"
- *Student Handout 3.1b:* "Inquiry in Tutorials"
- *Student Handout 3.1c:* "Roles in Collaborative Learning Groups"
- *Student Handout 3.1d:* "Tutorial Practice Problems"
- *Student Handout 3.1e:* "Sample Tutorial Notes"
- Make one set of "Tutorial Practice Problems" for each group of six students.

Instructions

- Model good, higher-level questions for tutorials.
- Provide students with an opportunity to practice writing tutorial questions.
- Encourage students to use their textbooks and other resources to assist them in writing higher-level questions. Assist students in locating higher-level questions in their textbooks by identifying helpful icons and textual cures.
- Distribute and review *Student Handouts 3.1a:* "Tutorial Learning Process," *3.1b:* "Inquiry in Tutorials" and *3.1c:* "Roles in Collaborative Learning Groups."
- Ask students to take Cornell Notes during the tutorial.
- Lead the students in a Fishbowl activity to demonstrate tutorials. You will play the role of the tutor. If you have tutors, they should observe you modeling this role as part of their training.
- Choose five or six students to take part in the Fishbowl demonstration.
- Ask the students participating in the Fishbowl to sit in a U-shape and face the white board. *Note:* Tutorial groups should never exceed seven students. The rest of the class will observe.
- Proceed with the Fishbowl by selecting a student presenter. Ask the student come to the board and write down his/her tutorial question.
- Group members should take notes as the student presenter is at the board. The teacher/tutor should take notes for the student presenter.

- Model the inquiry questioning described on *Student Handout 3.1b:* "Inquiry in Tutorials." Take the lead in the questioning at the beginning of the demonstration.
- As you continue with the demonstration, coach the group members to ask the student presenters questions.
- Demonstrate how the role of the tutor/teacher or peer tutor is to encourage and promote the collaboration and the inquiry used in the tutorial process.
- Group members continue to take notes throughout the demonstration and they participate in the questioning as much as possible.
- Ask student presenters to finish up their question by reflecting on the process used to arrive at the solution.
- At the end of the demonstration, ask all group members to write a reflection and summary on their tutorial notes about the learning from the session.
- End the Fishbowl demonstration by having a class discussion about the roles and responsibilities of each person in the group.
- Organize the students into groups of six students each. Select a group leader/peer tutor for each group to facilitate the group's process.
- Distribute *Student Handout 3.1d*, a set of six "Tutorial Practice Problems," to each group.
- Have the students follow the same process they saw in the Fishbowl. Remind them to use questions to help the student presenter think about his/her question. Remind them that they should not give answers in tutorials.
- Move from group to group modeling good questioning and coaching the students with their collaboration and inquiry skills.
- Stop the groups at least 10 minutes before the period ends. Have one person from each group briefly share with the class some learning that took place in that group.
- Ask students to write a reflective summary and show what they learned in the tutorial.
- Provide frequent opportunities for students to practice collaborative group skills in tutorials.
- *Challenge:* Distribute *Student Handout 3.1e:* "Sample Tutorial Notes." Ask students to improve the study questions and write a reflective summary for the tutorial notes.

Higher-Level Questions

Level Two

- How are Collaborative Tutorials the same or different than traditional tutorials?

Level Three

- What are the advantages of Collaborative Tutorials?

Formative Assessment

- Were student tutorial questions written at a higher level?
- Did students work collaboratively in finding solutions to tutorial questions?
- Did students take Cornell Notes on all tutorial questions?
- Were student reflections completed accurately?

Tutorial Learning Process

1. For homework the night before tutorials, write two questions from your classwork, text, or homework. *Note:* These questions should require higher-level processing and should not be "first level" questions.

2. The teacher/tutors or peer tutors should collect your tutorial questions as you enter the room.

3. The teacher/tutors or peer tutors should form tutorial groups based on the content of your questions. These groups should have four to seven students in them. The seating configuration should be a semi-circle and it will facilitate your communication if you face the board on which the student presenter illustrates their problem.

4. The tutor/teacher or peer tutor should be positioned with the seated members of the group and should facilitate the process of selecting a student presenter.

5. The student presenter should write his/her question on the board and explain to the seated members of the group the difficulty he/she is having. He/she should expect the seated members of the group to ask questions that clarify their own understanding of the question/problem, questions that check for understanding, and questions that probe deeper into possible approaches to solving the problem. When the presenter understands the problem with greater clarity, he/she should then communicate to the group this understanding.

6. The seated members of the group are responsible for helping the presenter think about the problem by asking questions. They are not responsible for finding the solution or necessarily leading the presenter to a solution. They should however, ask questions to clarify their understanding and to push the thinking of the presenter.

7. The teacher/tutor or peer tutor should facilitate the inquiry aspect of this tutorial process by guiding the seated members of the group with questions and modeling questions that they might ask the student presenter. The teacher/tutor or peer tutor should remind the seated members of the group to focus on the presenter's thinking, not the solution to the problem.

8. The teacher/tutor or peer tutor should remind the seated members of the group to take Cornell Notes on all questions.

9. The teacher/tutor should take notes for the student presenter.

10. Near the end of the tutorial session, all members of the group should write a summary/reflection of their learning (content and/or process). You may share these short writings with another member of your group if time permits.

Inquiry in Tutorials

You will often be asked to serve in the role of a peer tutor. The processes and questions provided in this handout will help guide you in your role as a peer tutor and also in your role as a participant or student presenter.

Student Expectations for Tutorials

- The text for inquiry may come from ideas and notes in your learning logs and notes or materials from your math class.
- You will be expected to come to your tutorial group with questions already formulated.
- You will be provided with an opportunity to exchange responses and collaborate in a search for understanding. The strength of the group process rests on the belief that the group can arrive together at some understanding that would not be arrived at independently.

Following is a list of general questions that you may ask in your role as a peer tutor or as a tutorial group participant to help guide the learning of the student presenter:

Understanding the Problem

- What level question have you asked? If a level one question has been offered, assist the group in raising the level of question. Use the textbook and other resources to ensure that a higher-level is being addressed.
- What is this problem about?
- What can you tell us about it?
- Can you explain the problem in your own words?
- What do you know about this part?
- Is there something that we can eliminate or that is missing?
- What assumptions do we have to make?
- How would you explain what we know right now?

Strategies: Thinking It Through

- What have you tried? What steps did you take?
- Do you have a system or strategy?
- What information do you have?
- How did you organize the information?
- What didn't work?
- Have you tried ... (guess and check, list, diagrams, etc.)?
- Where could we find out more information about that?
- Let's look at your notes.
- Let's see if we can break it down. What would the parts be?
- Have you tried making a guess?

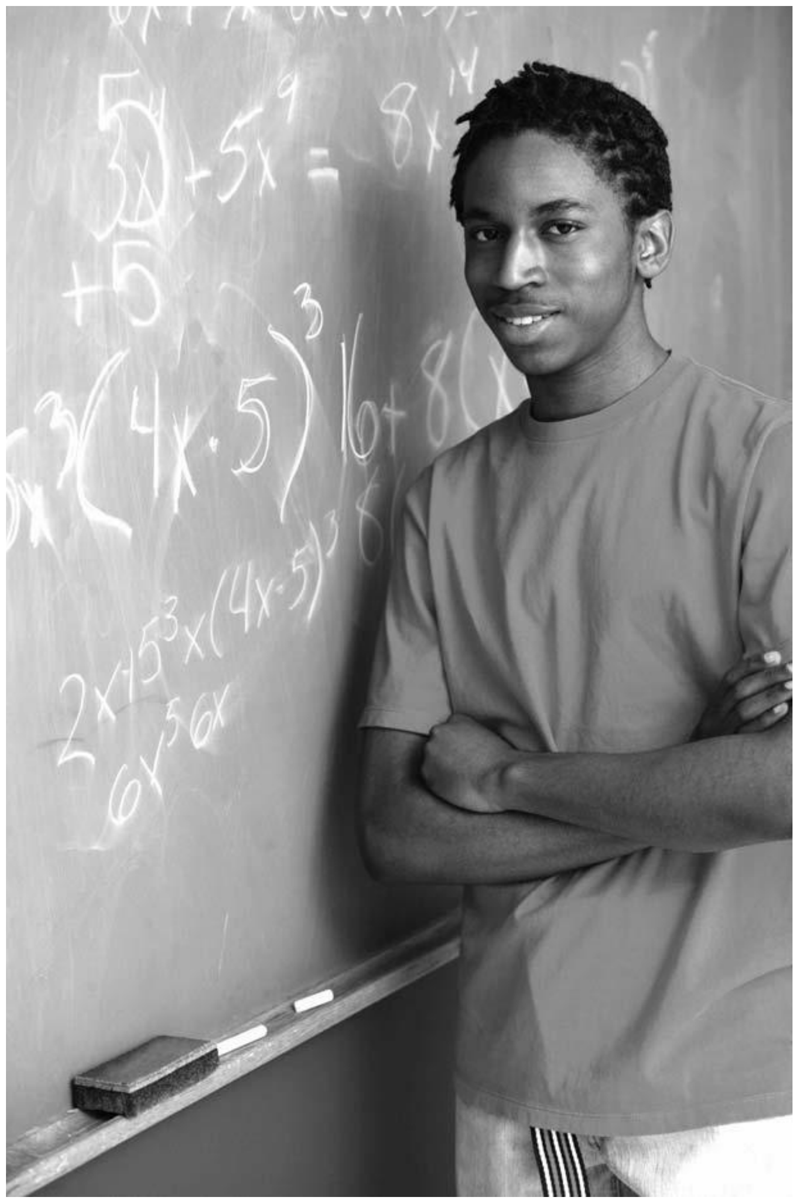
- Can you think of an easier but similar problem?
- What do you think comes next?
- What do you need to do next?

Checking the Solution

- Is your solution reasonable?
- How could you check your answer?
- Is that the only possible answer?
- Is there another way to do this problem?
- How do you know you have completed the problem?

Presenting the Solution

- Is your explanation clear and concise?
- Is there a general rule?
- Did you include charts, graphs, or diagrams in your explanation?
- Can anyone explain it in a different way?
- Is there a "real-world" situation where this could be used?
- Could your method of solving the problem work for the other problems?
- What were some things you learned from this problem?



Roles in Collaborative Learning Groups

Collaborative learning groups are the cornerstone of building successful tutorials. You will be working together while taking responsibility for your learning, as well as the group's learning. By working in small groups, you will have the opportunity to ask higher-level questions as you work together to explore and answer questions. Collaborative groups will strengthen and enhance your listening, thinking, speaking, and writing skills. The collaborative process will provide an opportunity to discover new ideas and take ownership of your learning because you will be actively involved. For true collaboration, it is not essential that all members of the group master the same concepts at the same time. The members of the group will have strengths in a variety of different areas. Depending on the strengths of the individual group members, the collaborative group will create a positive interdependence and productiveness. The teacher/tutor or peer tutor will serve as a facilitator and coach. It is important that all members of the tutorial group understand their role as an active participant in the collaborative tutorial process.

Teacher/tutor or peer tutor's role in the collaborative process:

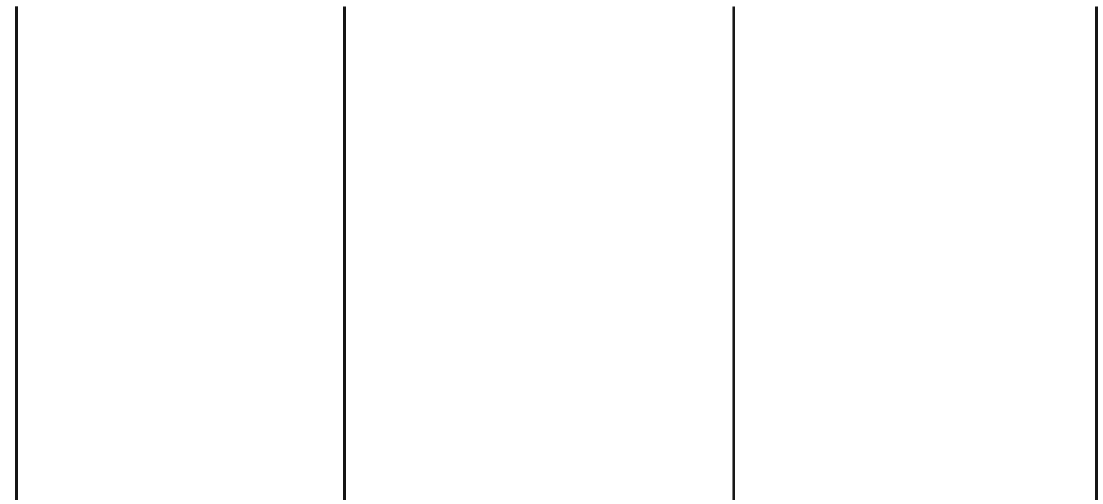
- Encourage group members to respect the ideas/thinking of others
- Model use of inquiry to allow group members to gain a deeper understanding
- Facilitate a balance of shared participation among group members
- Prompt members of the group to use WICR to summarize learning
- Coach members of the group to ask higher-level questions of each other in order to gain a deeper understanding of their rigorous content
- Ensure a safe environment where members of the group are free to ask for clarification of the content

Student's role in the collaborative process:

- Formulate and write higher-level questions in preparation for the tutorial group
- Respect ideas/thinking of others in the group
- Use inquiry to gain a deeper understanding of the content being discussed
- Actively participate in the group by listening, asking questions, answering questions, and taking Cornell Notes
- Use WICR in the collaborative process
- Create an environment where group members feel comfortable and safe to ask questions and seek clarification of content
- Communicate openly about the group experience

Five Marks Problem

Question: How can you add five more marks to make ten?

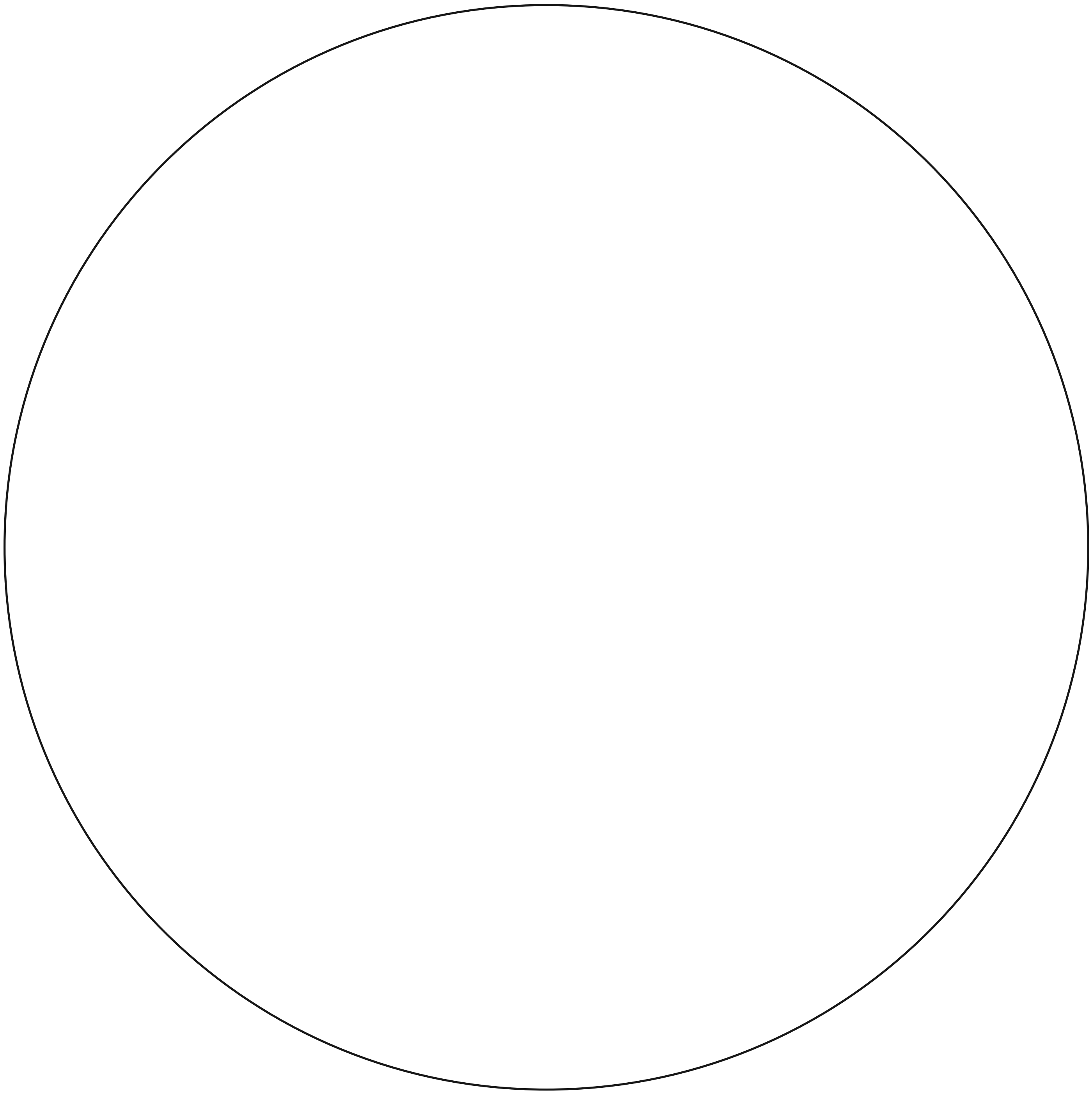


Answer to Amoeba Question

It will take the single amoeba three hours and three minutes to fill the jar. Once the amoeba in the first jar has reproduced itself (a process that takes three minutes), the jar is at the same point at which the second jar started. The only difference is that the amoeba in the first jar is three minutes behind the amoebas in the second jar.

Circle Problem

Question: What is the maximum number of parts into which a circle may be divided by drawing four straight lines?



Answer to the Water Lily Question

The lake is half covered on the fifty-ninth day. Since the water lilies double each day, the lake is half covered the day before it is fully covered.

Amoeba Problem

There are two jars of equal capacity. In the first jar there is one amoeba. In the second jar there are two amoebas.

An amoeba can reproduce itself in three minutes. It takes the amoebas in the second jar three hours to fill the jar to capacity.

Question: How long does it take the one amoeba in the first jar to fill the jar to capacity?

Answer to the Jamais/Toujours Question

1. Make the single question a nonsense question, such as, "Are you a rhinoceros?" Clearly, the individual who claims to be a rhinoceros is from Jamais.

OR

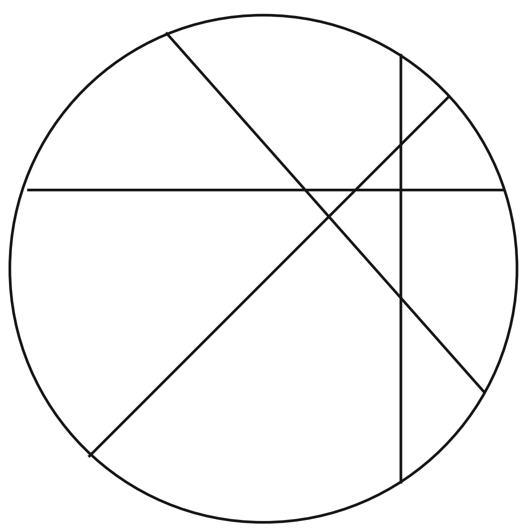
2. Ask any question that you can verify, such as, "Is it raining?"

Jamais/Toujours Problem

You know that the inhabitants of Jamais always lie, while the inhabitants of Toujours always tell the truth. You meet a man who you know comes from either Jamais or Toujours. You want to know which village he comes from.

Question: How can you find out by asking him only one question?

Answer to the Circle Question



Eleven parts may be formed with the four lines. The key is that each successive line must divide as many parts as possible.

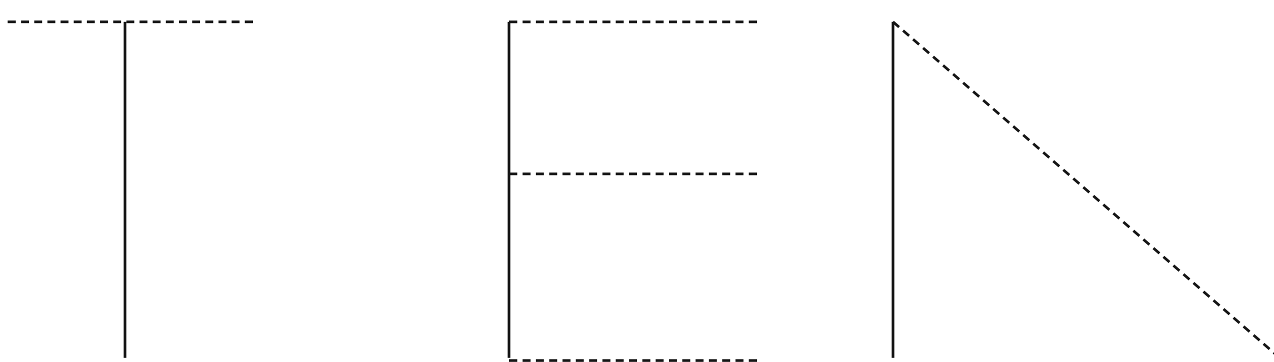
Rope Ladder Problem

A ship is at anchor. Over its side hangs a rope ladder with rungs a foot apart.

The tide rises at the rate of 8 inches per hour.

Question: At the end of 6 hours, how much of the rope ladder will remain above the water, assuming that 8 feet were above the water when the tide began to rise?

Answer to the Five Marks Question



(Two other solutions are also possible. Can you find them?)

Water Lily Problem

Water lilies on a certain lake double in area every twenty-four hours. From the time the first water lily appears until the lake is completely covered takes sixty days.

Question: On what day is the lake half covered?

Answer to the Rope Ladder Question

Since the ship is afloat, the water level in relation to the ship is always the same. Therefore, eight feet of the rope ladder are above the water at the end, just as at the beginning.

Sample Tutorial Notes

Tutorial oct 25,

How can you solve a function by grouping?

$f(x) = 2x^3 - 3x^2 - 8x + 12$

$= (2x^3 - 3x^2) - (8x + 12)$

$x^2(2x-3) - 4(2x+3)$

$(2x-3)\quad(x^2-4)$

$(2x-3)\quad(x-2)(x+2) = 0$

$2x - 3 = 0$

$+3 \quad +3$

$x = 3/2, -2, 2$

$\frac{2x}{2} = \frac{3}{2}$

x is interception

The multiples Principles can be used if theyr is a percentage decrease suppose these is a 20 percentage discount. Again let x be the original cost.

What u the new price in term of x?

A percentage put into an equation to get a number.

new price $= x - (.20x)$

Ex: new shoe = $ 101.00

discount 20%

old shoe $= 101.00 - [.20(101.00)]$

101.00
× .20
00000
20200

101.00
− 20.2
80.8

old shoe = $80.8

3.2: Clustering

Topic

- Exploring associations and connections as a prewriting strategy

Objectives

Students will:

- Work collaboratively to make as many associations and connections as possible to a proposed topic or concept
- Write "Clustering" essays
- Edit peer writing
- Follow the writing process to refine and improve writing skills
- Demonstrate mastery of specific technical writing skills

Timeline

- One 50-minute class period to practice clustering

WICR Strategies

- Writing to Learn
  - — Use a prewriting activity to activate background knowledge
  - — Use the groups associations and connections as a guide for writing a short paragraph
- Inquiry
  - — Explore creative associations and connections
- Collaboration
  - — Work in small groups to brainstorm associations and connections to a proposed topic or concept
  - — Work with a peer to refine and improve writing
- Reading to Learn
  - — Read and edit peer writing

NCTM Standards

Communication

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- analyze and evaluate the mathematical thinking and strategies of others; and
- use the language of mathematics to express mathematical ideas precisely.

Connections

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- recognize and use connections among mathematical ideas;
- understand how mathematical ideas interconnect and build on one another to produce a coherent whole; and
- recognize and apply mathematics in contexts outside of mathematics.

Rationale

Based on the premise that working with the natural rhythms of the brain to create writing produces work that is rich in meaning, clustering is a nonlinear brainstorming process that helps writers discover the ideas and patterns of organization that characterize strong writing. Because clustering often produces material that is abundant in memories, metaphor and wholeness, it is an exciting technique for prompting creative writing. Its application to the academic terrain of technical writing (or even test review) is equally impressive.

Vertical Alignment

- Clustering can be used as a prewriting or review activity at any level.

Materials/Preparation

- Chart paper
- Color markers
- "Technical Writing Tips for Mathematics" (See the *Introduction to Writing in Mathematics*)

Instructions

- Select a word or concept to serve as a nucleus, write it on the board, and circle it.
- Ask students to call out as many associations as they can think of and add those to the board, circling them and connecting them to the word(s) that prompted them.
- Help students make the connection between clustering and writing by directing them to begin writing at any point when they feel they have a direction for writing. Ask students to demonstrate a specific technical writing skill. For examples of technical writing skills that can be incorporated, see the "Technical Writing Tips for Mathematics" in the *Introduction to Writing in Mathematics.*
- Share cluster writings with a partner, groups or the entire class.
- Provide students with an opportunity to read and offer editorial commentary to the writing of their peers.
- Reinforce the importance of the writing process by providing time for students to write second drafts of their cluster writing.
- After students understand Clustering, introduce Clustering as a small group activity. It can be done as a timed contest with each group trying to make the most connections/associations.
  - — Divide the students into small groups of four or five students each.
  - — Distribute chart paper and color markers.

- — Designate a recorder for each group.
- — Designate a time limit.
- — Determine a winning group by counting the number of unique associations and connections.
- — Provide time for writing and editing.
- — Assign third drafts as homework. Ask students to demonstrate a specific technical writing skill.

Higher-Level Questions

Level Two

- What is the most unique/oblique connection or association that was made?

Level Three

- Are some associations/connections more helpful in organizing writing?

Formative Assessment

- Assess the variety and accuracy of associations and connections.
- Review student writing and editing skills.
- Compare first, second, and third drafts.

3.3: Algebra Charades

Topic

- Transformations of linear, quadratic, absolute value, square root, cubic, logarithmic, exponential and rational parent functions

Objectives

Students will:

- Explore transformations of functions through a collaborative, kinesthetic activity
- Identify and write symbolic representations of functions

Timeline

- 20–30 minutes for students to write a transformation, plan and play "Algebra Charades"

WICR Strategies

- Writing to Learn
  - — Write symbolic representations of functions
- Collaboration
  - — Work in teams to present functions using algebra aerobics, while the rest of the class determines which function is represented

NCTM Standards

Focal Point Grade 6

Algebra: Writing, interpreting, and using mathematical expressions and equations

Focal Point Grade 8

Algebra: Analyzing and representing linear functions

Algebra

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- represent and analyze mathematical situations and structures using algebraic symbols; and
- analyze change in various contexts.

Geometry

Instructional programs from pre-kindergarten through grade 12 should enable all students to apply transformations and use symmetry to analyze mathematical situations.

Communication

Instructional programs from pre-kindergarten through grade 12 should enable all students to organize and consolidate their mathematical thinking through communication.

Representation

Instructional programs from pre-kindergarten through grade 12 should enable all students to use representations to model and interpret physical, social, and mathematical phenomena.

Rationale

In the *Write Path I: Mathematics* activity "Algebra Aerobics," students engaged in representing transformations of parent graphs of functions with their arms and legs. In *"Algebra Charades,"* the activity is extended in a variety of ways. Students work in groups to write and represent functions that need more than one person to model. One student acts as the coordinate plane with extended arms representing the x-axis. Other group members act out various aspects of the graphical representation of the function and a transformation of the graph. The other groups work together to determine and write the function and its transformation. The purpose of this activity is to provide kinesthetic practice and to assess students on their knowledge of parent functions and transformations.

Vertical Alignment

- This activity can be done at any level when a new parent graph is introduced. Students in grades 6–8 can represent linear, quadratic, and cubic equations. As students move up through high school mathematics they encounter new functions and their graphs and can engage in "Algebra Charades" as an extension to their study of these new functions, including square root, power, rational, trigonometric and other higher-level functions.

Materials/Preparation

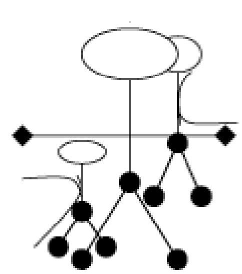
- Index cards or paper for each group
- *Optional:* Mini dry-erase boards

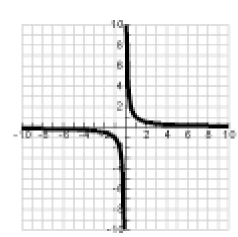
Instructions

- Place students in collaborative groups of three to four.
- Ask each group to write a parent function and a simple transformation of a parent function on a piece of paper or index card.
- Ask the groups to plan out their charades. Set a short time limit.
- Each group then acts out their charades to the class one group at a time. Groups that are guessing should write their guesses on either a sheet of paper or on a mini dry-erase board. Each group to guess the correct function and translation earns a point for that round.
- *(Optional)* The teacher can provide prizes or some other reward for best charade, best writing of equation, and for the group with the most correct guesses.

Example: Three students acting out $y = {}^{1}/_{x}$

- One student acts as the x and y axes, while two other students each represent part of the function.





***Optional:* Extending Algebra Charades to Interval Notation**

- Teams of four students can act out interval notation such as $-3 \leq x < 5$.
  - — The student acting as negative 3 wraps their arms around themselves depicting the closed endpoint.
  - — The student acting as positive 5 makes a circle with their arms depicting the open endpoint.
  - — The other two students stand next to the two students depicting endpoints, pointing with their arms in the appropriate direction.
  - — This activity can be done with a number line drawn on a whiteboard or on the floor.

Higher-Level Questions

Level Two

- Why do some groups need more than two people to represent their function?

Level Three

- How many people would be required to represent a given function with a given translation?

Formative Assessment

- Monitor the use of the mathematics academic language students use in their groups associated with the course-level functions and translations. Are students using words like axis, intercept, asymptote, and concave up or down?

3.4: Complicating Equations

Topic

- The role of inverse operations in solving linear equations

Objectives

Students will:

- Begin with the solution of a linear equation and "complicate it" to arrive at the original equation
- Write descriptions of the steps they used to complicate and solve the equation

Timeline

- 20–30 minutes for students to identify the inverse operations that are related in both complicating and solving equations

WICR Strategies

- Write to Learn
  - — Write linear equations for a given solution
  - — Write descriptions of inverse operations
  - — Write a summary/reflection describing the role of inverse operations in solving linear equations
- Collaboration
  - — Work in collaborative learning groups to complete the activity

NCTM Standards

Focal Point Grade 6

Algebra: Writing, interpreting, and using mathematical expressions and equations

Focal Point Grade 7

Number and Operations and Algebra: Developing an understanding of operations on all rational numbers and solving linear equations

Focal Point Grade 8

Analyzing and representing linear functions and solving linear equations and systems of linear equations

Rationale

One of the fundamental concepts we use to solve equations is the idea of using inverse operations to "undo" what is happening in the equation. If we have the expression ***4x-7*** on one side of an equation, we look at the ***-7*** and decide that we can undo the ***-7*** by adding ***7*** to both sides of the equation. We then look at ***4x*** as ***"4 times x"*** and choose to divide both sides by ***4*** in order to solve for ***x***. In *"Complicating Equations,"* we explore the use of inverse operations as a tool for solving linear equations. Students begin with the solution to an equation, then "complicate it" by applying one of the four operations with a number to both sides, recording the descriptions of their complications on *Student Handout 3.4a:* "Complicating Functions" as they go. Students then trade their complicated equation with a partner. The partner solves the complicated equation by looking at the clues located in the description. As the equation is solved, the solving descriptions are recorded on the same handout. Finally, students identify the inverse operations that are related in both complicating and solving the equation. The focus here is on understanding the role that inverse operations plays in solving linear equations.

Vertical Alignment

- There are many topics in secondary school mathematics where complicating a simple form can play an important role in understanding the underlying mathematics. Other topics include:
  - *Parent Functions*
  - *Transformations of Graphs of Parent Functions*
  - *Function Composition*
  - *The Chain Rule in Calculus*

Materials/Preparation

- *Student Handout 3.4a:* "Complicating Functions"
- Cornell note-paper

Instructions

- Arrange the class into partner groups.
- Distribute *Student Handout 3.4a:* "Complicating Functions."
- Model the activity by completing one of the "Complicating Functions" activity tables on the board or overhead.
- Discuss inverse operations and the role they play in solving linear equations.
- Ask students to copy the example into their Cornell Notes.
- Ask students to write a definition for "inverse operations" into their Cornell Notes.
- Ask students to brainstorm examples of inverse operations and record them into their Cornell Notes.
- To begin the activity, ask the partner groups to complete the first column "Complicating the Equation" and the second column "Complicating Description" on *Student Handout 3.4a.*
- When each student is finished, ask the partners to trade handouts. The last cell in the first column "Complicating the Equation" is copied into the first cell of the fourth column "Solving the Equation." (See the example on page 2 of *Student Handout 3.4a*).
- Ask students to solve their partner's equation and to record their "Solving Descriptions" as they work.
- Remind students to look at the "Complicating Descriptions" for clues and apply what they know about inverse operations to help them in solving their partner's equation.
- The last step is to connect the related cells in the description cells. Descriptions are related if they are inverse operations on the same numbers.
- Ask each student to explain their solutions to their partner's equation.

- Repeat the activity in the second table on *Student Handout 3.4a.*
- In the summary/reflection of their Cornell Notes, ask students to describe the importance of inverse operations in solving equations.

Note: The example given in *Student Handout 3.4a* is for a basic two-step equation. As students encounter more complex linear equations, you can use this activity again for equations that take up to four steps to solve.

Higher-Level Questions

Level Two

- Compare the terms "inverse" and "opposite" when used in mathematics. How are they the same? How are they different?

Level Three

- Write a statement that generalizes the use of inverse operations in solving equations.

Formative Assessment

- Monitor student conversation during the activity. Are students using "inverse" and "operation" as they discuss their equations with their partner?
- Do your students know the difference between "inverse" and "opposite?" Do they misuse the terms?

Complicating Equations

Complicating the Equation	Complicating Description	Solving Description	Solving the Equation

Complicating the Equation	Complicating Description	Solving Description	Solving the Equation

Complicating Equations *Example*

Complicating the Equation	Complicating Description	Solving Description	Solving the Equation
$x = 3$			$-2x + 13 = 7$
$-2x = -6$	**Multiply both sides by -2**	**Subtract 13 from both sides**	$-2x = -6$
$-2x + 13 = 7$	**Add 13 to both sides**	**Divide both sides by 2**	$x = 3$

3.5: Four-Color Functions

Topic

- Multiple representations of functions

Objective

- Students will work collaboratively to investigate multiple representations of functions

Timeline

- One 50-minute class period to design and play a “Four-Color Functions” card game

WICR Strategies

- Inquiry
  — Use of inquiry strategies to match various representations of functions
- Collaboration
  — Work in collaborative groups
- Reading to Learn
  — Read and interpret written descriptions of functions

NCTM Standards

Algebra

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- understand patterns, relations, and functions;
- represent and analyze mathematical situations and structures using algebraic symbols;
- use mathematical models to represent and understand quantitative relationships;
- analyze change in various contexts;
- develop and evaluate inferences and predictions that are based on data; and
- understand and apply basic concepts of probability.

Problem Solving

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- build new mathematical knowledge through problem solving;
- solve problems that arise in mathematics and in other contexts;
- apply and adapt a variety of appropriate strategies to solve problems; and
- monitor and reflect on the process of mathematical problem solving.

Reasoning and Proof

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- recognize reasoning and proof as fundamental aspects of mathematics;
- make and investigate mathematical conjectures;
- develop and evaluate mathematical arguments and proofs; and
- select and use various types of reasoning and methods of proof.

Communication

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- analyze and evaluate the mathematical thinking and strategies of others; and
- use the language of mathematics to express mathematical ideas precisely.

Connections

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- recognize and use connections among mathematical ideas;
- understand how mathematical ideas interconnect and build on one another to produce a coherent whole; and
- recognize and apply mathematics in contexts outside of mathematics.

Representation

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- create and use representations to organize, record, and communicate mathematical ideas;
- select, apply, and translate among mathematical representations to solve problems; and
- use representations to model and interpret physical, social, and mathematical phenomena.

Rationale

Understanding and interpreting multiple representations of functions is a foundation skill for the exploration of higher mathematics. In *"Four-Color Functions,"* students develop and refine collaborative group processing strategies that will serve them well throughout their educational experience.

Vertical Alignment

- Young students may struggle with the functional notation. However, experience and practice in the interpretation of multiple representations of functions will serve them well as they advance in their mathematical studies. The "Four-Color Function" activity can easily be adapted to include less complex and more accessible functions appropriate for the younger student.

Materials/Preparation

- *Student Handout 3.5a:* "Four-Color Activity for Function"
- *Teacher Reference Sheet 3.5b:* "Four-Color Activity for Function *Answer Key"*

- *Teacher Reference Sheet 3.5c:* “Activity Cards”
- Copy four sets of activity cards onto four different colors of paper or cardstock and then cut them out. Make enough sets so that each group of three to four students will have a complete set of each of the four colored cards.

Instructions

- Briefly review the four possible representations of functions and the written notation for functions.
- Divide the class into groups of three to four students each.
- Distribute a complete set of each color cards to each group.
- Distribute *Student Handout 3.5a:* “Four-Color Activity for Function.”
- Provide students with time to work collaboratively to match the four representations of each function.
- Ask each group of students to design game cards that illustrate two to three representations of a function.
- Ask groups to exchange their constructed cards and complete the missing representation(s).
- Encourage students to select examples created by the class to be used in a formative assessment.
- Debrief the activity by highlighting key successful strategies and pitfalls.
- Provide students with time to record their learning in their learning log or in their Cornell Notes.

Higher-Level Questions

Level Two

- What common characteristics do all of the functions presented in the activity share?
- Contrast the four representations of continuous functions verses discontinuous functions. How are they the same? How are they different?

Level Three

- Create four representations of the same concept for something you have studied in mathematics in the past. What types of relationships would you find in the representations?

Formative Assessment

- How well did students work in their collaborative groups?
- Did students demonstrate the ability to match the four representations of functions?
- Were students able to construct examples of functions and show four representations?

Four-Color Activity for Function

Description	Function	Table	Graph
D1	______	______	______
D2	______	______	______
D3	______	______	______
D4	______	______	______
D5	______	______	______
D6	______	______	______
D7	______	______	______
D8	______	______	______
D9	______	______	______
D10	______	______	______

Four-Color Activity for Function *Answer Key*

Description	Function	Table	Graph
D1	F2	T5	G9
D2	F1	T2	G6
D3	F10	T7	G7
D4	F5	T1	G4
D5	F9	T10	G2
D6	F7	T8	G1
D7	F8	T3	G10
D8	F3	T9	G3
D9	F6	T4	G5
D10	F4	T6	G8

Activity Cards

D1 Garrett waited three minutes before starting to fill the bathtub, but after two minutes of adding water to the tub, he decided to drain the water.	**D2** After six minutes of adding water, Camryn noticed that the bathtub was full and turned off the water.
D3 Since the bathtub was already full, Austin used the first four minutes to bathe and started draining the water after that.	**D4** Christine does not like the bathtub full of water so she drained some out during the first four minutes.
D5 After five minutes, Evan added more water to the bathtub for two minutes and then drained some out for three minutes.	**D6** Elle added water to the bathtub for the first two minutes, stopped the water for six minutes while she answered the telephone, and then drained water for two minutes.
D7 Matt added water to the bathtub for five minutes, then drained some for one minute before he added some more water for two minutes.	**D8** Aaron added water for the first four minutes, took a bath for three minutes, and drained water out for three minutes.
D9 Madison was interrupted before her bath so she did not start adding water until four minutes had passed.	**D10** Bryce drained some water for two minutes, rested for four minutes, and then added some more water for two minutes. Then he let the water drain out for two minutes.

F2

$$f(x) = \begin{cases} 0, & 0 \le x \le 3 \\ 40x - 120, & 3 < x \le 5 \\ -20x + 180, & 5 < x \le 9 \\ 0, & 9 < x \le 10 \end{cases}$$

F1

$$f(x) = \begin{cases} (\frac{40}{3})x, & 0 \le x \le 6 \\ 80, & 6 < x \le 10 \end{cases}$$

F10

$$f(x) = \begin{cases} 140, & 0 \le x \le 4 \\ -10x + 180, & 4 < x \le 10 \end{cases}$$

F5

$$f(x) = \begin{cases} -15x + 140, & 0 \le x \le 4 \\ 80, & 4 < x \le 10 \end{cases}$$

F9

$$f(x) = \begin{cases} 100, & 0 \le x \le 5 \\ 20x, & 5 < x \le 7 \\ -20x + 280, & 7 < x \le 10 \end{cases}$$

F7

$$f(x) = \begin{cases} 30x, & 0 \le x \le 2 \\ 60, & 2 < x \le 8 \\ -20x + 220, & 8 < x \le 10 \end{cases}$$

F8

$$f(x) = \begin{cases} 24x, & 0 \le x \le 5 \\ -40x + 320, & 5 < x \le 6 \\ 20x - 40, & 6 < x \le 8 \\ 120, & 8 < x \le 10 \end{cases}$$

F3

$$f(x) = \begin{cases} 20x, & 0 \le x \le 4 \\ 80, & 4 < x \le 7 \\ -20x + 220, & 7 < x \le 10 \end{cases}$$

F6

$$f(x) = \begin{cases} 0, & 0 \le x \le 4 \\ 30x - 120, & 4 < x \le 10 \end{cases}$$

F4

$$f(x) = \begin{cases} -30x + 120, & 0 \le x \le 2 \\ 60, & 2 < x \le 6 \\ 20x - 60, & 6 < x \le 8 \\ -30x + 340, & 8 < x \le 10 \end{cases}$$

T5

x	0	1	2	3	4	5	6	7	8	9	10
f(x)	0	0	0	0	40	80	60	40	20	0	0

T2

x	0	3	6	7	8	9	10
f(x)	0	40	80	80	80	80	80

T7

x	0	1	2	3	4	5	6	7	8	9	10
f(x)	140	140	140	140	140	130	120	110	100	90	80

T1

x	0	1	2	3	4	5	6	7	8	9	10
f(x)	140	125	110	95	80	80	80	80	80	80	80

T10

x	0	1	2	3	4	5	6	7	8	9	10
f(x)	100	100	100	100	100	100	120	140	120	100	80

T8

x	0	1	2	3	4	5	6	7	8	9	10
f(x)	0	30	60	60	60	60	60	60	60	40	20

T3

x	0	1	2	3	4	5	6	7	8	9	10
f(x)	0	24	48	72	96	120	80	100	120	120	120

T9

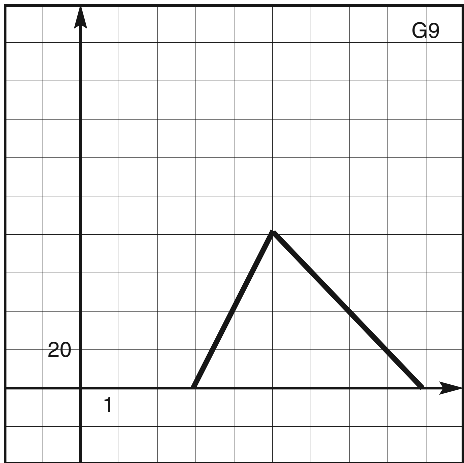
x	0	1	2	3	4	5	6	7	8	9	10
f(x)	0	20	40	60	80	80	80	80	60	40	20

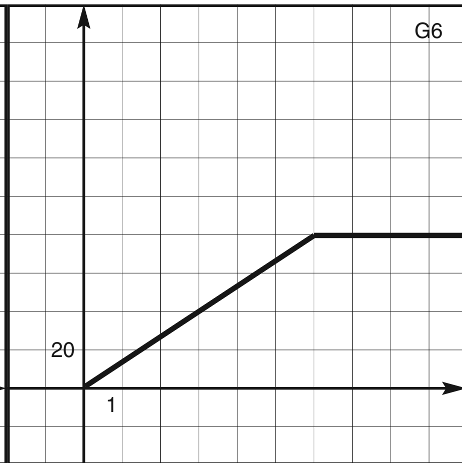
T4

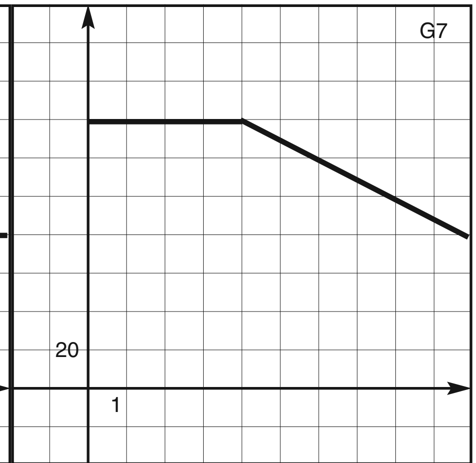
x	0	1	2	3	4	5	6	7	8	9	10
f(x)	0	0	0	0	0	30	60	90	120	150	180

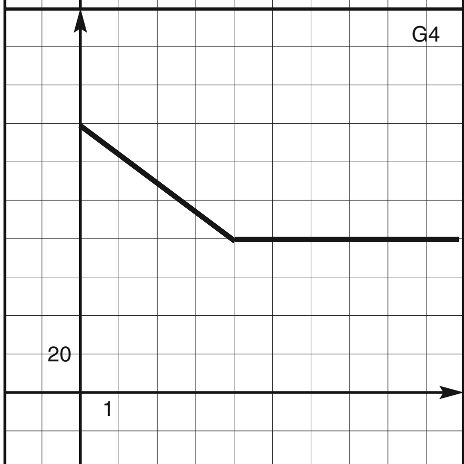
T6

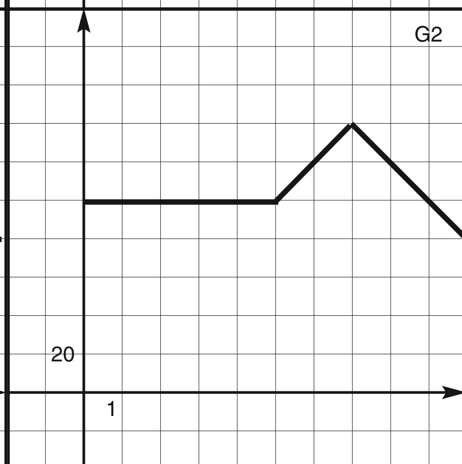
x	0	1	2	3	4	5	6	7	8	9	10
f(x)	120	90	60	60	60	60	60	80	100	70	40

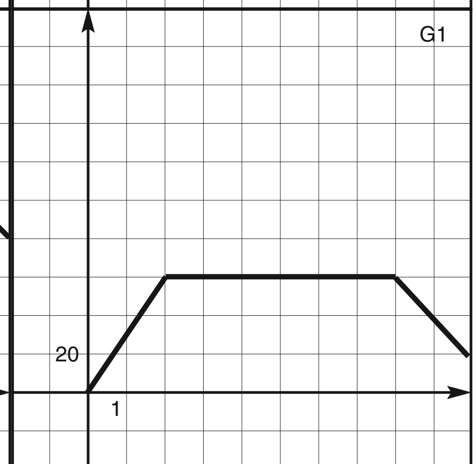

G9
20
1

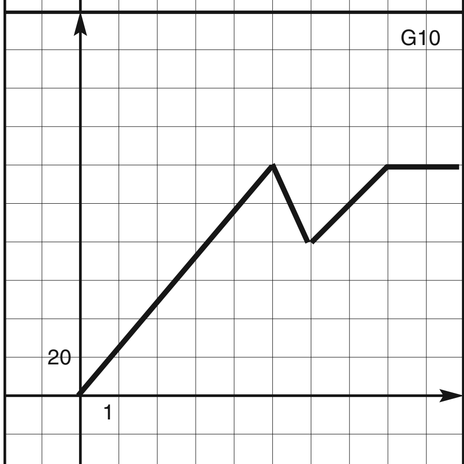

G6
20
1

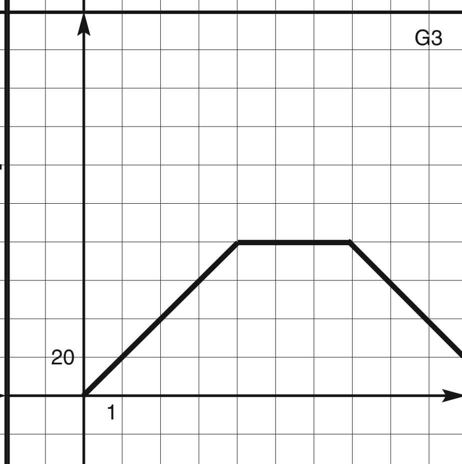

G7
20
1

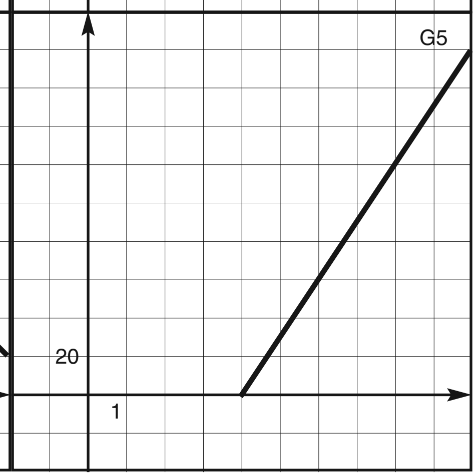

G4
20
1

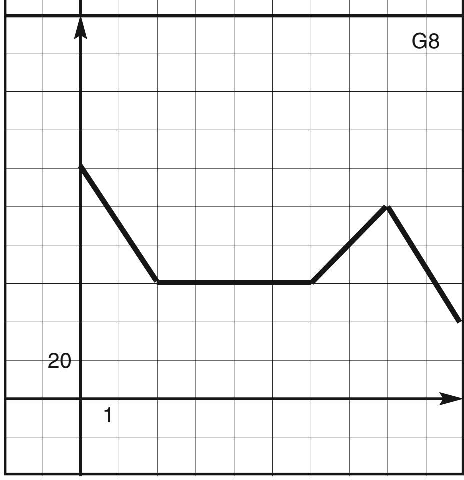

G2
20
1


G1
20
1


G10
20
1


G3
20
1


G5
20
1


G8
20
1

3.6: Investigating Area under the Curve

Topic

- Estimating area

Objective

- Students will use several methods for estimating the area bounded by a function and the *x* and *y* axis

Timeline

- One 50-minute class period to investigate the area under a curve

WICR Strategies

- Writing to Learn
  — Explain solution processes
- Inquiry
  — Investigate various methodologies for finding area
- Collaboration
  — Work in collaborative groups to investigate solution processes

NCTM Standards

Focal Point Grade 7

Measurement and Geometry and Algebra: Developing an understanding of and using formulas to determine surface areas and volumes of three-dimensional shapes

Focal Point Grade 8

Geometry and Measurement: Analyzing two- and three-dimensional space and figures by using distance and angle

Measurement

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- understand measurable attributes of objects and the units, systems, and processes of measurement; and
- apply appropriate techniques, tools, and formulas to determine measurements.

Problem Solving

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- build new mathematical knowledge through problem solving;
- solve problems that arise in mathematics and in other contexts;
- apply and adapt a variety of appropriate strategies to solve problems; and
- monitor and reflect on the process of mathematical problem solving.

Communication

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- analyze and evaluate the mathematical thinking and strategies of others; and
- use the language of mathematics to express mathematical ideas precisely.

Connections

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- recognize and use connections among mathematical ideas;
- understand how mathematical ideas interconnect and build on one another to produce a coherent whole; and
- recognize and apply mathematics in contexts outside of mathematics.

Rationale

Working in collaborative groups and learning through inquiry are critical skills that students can master only by being provided numerous opportunities for practice. The "*Investigating Area under the Curve"* activity provides students at all levels with an accessible problem that will provide a solid foundation for exploration of integrals in higher-level mathematics.

Vertical Alignment

- The first part of the "Investigating Area under the Curve" activity can easily be introduced to very young children and expanded to include increasingly more complex functions and processes as students mature.

Materials/Preparation

- Color markers
- Grid paper ($^1/_2$ inch square)
- *Student Handout 3.6a:* "Investigating Area under the Curve" (8 pages)
- *Teacher Reference Sheet 3.6b:* "Investigating Area under the Curve *Answer Key*"
- *Overhead Transparency 3.6c:* "Investigating Area under the Curve" master graph

Instructions

- Divide students into collaborative groups of three or four students.
- Distribute the *Student Handout 3.6a* packet: "Investigating Area under the Curve."
- Provide students with time to review the directions for Part 1.
- Model the beginning of the activity by finding the area of a hand.
- Provide students with time to work collaboratively to complete Parts 2–7. Teachers may wish to model Part 2 using *Overhead Transparency 3.6c.*

- Allow time to debrief.
- Encourage students to complete their Learning Logs or Cornell Notes to capture their learning.

Higher-Level Questions

Level Two

- Which geometric shape provided a better estimation of the area under the curve? Explain.

Level Three

- What must be considered when determining the number of intervals needed for a close estimate of the true area?

Formative Assessment

- How effectively did students work in their collaborative groups?
- Were the estimates at which students arrived accurate?
- Were students able to accurately answer the questions posed on the student handouts?

Investigating Area under the Curve

Part 1:

1. Trace the outline of your hand onto grid paper.
2. Color the squares that lie fully inside the outline of your hand. (If any part of the square is outside the outline, don't color it.)
3. In another color, color the squares that lie on the outline of your hand. If any part of the square touches the outline, color the ENTIRE square.
4. (a) ________ Number of squares fully inside outline

   (b) ________ Number of squares partially inside outline

   (c) ________ Total number of squares colored
5. What does your answer to part (a) represent?
6. What does your answer to part (c) represent?
7. Write a ratio relating the number of squares to the number of inches.
8. Use this ratio to convert the number of squares you got for question 4, parts (a) and (c), to square inches.

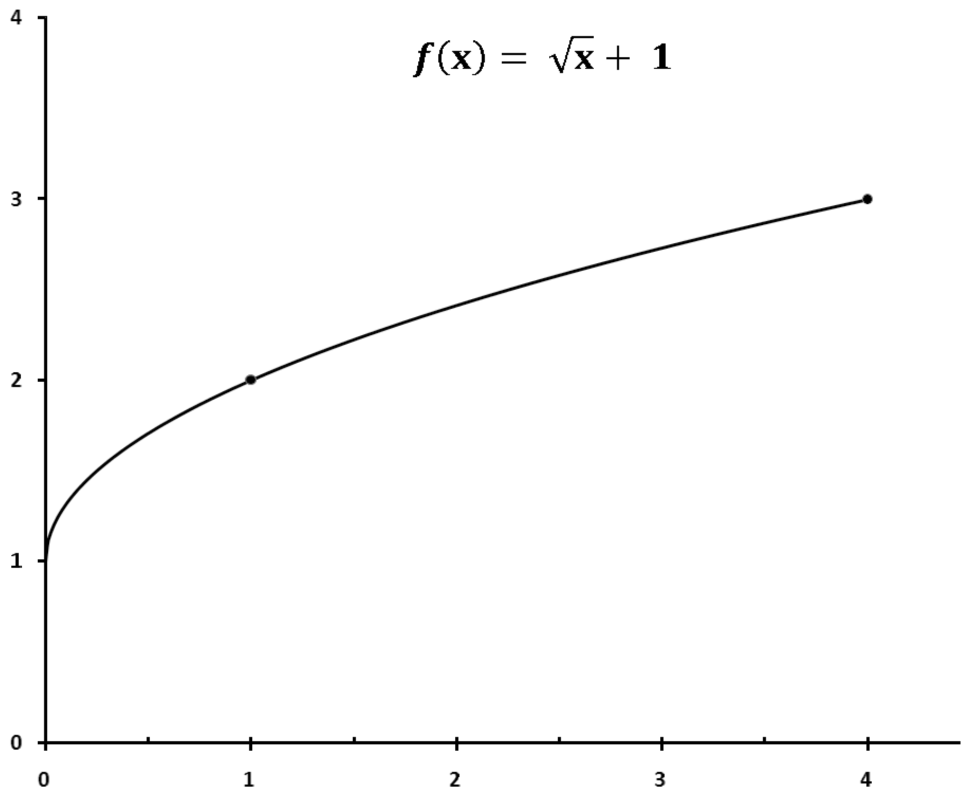
(a) _________ square inchs

(c) _________ square inchs

9. Use your two answers from question 8 to estimate the area of your hand in square inches.
10. Compare your estimate to that of your classmates. Does your estimate make sense? Why?

Part 2:

1. Divide the following graph of $f(x) = \sqrt{x} + 1$ from 0 to 4 into four equal intervals.

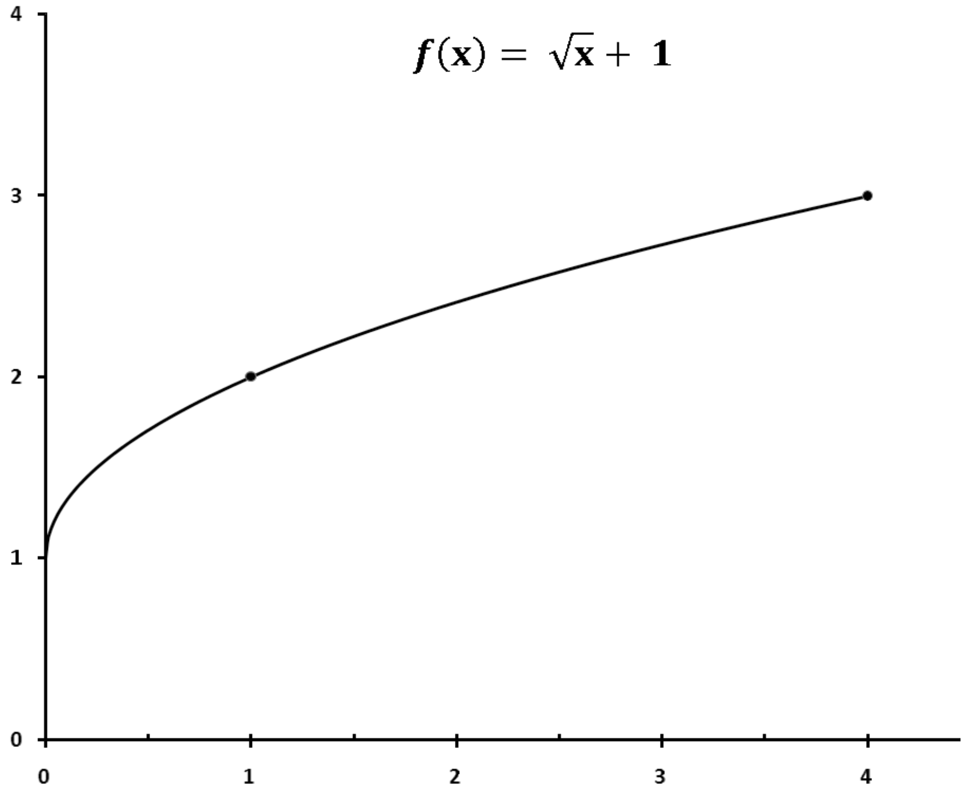


2. Form four rectangles by drawing a horizontal line from the left of the interval to the right.
3. What is the width of each rectangle?
4. What is the height of the first rectangle?
   the second rectangle?
   the third rectangle?
   the fourth rectangle?
5. What is the area of the first rectangle?
   the second rectangle?
   the third rectangle?
   the fourth rectangle?
6. What is the total area of the four rectangles?

7. Do you think the answer to question 6 is an over estimate or under estimate to the area under the curve? Why?

Part 3:

1. Divide the following graph of $f(x) = \sqrt{x} + 1$ from 0 to 4 into four equal intervals.

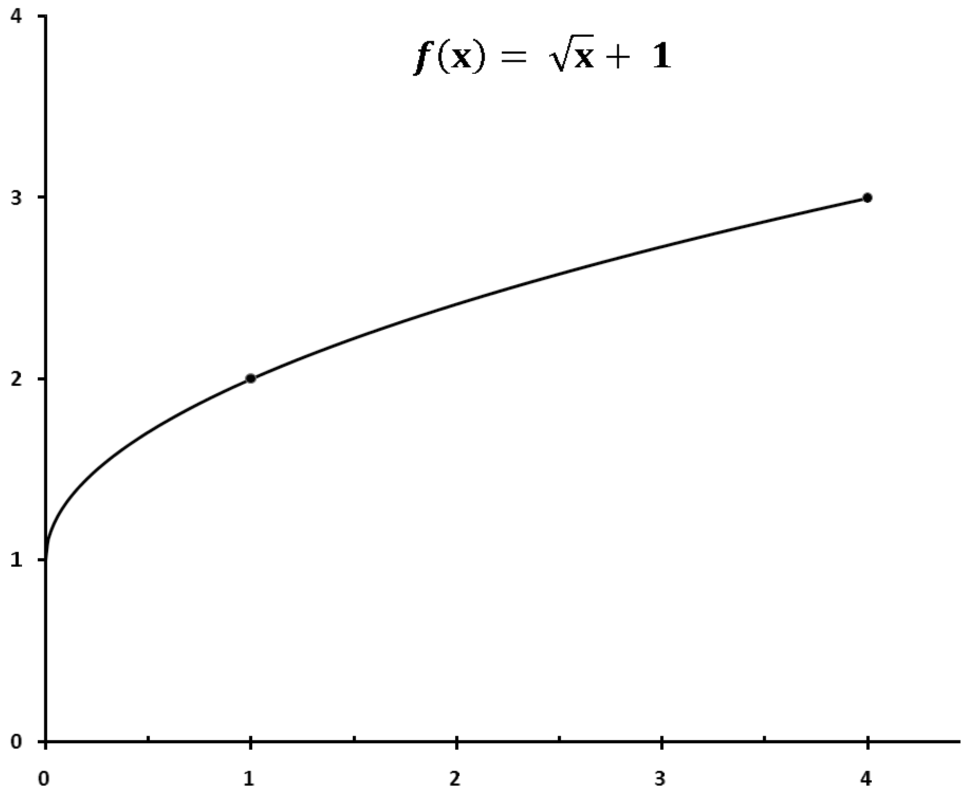


2. Form four rectangles by drawing a horizontal line from the right of the interval to the left. You will have to draw a vertical line to complete each rectangle.
3. What is the width of each rectangle?
4. What is the height of the first rectangle?
   the second rectangle?
   the third rectangle?
   the fourth rectangle?
5. What is the area of the first rectangle?
   the second rectangle?
   the third rectangle?
   the fourth rectangle?
6. What is the total area of the four rectangles?

7. Do you think the answer to question 6 is an over estimate or under estimate to the area under the curve? Why?

Part 4:

1. Divide the following graph of $f(x) = \sqrt{x} + 1$ from 0 to 4 into four equal intervals.

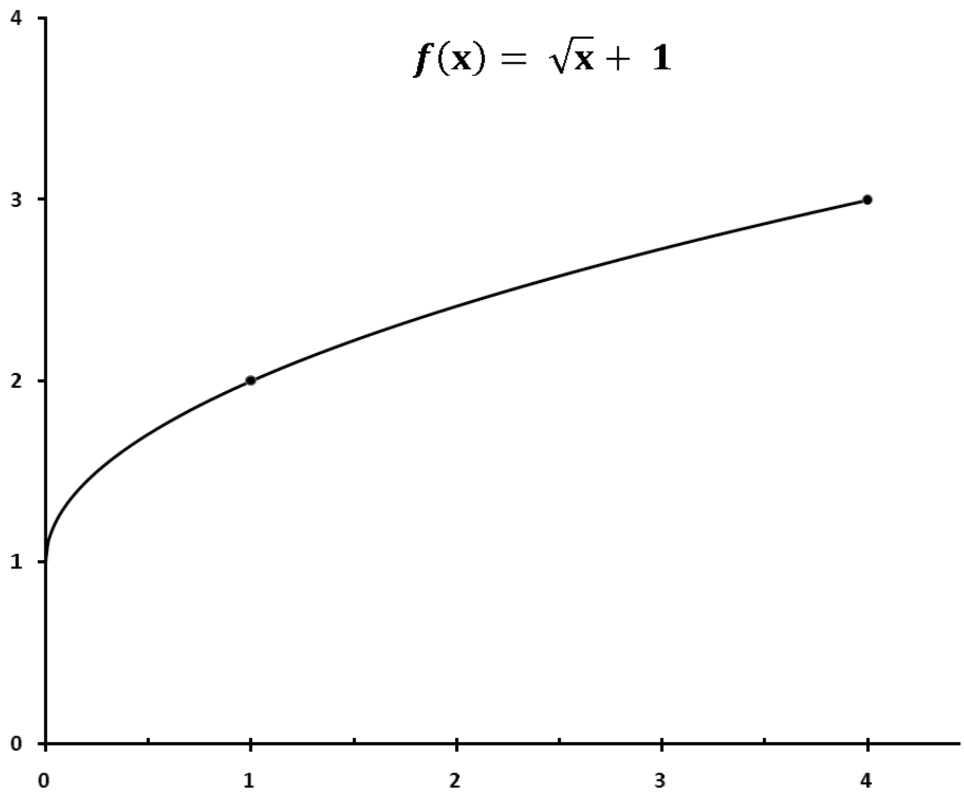


2. Form four rectangles by drawing a horizontal line from the center (midpoint) of the interval to the left and right. You will have to draw a vertical line to complete each rectangle.
3. What is the width of each rectangle?
4. What is the height of the first rectangle?
   the second rectangle?
   the third rectangle?
   the fourth rectangle?
5. What is the area of the first rectangle?
   the second rectangle?
   the third rectangle?
   the fourth rectangle?
6. What is the total area of the four rectangles?

7. Do you think the answer to question 6 is an over estimate or under estimate to the area under the curve? Why?

Part 5:

1. Divide the following graph of $f(x) = \sqrt{x} + 1$ from 0 to 4 into four equal intervals.



2. Form four trapezoids by drawing a slanted line from the left of the interval to the right.
3. What is the height of each trapezoid?
4. What is the height of the first trapezoid?

   Remember, the height is defined as the distance between the bases (the parallel lines).

   the second trapezoid?

   the third trapezoid?

   the fourth trapezoid?
5. What is the area of the first trapezoid?

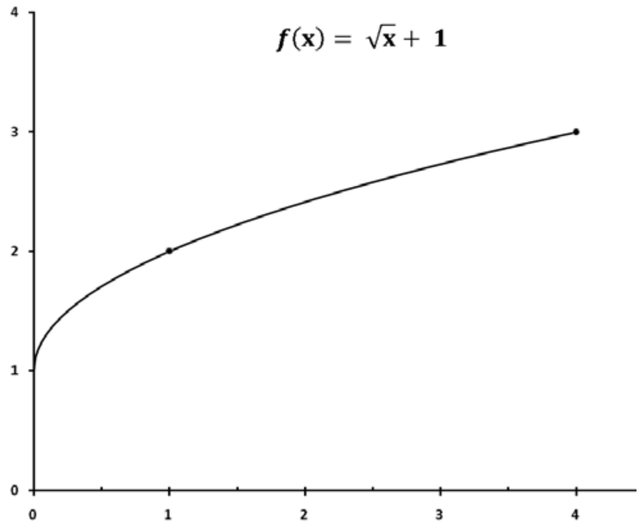
   the second trapezoid?

   the third trapezoid?

   the fourth trapezoid?
6. What is the total area of the four trapezoids?
7. Do you think the answer to question 6 is an over estimate or under estimate to the area under the curve? Why?

Part 6:

1. Divide the following graph of $f(x) = \sqrt{x} + 1$ from 0 to 4 into eight equal intervals.



2. Form eight rectangles by drawing a horizontal line from the left of the interval to the right.
3. What is the width of each rectangle?
4. What is the height of the first rectangle?
   the second rectangle?
   the third rectangle?
   the fourth rectangle?
   the fifth rectangle?
   the sixth rectangle?
   the seventh rectangle?
   the eighth rectangle?
5. What is the area of:
   the first rectangle?
   the second rectangle?
   the third rectangle?
   the fourth rectangle?
   the fifth rectangle?
   the sixth rectangle?
   the seventh rectangle?
   the eighth rectangle?
6. What is the total area of the eight rectangles?

7. Do you think the answer to question 6 is an over estimate or under estimate to the area under the curve? Why?

Part 7:

1. Write the estimates you got from parts 2–6.

   Part 2: Part 5:

   Part 3: Part 6:

   Part 4:

2. Which estimate do you think is the best estimate for the area under the curve formed by $f(x) = \sqrt{x} + 1$? Why?

3. The true area under the curve is a $9\frac{1}{3}$. Why do you think the trapezoidal estimate was so close?

4. How did subdividing the area into more intervals improve the estimate?

5. What would happen if the area was divided into many more intervals?

6. Write the sum of rectangular areas in Part 2 in summation notation.

   In Part 3:

   In Part 6:

7. $\sum_{i=1}^{n} f(x_i)\Delta x$ is the formula for calculating an estimate of the area under a curve from *a* to *b* (or the interval [*a, b*]), and is called the Riemann Sum. How would you define *n*? $f(x_i)$? Δx?

8. $\frac{\Delta x}{2}$ $[f(x_0) + 2f(x_1) + 2f(x_2) + ... + 2f(x_{n\text{-}1}) + f(x_n)]$ is called the Trapezoid Rule and is used for approximating the area under a curve using trapezoids. Explain, based on your answer in Part 5, question 5, what each term in the formula represents.

9. $\int_{b}^{a} f(x)dx$ is the formula for calculating the area under a curve, *f(x)*, from *a* to *b*. What does a and b represent?

What does *f(x)* represent?

What does *dx* represent?

What does ∫ represent?

10. Use $\int_{b}^{a} f(x)dx$ to find the area under $f(x) = \sqrt{x} + 1$ from 0 to 4.

Investigating Area under the Curve *Answer Key*

Part 1:

1. Trace the outline of your hand onto grid paper.
2. Color the squares that lie fully inside the outline of your hand. (If any part of the square is outside the outline, don't color it.)
3. In another color, color the squares that lie on the outline of your hand. If any part of the square touches the outline, color the ENTIRE square.
4. (a) ________ Number of squares fully inside outline

   (b) ________ Number of squares partially inside outline

   (c) ________ Total number of squares colored
5. What does your answer to part (a) represent?

   This is a lower-bound estimate to the area of your hand.
6. What does your answer to part (c) represent?

   This is the upper-bound estimate to the area of your hand.
7. Write a ratio relating the number of squares to the number of inches.

$$\mathbf{\frac{1\ inch^2}{4\ squares}}$$

8. Use this ratio to convert the number of squares you got for question 4, parts (a) and (c), to square inches.

   (a) _________ square inches

   (c) _________ square inches

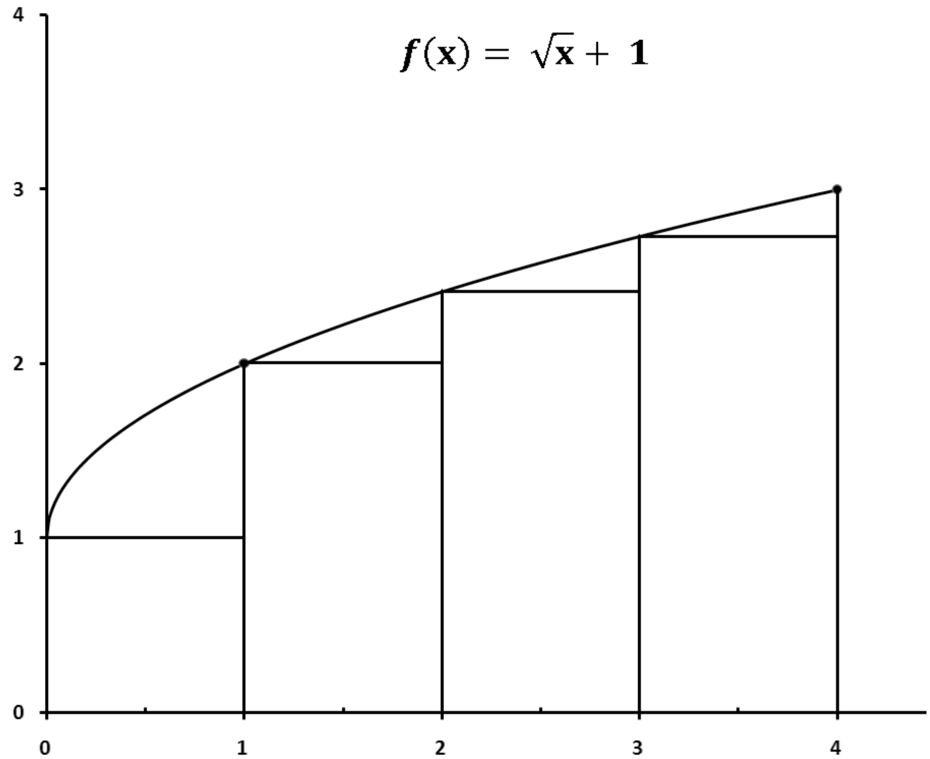
   $\mathbf{\frac{1\ inch^2}{4\ squares}}$ **x number of squares** **(Take the number of squares and divide by 4.)**

9. Use your two answers from question 8 to estimate the area of your hand in square inches.

10. Compare your estimate to that of your classmates. Does your estimate make sense? Why?

Part 2:

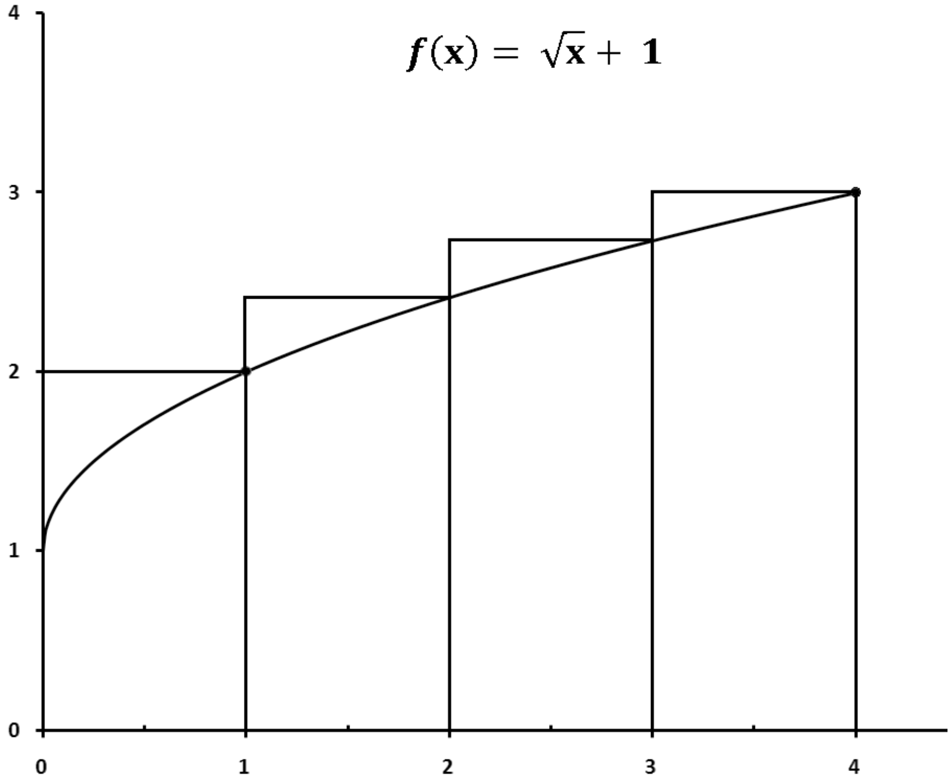
1. Divide the following graph of $f(x) = \sqrt{x} + 1$ from 0 to 4 into four equal intervals.



2. Form four rectangles by drawing a horizontal line from the left of the interval to the right.
3. What is the width of each rectangle? **1**
4. What is the height of the first rectangle? $\mathbf{f(0) = \sqrt{0} + 1 = 1}$
   the second rectangle? $\mathbf{f(1) = \sqrt{1} + 1 = 2}$
   the third rectangle? $\mathbf{f(2) = \sqrt{2} + 1 \approx 2.414}$
   the fourth rectangle? $\mathbf{f(3) = \sqrt{3} + 1 \approx 2.732}$
5. What is the area of the first rectangle? $\mathbf{A_1 = (1)(1) = 1}$
   the second rectangle? $\mathbf{A_2 = (1)(2) = 2}$
   the third rectangle? $\mathbf{A_3 = (1)(2.414) = 2.414}$
   the fourth rectangle? $\mathbf{A_4 = (1)(2.732) = 2.732}$
6. What is the total area of the four rectangles?
   $\mathbf{A_1 + A_2 + A_3 + A_4 = 1 + 2 + 2.414 + 2.732 = 8.146}$
7. Do you think the answer to question 6 is an over estimate or under estimate to the area under the curve? Why?
   It is an under estimate because the rectangles all lie within the curve.

Part 3:

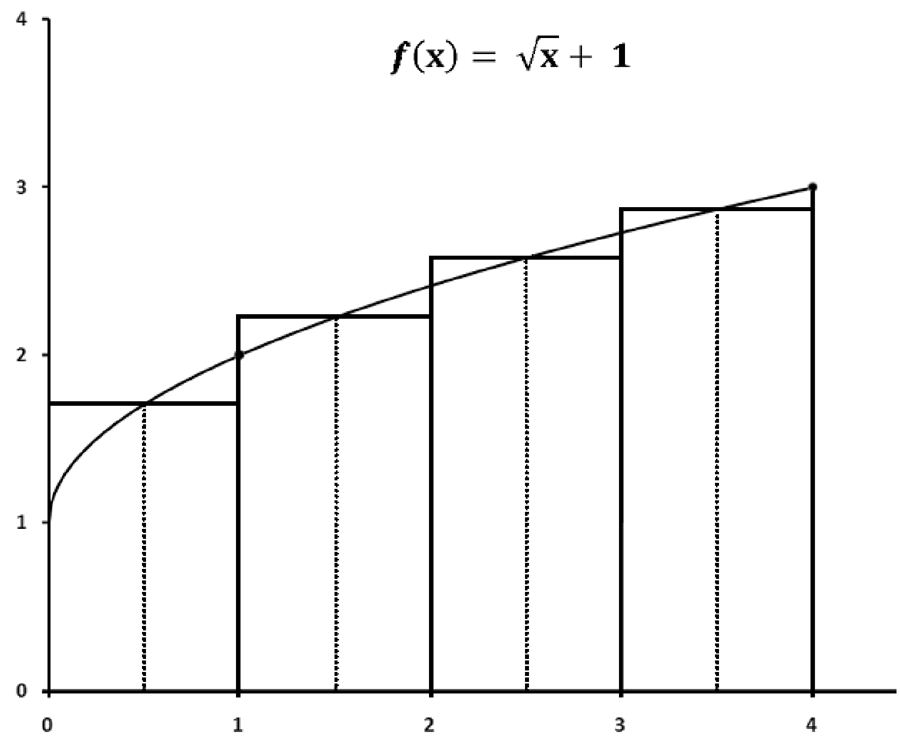
1. Divide the following graph of $f(x) = \sqrt{x} + 1$ from 0 to 4 into four equal intervals.



2. Form four rectangles by drawing a horizontal line from the right of the interval to the left. You will have to draw a vertical line to complete each rectangle.
3. What is the width of each rectangle? **1**
4. What is the height of the first rectangle? $\mathbf{f(1) = \sqrt{1} + 1 = 2}$
   the second rectangle? $\mathbf{f(2) = \sqrt{2} + 1 \approx 2.414}$
   the third rectangle? $\mathbf{f(3) = \sqrt{3} + 1 \approx 2.732}$
   the fourth rectangle? $\mathbf{f(4) = \sqrt{4} + 1 = 3}$
5. What is the area of the first rectangle? $\mathbf{A_1 = (1)(2) = 2}$
   the second rectangle? $\mathbf{A_2 = (1)(2.414) = 2.414}$
   the third rectangle? $\mathbf{A_3 = (1)(2.732) = 2.732}$
   the fourth rectangle? $\mathbf{A_4 = (1)(3) = 3}$
6. What is the total area of the four rectangles?
   $\mathbf{A_1 + A_2 + A_3 + A_4 = 2 + 2.414 + 2.732 + 3 = 10.146}$
7. Do you think the answer to question 6 is an over estimate or under estimate to the area under the curve? Why?
   It is an over estimate because some of each of the four rectangles lie outside the curve.

Part 4:

1. Divide the following graph of $f(x) = \sqrt{x} + 1$ from 0 to 4 into four equal intervals.



2. Form four rectangles by drawing a horizontal line from the center (midpoint) of the interval to the left and right. You will have to draw a vertical line to complete each rectangle.
3. What is the width of each rectangle? **1**
4. What is the height of the first rectangle? **$f(0.5) = \sqrt{0.5} + 1 \approx 1.707$**

   the second rectangle? **$f(1.5) = \sqrt{1.5} + 1 \approx 2.225$**

   the third rectangle? **$f(2.5) = \sqrt{2.5} + 1 \approx 2.581$**

   the fourth rectangle? **$f(3.5) = \sqrt{3.5} + 1 \approx 2.871$**
5. What is the area of the first rectangle? **$A_1 = (1)(1.707) = 1.707$**

   the second rectangle? **$A_2 = (1)(2.225) = 2.225$**

   the third rectangle? **$A_3 = (1)(2.581) = 2.581$**

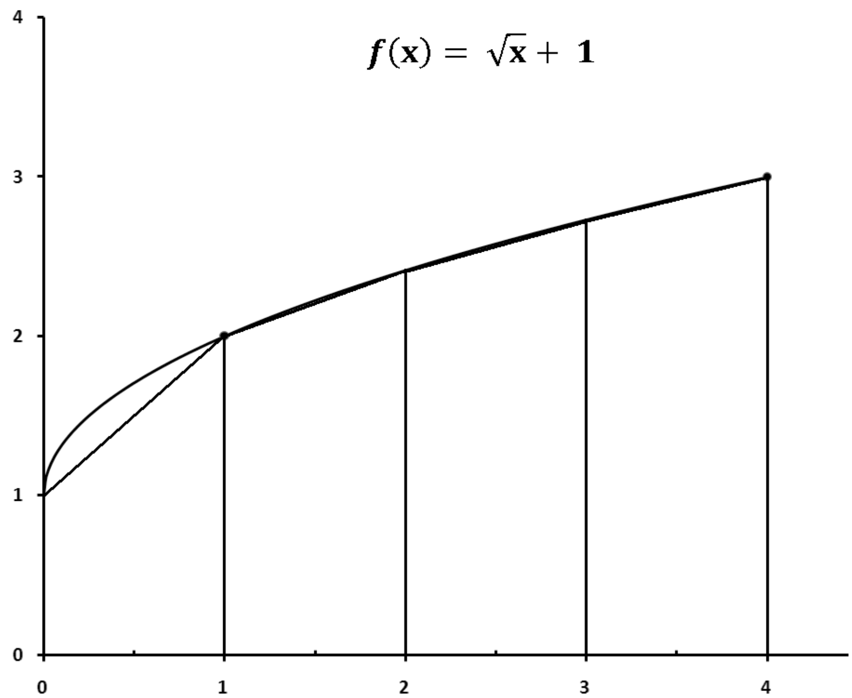
   the fourth rectangle? **$A_4 = (1)(2.871) = 2.871$**
6. What is the total area of the four rectangles?

   $A_1 + A_2 + A_3 + A_4 = 1.707 + 2.225 + 2.581 + 2.871 = 9.384$
7. Do you think the answer to question 6 is an over estimate or under estimate to the area under the curve? Why?

   This is hard to tell, but it appears that there is slightly more rectangle on the outside (especially from 0 to 0.5) than on the inside.

Part 5:

1. Divide the following graph of $f(x) = \sqrt{x} + 1$ from 0 to 4 into four equal intervals.



2. Form four trapezoids by drawing a slanted line from the left of the interval to the right.
3. What is the height of each trapezoid? **1**

   Remember, the height is defined as the distance between the bases (the parallel lines).
4. What are the two bases of the first trapezoid? $\mathbf{f(0) = \sqrt{0} + 1 = 1,\ \ f(1) = \sqrt{1} + 1 = 2}$

   the second trapezoid? $\mathbf{f(1) = \sqrt{1} + 1 = 2,\ \ f(2) = \sqrt{2} + 1 \approx 2.414}$

   the third trapezoid? $\mathbf{f(2) = \sqrt{2} + 1 = 2.414,\ \ f(3) = \sqrt{3} + 1 \approx 2.732}$

   the fourth trapezoid? $\mathbf{f(3) = \sqrt{3} + 1 \approx 2.732,\ \ f(4) = \sqrt{4} + 1 = 3}$
5. What is the area of the first trapezoid? $\mathbf{A_1 = (0.5)(1)(1 + 2) = 1.5}$

   the second trapezoid? $\mathbf{A_2 = (0.5)(1)(2 + 2.414) = 2.207}$

   the third trapezoid? $\mathbf{A_3 = (0.5)(1)(2.414 + 2.732) = 2.573}$

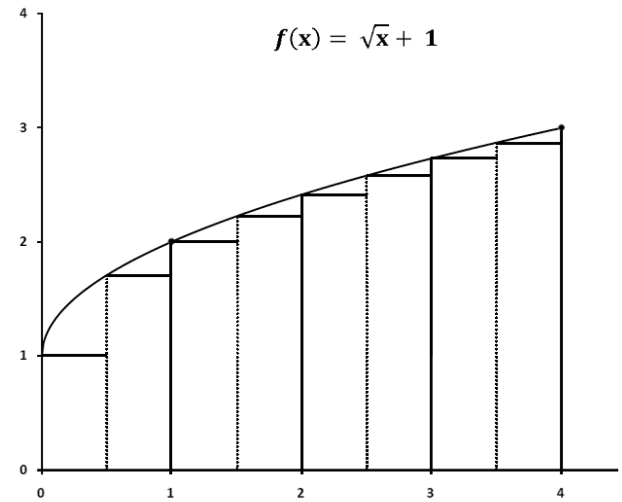
   the fourth trapezoid? $\mathbf{A_4 = (0.5)(1)(2.732 + 3) = 2.866}$
6. What is the total area of the four trapezoids?

   $\mathbf{A_1 + A_2 + A_3 + A_4 = 1.5 + 2.207 + 2.573 + 2.866 = 9.146}$
7. Do you think the answer to question 6 is an over estimate or under estimate to the area under the curve? Why?

   It appears to be a slight under estimate because there is some missing area from 0 to 1.

Part 6:

1. Divide the following graph of $f(x) = \sqrt{x} + 1$ from 0 to 4 into eight equal intervals.



2. Form eight rectangles by drawing a horizontal line from the left of the interval to the right.
3. What is the width of each rectangle? **0.5**
4. What is the height of the first rectangle? $\mathbf{f(0) = \sqrt{0} + 1 = 1}$

the first rectangle?	$f(0) = \sqrt{0} + 1 = 1$
the second rectangle?	$f(0.5) = \sqrt{0.5} + 1 \approx 1.707$
the third rectangle?	$f(1) = \sqrt{1} + 1 = 2$
the fourth rectangle?	$f(1.5) = \sqrt{1.5} + 1 \approx 2.225$
the fifth rectangle?	$f(2) = \sqrt{2} + 1 \approx 2.414$
the sixth rectangle?	$f(2.5) = \sqrt{2.5} + 1 \approx 2.581$
the seventh rectangle?	$f(3) = \sqrt{3} + 1 \approx 2.732$
the eighth rectangle?	$f(3.5) = \sqrt{3.5} + 1 \approx 2.871$

5. What is the area of:

the first rectangle?	$A_1 = (0.5)(1) = 0.5$	the fifth rectangle?	$A_5 = (0.5)(2.414) = 1.207$
the second rectangle?	$A_2 = (0.5)(1.707) = 0.854$	the sixth rectangle?	$A_6 = (0.5)(2.581) = 1.291$
the third rectangle?	$A_3 = (0.5)(2) = 1$	the seventh rectangle?	$A_7 = (0.5)(2.732) = 1.366$
the fourth rectangle?	$A_4 = (0.5)(2.225) = 1.113$	the eighth rectangle?	$A_8 = (0.5)(2.871) = 1.436$

6. What is the total area of the eight rectangles?

$$A_1 + A_2 + A_3 + A_4 + A_5 + A_6 + A_7 + A_8 = 0.5 + 0.854 + 1 + 1.113 + 1.207 + 1.291 + 1.366 + 1.436 = 8.767$$

7. Do you think the answer to question 6 is an over estimate or under estimate to the area under the curve? Why?

   It is an under estimate because there is missing area in all intervals.

Part 7:

1. Write the estimates you got from parts 2–6.

Part 2:	**8.146**	Part 5:	**9.146**
Part 3:	**10.146**	Part 6:	**8.767**
Part 4:	**9.384**		

2. Which estimate do you think is the best estimate? Why?

   The midpoint estimate in part 4 is the best estimate because the rectangles best "fit" the area. The area that was lost under the curve was made up for by the area above the curve.

3. The true area under the curve is a $9\frac{1}{3}$. Why do you think the trapezoidal estimate was so close?

   The estimate was good because the slanted line ran very close to the actual curve. The only part that was under was with the first trapezoid.

4. How did subdividing the area into more intervals improve the estimate?

   Subdividing the area into more subintervals improved the estimate by making the rectangles "fit" the area better. Although it was still an underestimate, there was not as much missing area.

5. What would happen if the area was divided into many more intervals?

   We would get a truer estimate of the area, because there would be less missing area below the curve or less area above the curve.

6. Write the sum of rectangular areas in Part 2 in summation notation. $\sum_{i=1}^{4} f(i-1)\left(\frac{4-0}{4}\right)$

   In Part 3: $\sum_{i=1}^{4} f(i)\left(\frac{4-0}{4}\right)$

   In Part 6: $\sum_{i=1}^{8} f\left(\frac{i-1}{2}\right)\left(\frac{4-0}{8}\right)$

7. $\sum_{i=1}^{n} f(x_i)\Delta x$ is the formula for calculating an estimate of the area under a curve from a to b (or the interval [a, b]), and is called the Riemann Sum. How would you define n? $f(x_i)$? Δx?

   - **n is the number of intervals**
   - **$f(x_i)$ is the function defined at each point, x_i, which are the x coordinates of the endpoints of each subinterval.**
   - **Δx is the width of each rectangle, or width of each subinterval. It can be found by dividing the difference of the endpoints (b – a) by n.**

8. $\frac{\Delta x}{2}[f(x_0) + 2f(x_1) + 2f(x_2) + ... + 2f(x_{n-1}) + f(x_n)]$ is called the Trapezoid Rule and is used for approximating the area under a curve using trapezoids. Explain, based on your answer in Part 5, question 5, what each term in the formula represents.

   - **Δx represents the height of each trapezoid, which is found by calculating the width of each interval. $\Delta x = \frac{b - a}{n}$.**
   - **The 2 dividing the Δx represents the $\frac{1}{2}$ in the trapezoid area formula.**
   - **The x coordinates of the endpoints of the interval, x_0 and x_n, are not doubled when put in the function because those values are only used once when calculating the bases of the trapezoids. All of the x_i's are used twice because of shared bases.**

9. $\int_b^a f(x)dx$ is the formula for calculating the area under a curve, f(x), from a to b. What does a and b represent?

   What does a and b represent? **The endpoints of the interval.**

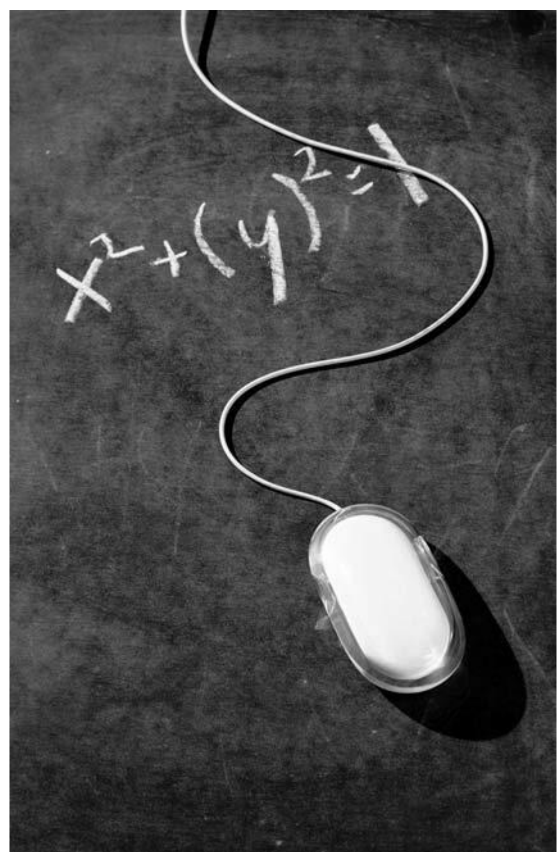
   What does f(x) represent? **f(x) represents the height of each rectangle.**

   What does dx represent? **dx, like Δx represents the width of each rectangle. With dx, these widths are extremely small.**

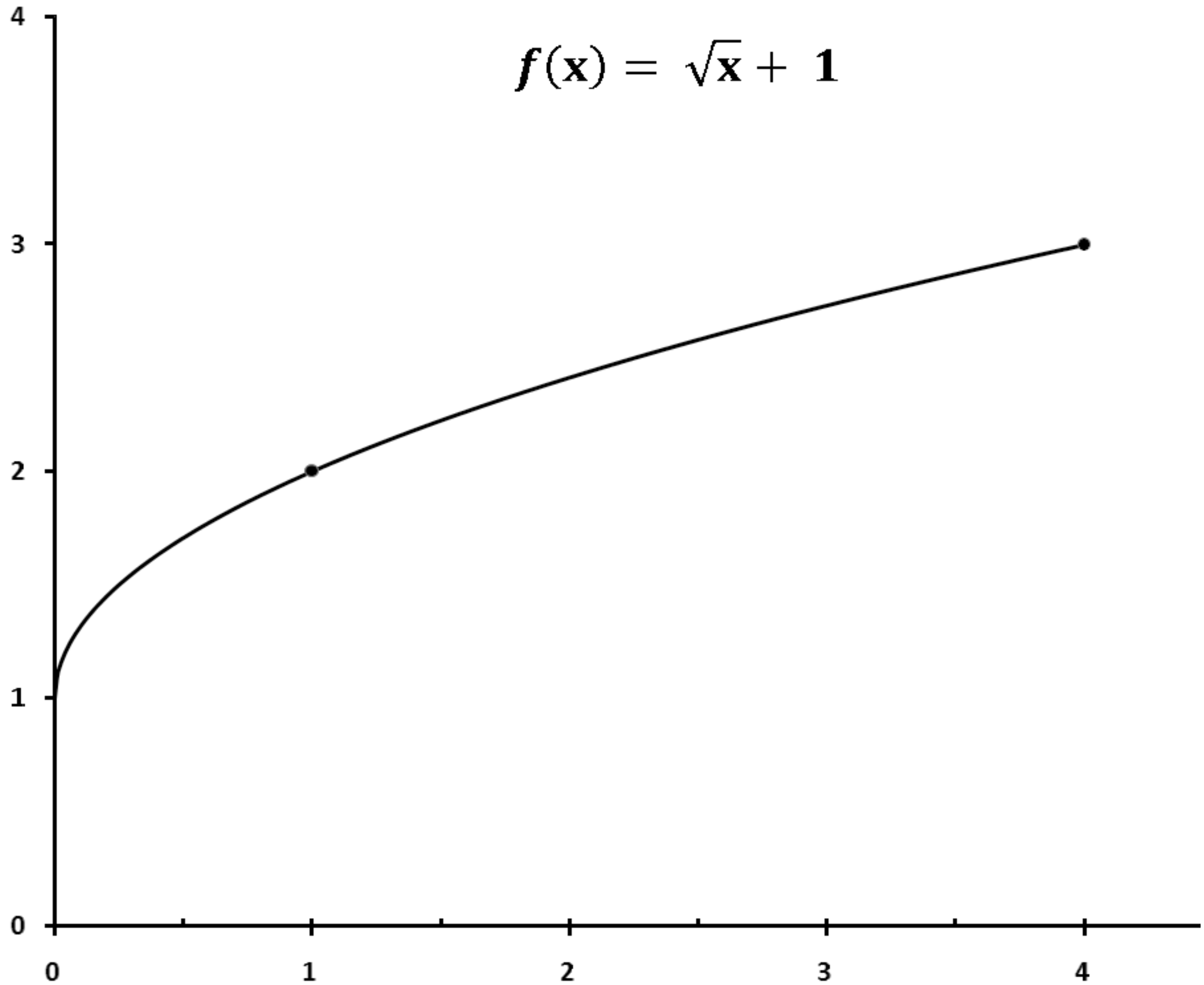
   What does ∫ represent? **∫ represents the summation of the infinite amount of rectangles that are used to find the true area under the curve.**

10. Use $\int_b^a f(x)dx$ to find the area under $f(x) = \sqrt{x} + 1$ from 0 to 4.

   $\int_0^4 (\sqrt{x} + 1)dx = \left[\frac{2}{3}x^{3/2} + x\right]_0^4 = \frac{2}{3}(4)^{3/2} + 4 = \frac{2}{3}(8) + 4 = 9\frac{1}{3}$



Investigating Area under the Curve



3.7: Deriving the Quadratic Formula

Topic

- Derivation of the quadratic formula by completing the square on the general form of the quadratic equation

Objective

- Students will work in collaborative learning groups to sequence the symbolic representations, descriptions, and reasons for the steps in deriving the quadratic formula

Timeline

- One 50–60-minute class period to sequence the steps, descriptions, and reasons for a derivation

WICR Strategies

- Inquiry
  — Investigate the derivation of the quadratic formula
- Collaboration
  — Work in collaborative learning groups to complete the activity

NCTM Standards

Reasoning and Proof

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- recognize reasoning and proof as fundamental aspects of mathematics;
- make and investigate mathematical conjectures;
- develop and evaluate mathematical arguments and proofs; and
- select and use various types of reasoning and methods of proof.

Communication

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- analyze and evaluate the mathematical thinking and strategies of others; and
- use the language of mathematics to express mathematical ideas precisely.

Rationale

For most Algebra students, the quadratic formula is a magical tool that is used for finding the solutions of a quadratic equation. "*Deriving the Quadratic Formula"* is an activity designed to give students the opportunity to understand that the formula is not merely a mathematician's magic trick; it is the logical outcome of solving the general form of the quadratic equation for x. The main tool for the derivation, completing the square, is a skill that is often taught as an isolated task. Math students typically don't connect the tool to an application until the study of conic sections in Algebra II. By working together to sequence the steps, descriptions, and reasons for the derivation, students will not only learn the mechanics of the task, they will gain an appreciation for the importance of completing the square in the study of Algebra and many other topics in higher-level mathematics.

Vertical Alignment

- Although "Deriving the Quadratic Formula" is an activity designed for Algebra I and II students, the concept behind the activity can be applied to a wide range of mathematics at all levels in grades 6–12. Any task that can be arranged in a sequence of steps can be turned into a sequencing and matching activity like "Deriving the Quadratic Formula."

Materials/Preparation

- *Teacher Reference Sheet 3.7a:* "Deriving the Quadratic Formula Activity Cards"
- Using *Teacher Reference Sheet 3.7a,* make one set of activity cards for each group. Activity cards should be copied onto card stock. Laminating standard copy paper is also an effective way to produce the card sets.
- The order of the cards should be mixed prior to the activity.

Instructions

- This activity should occur after students have studied the derivation and use of the quadratic formula.
- Arrange the class into collaborative groups of three to four.
- Distribute the activity card sets to the groups.
- Explain to the groups that there are two aspects to the activity. The first task is to sequence the steps (S cards), starting with the general form of the quadratic equation, and ending with the quadratic formula. The second task is to match the descriptions (D cards) and reasons (R cards) with each step.
- If students are having difficulty with sequencing the steps, guide them to look for Step cards that are similar, asking themselves what has changed from one card to the next.
- Once students are finished, bring the groups together to debrief the activity. Make sure that all of the groups have sequenced and matched the three sets of cards correctly.
- To wrap up the activity, have students copy the derivation of the quadratic formula into their Cornell Notes. The summary/reflection should describe the tools they used to derive the formula.

Note: You can use *Teacher Reference Sheet 3.7a:* "Deriving the Quadratic Formula Activity Cards" as an answer key. The cards are matched and ordered in the proper sequence.

Higher-Level Questions

Level Two

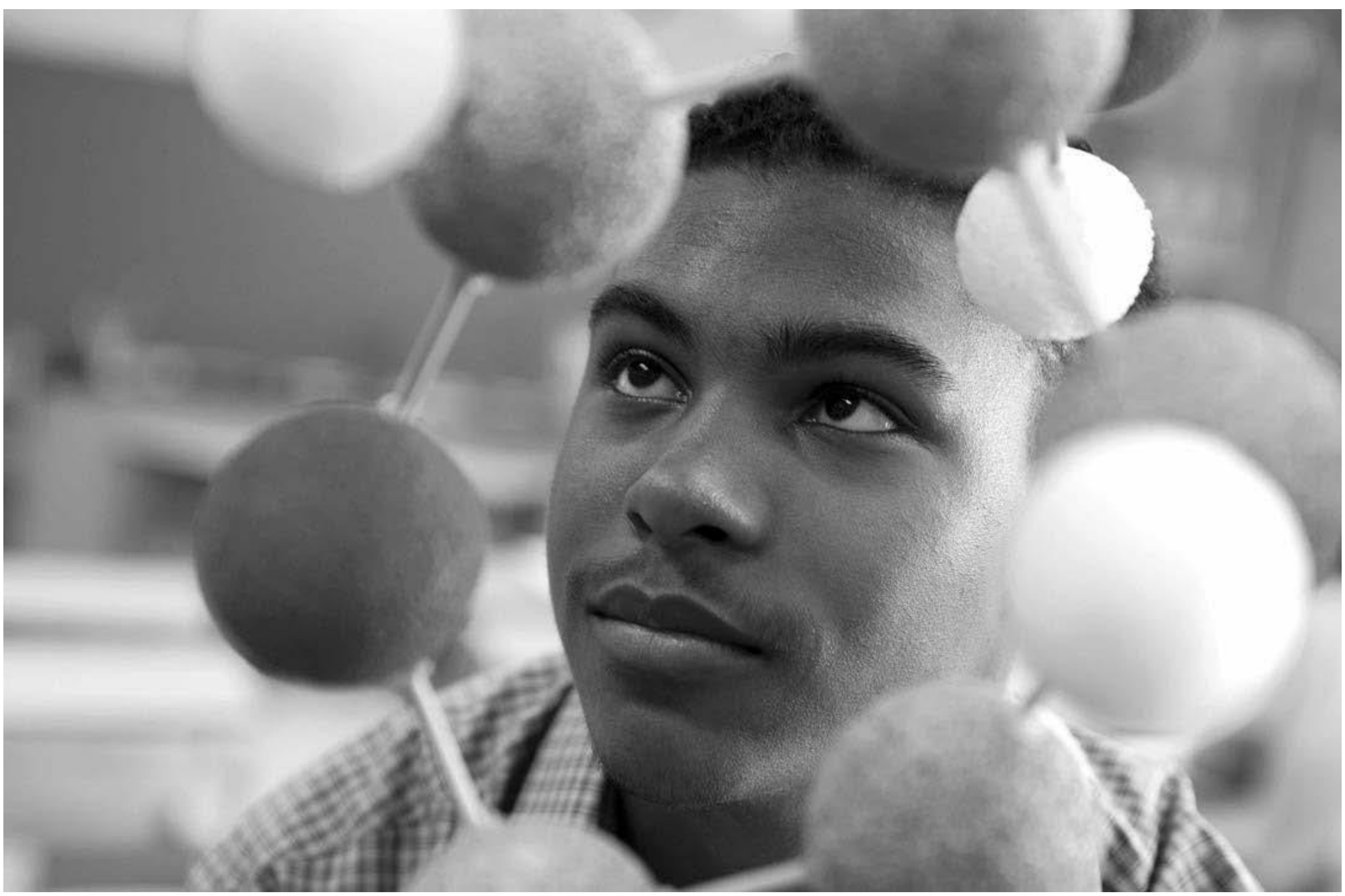
- Compare the steps in the derivation of the quadratic formula to the steps you take to solve a quadratic equation by completing the square.

Level Three

- Is it possible to "complete the cube?" Predict what you would have to do to "complete the cube."

Formative Assessment

- Did your students have a deep enough understanding of completing the square to access the activity?
- Do you need to review completing the square?
- Do your students know how to solve equations like $(x + 2)^2 = 16$, by taking the square root of both sides?



Deriving the Quadratic Formula Activity Cards

S6 $ax^2 + bx + c = 0$	**D3** Begin with the Standard Form of a Quadratic Equation.	**R7** We need the Standard Form so we can solve the equation for *x*.
S4 $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$	**D6** Divide each term of the equation by *a*.	**R1** We need the lead coefficient to equal 1 so we can complete the square.
S8 $x^2 + \frac{b}{a}x = -\frac{c}{a}$	**D10** Subtract the constant term from both sides of the equation.	**R3** We will complete the square on $x^2 + \frac{b}{a}x$.
S1 $x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{b^2}{4a^2} - \frac{c}{a}$	**D2** Divide the coefficent of the *x* term by two, square it, and add to both sides.	**R10** By adding $\frac{b^2}{4ac}$ to the left side, we have created a perfect square trinomial.
S10 $\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}$	**D5** Write the left-side in factored form.	**R6** Once the left side is factored, we can take the square root of both sides to solve for *x*.
S7 $\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$	**D1** Add the fractions on the right side of the equation.	**R2** We need to simplify the right side prior to taking the square root of both sides.

S3 $x + \frac{b}{2a} = \pm \sqrt{\frac{b^2 - 4ac}{4a^2}}$	**D9** Take the square root of both sides of the equation.	**R8** Taking the square root of both sides leaves us with a linear equation with *x* on the left side.
S9 $x = -\frac{b}{2a} \pm \sqrt{\frac{b^2 - 4ac}{4a^2}}$	**D4** Subtract $\frac{b}{2a}$ from both sides of the equation.	**R5** Subtract $\frac{b}{2a}$ from both sides leaves *x* by itself on the left side.
S5 $x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$	**D8** Rewrite $\sqrt{4a^2}$ as $2a$.	**R4** We now have common denominators for the terms on the right side.
S2 $x = -\frac{b \pm \sqrt{b^2 - 4ac}}{2a}$	**D7** Add the fractions on the right side of the equation.	**R9** Adding the fractions on the right side gives us the Quadratic Formula!

"Already, the teachers, staff, and administrators hail the program and comment regularly about its significantly positive impact. But most important, AVID is not just a program intended to address the needs of students selected for an AVID class. Rather, it is a schoolwide reform effort that is intended to reach all students by providing additional support and revising the ways in which professional staff address the needs of students. With our planned expansion of the project, Baltimore County Public Schools is on the way to becoming the flagship system for AVID in Maryland."

—Dr. Joe A. Hairston, Superintendent
Baltimore County Schools, Maryland

UNIT FOUR: READING IN MATHEMATICS

Introduction to Reading in Mathematics

Reading to Learn in Mathematics

The mathematics classroom should incorporate strategies that can help students become more effective readers of mathematics. In the same way that a student needs to be taught the special skills needed to read poetry, fiction and non-fiction, the skills needed to comprehend a math textbook must also be explicitly taught. The math content teacher is often in a better position to teach the skills needed for reading a math textbook than any other teacher in the school. Math teachers have, by the nature of their work, developed effective skills for decoding the mathematics text. The skills that they have identified and honed over the years can and should be taught to students. Students need to be taught how to "read like a mathematician." It has become far too common for teachers to translate the text into graphs, tables, algorithms, or verbal explanations rather than giving students the tools and the opportunity to practice using those tools to decipher their texts. Rather than making the students dependent on the teacher to construct meaning from the textbook, the teacher can equip the students to be self-reliant and in the process provide them with powerful reading comprehension tools which they can use in furthering their understanding of mathematics.

The math text cannot be simply read. It must be "worked through." To do this, students need to understand critical structural characteristics of the math text. Math textbooks are written in a very terse or compact style. Every word counts. If a concept is missed, there is little chance of picking it up later. An "elegant" explanation, derivation, or proof in mathematics is the one that uses the fewest words and uses the words in the most precise way, making vocabulary acquisition critical to comprehension. Identifying new vocabulary and utilizing specific strategies such as concept maps, word walls, graphic organizers, semantic feature analysis, picture vocabulary cards, etc., will provide students with the tools they need for constructing meaning on their own.

Since each section in a mathematics text makes the assumption of having mastered the previous sections, there is no chance of just "catching the drift." There is very little redundancy, each word, symbol, or sentence has to be decoded prior to moving on to the next. While a student may read thirty to sixty pages of a novel in thirty minutes, in the same time the student may dwell on two to three lines in the math text. When reading mathematics, "reading slow *is* fast."

While every sentence in a math text is logically linked to a previous section and those sections that follow, it is not a linear reading experience. Math textbooks must be read in all directions, top to bottom, bottom to top, right to left, left to right, front to back, and back to front. There is usually something very wrong if an explanation, problem, or example is read only once. Each sentence and section must be thoroughly understood before moving ahead. The process of making sense of the text may include many iterations of scanning, rereading, cross referencing, attempting solutions, pausing and revisiting explanations, examples, illustrations, and glossaries.

Reading instruction must be scaffolded so that students develop strategies that help them become more confident with comprehension skills. Three factors are most helpful for ensuring successful comprehension: connecting to prior knowledge, understanding text structure, and using text-processing strategies.

Prior Knowledge

All readers bring what they already know to the piece they are reading. Readers compare information with their own experiences to assist in comprehension. When they encounter something new, they can make inferences based on their prior knowledge. Readers who have a greater range of prior knowledge will find comprehension easier than readers whose knowledge is more limited. For struggling readers, it is essential that the teacher provide some prior knowledge with new topics. Good teaching would suggest that prior to reading a teacher should ask questions that evoke anything a student might already know about the concept. Also, brainstorming as a whole class or in groups helps pool the information that students possess.

Text Structure

Understanding the pattern or structure of the math text can greatly improve the students' ability to construct meaning from the text. Most texts follow the prototypical pattern of statement, example(s), explanation, and practice. In addition, most include additional information in the margin or offset in colorfully illustrated boxes meant to engage student interest, activate background knowledge, and help make essential connections. Without direct explicit instruction and guidance, students often overlook and/or ignore these critical structural cues and vital reading aids.

Text-Processing Strategies

Students who use strategies to help them make sense of their reading while in the act and to synthesize their understanding at the end of their reading will have greater comprehension of and greater satisfaction with a text. Likewise, understanding how these strategies work and becoming aware of their own mental processes while reading (metacognition) can help students make informed and purposeful choices about how they read. They become aware of how reading a novel or a history text is different from reading a chapter in a math textbook. They recognize that what they do during reading and what they do to make sense of their understanding after they've read differs based on their purposes and the kind of text it is. Students also become aware that they need different processing strategies based on the difficulty or density of the text.

BEFORE READING	**DURING READING**	**AFTER READING**
• Think about prior knowledge related to the subject • Know the purpose for reading • Preview the text: look at the title, pictures, graphics	• Focus full attention on the material • Think aloud • Predict • Ask questions • Take notes/Draw Diagrams	• Create visuals to clarify meaning (tables, Venn diagrams, graphs, charts, etc.) • Summarize • Evaluate • Apply and practice what has been read

4.1: Text-Processing Strategies

Topic

- Reading comprehension

Objective

- Students will develop and practice text-processing strategies to increase their reading comprehension

Timeline

- 30-minutes for students to practice specific text-processing strategies

WICR Strategies

- Writing to Learn
  — Use a structured plan for writing notes while reading
- Collaboration
  — Work in collaborative groups to complete individual and group assignments and assessments
- Reading to Learn
  — Use text-processing strategies to increase reading comprehension

NCTM Standards

Communication

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- analyze and evaluate the mathematical thinking and strategies of others; and
- use the language of mathematics to express mathematical ideas precisely.

Connections

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- recognize and use connections among mathematical ideas;
- understand how mathematical ideas interconnect and build on one another to produce a coherent whole; and
- recognize and apply mathematics in contexts outside of mathematics.

Representation

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- create and use representations to organize, record, and communicate mathematical ideas;
- select, apply, and translate among mathematical representations to solve problems; and
- use representations to model and interpret physical, social, and mathematical phenomena.

Rationale

Students who are given an opportunity to practice text-processing strategies will become more self-reliant and confident. They will begin using their math text as an effective learning resource rather than a collection of practice problems. Most texts follow the prototypical pattern of statement, example(s), explanation, and practice. In addition, most include additional information in the margin or offset in colorfully illustrated boxes meant to engage student interest, activate background knowledge, and help make essential connections. Without direct explicit instruction and guidance, students often overlook and/or ignore these critical structural cues and vital reading aids. Students determine what is important in their reading based on the purpose of their reading. Teachers who provide a structure for students to follow in establishing purpose and an opportunity to practice synthesizing their thinking will enable students to develop critical text-processing skills.

Vertical Alignment

- Text-processing strategies for reading comprehension in mathematics should be introduced at early ages and refined as students mature.

Materials/Preparation

- *Student Handout 4.1a:* "Reading for a Purpose"
- Math text
- Cornell Notes

Instructions

- Assign a selection from the students' textbook or other mathematics text.
- Distribute and review *Student Handout 4.1a:* "Text-Processing Strategies."
- When students are first learning the strategies, assign and assess only one or two of the strategies at a time. As the students become familiar with the strategies, increase the number of strategies that are assigned and assessed.
- Provide students with time to work individually and in collaborative groups to practice the skills outlined.

Higher-Level Questions

Level Two

- Why does establishing a purpose contribute to improved reading comprehension?

Level Three

- Can you describe situations in which establishing a purpose for reading is more difficult?
- Which of the text-processing strategies listed in the student handout are more valuable than others?

Formative Assessment

- Assess the students' questions established prior to reading.
- Review the students' reading notes.
- Assess their reading comprehension informally and formally.

Text-Processing Strategies

Good readers determine what information is important in their reading based on the *purpose* for their reading. Math texts are an important resource in developing an understanding of mathematics. In most cases, mathematical texts follow a pattern of statement, example(s), explanation, and practice. They often include additional information in the margin or offset in colorfully illustrated boxes meant to spark the reader's interest, activate background knowledge, and help make essential connections. These critical structural cues are vital reading comprehension aids.

Summarized below are specific text-processing strategies that you can use before, during, and after reading to significantly improve your reading comprehension.

BEFORE READING	DURING READING	AFTER READING
• Think about prior knowledge related to the subject • Know the purpose for reading • Preview the text: look at the title, pictures, graphics	• Focus full attention on the material • Think aloud • Predict • Ask questions • Take notes • Draw diagrams	• Create visuals to clarify meaning (tables, Venn diagrams, graphs, charts, etc.) • Summarize • Evaluate • Apply and practice what has been read

Below are suggested ways to use your Cornell Notes to demonstrate your understanding of each of the reading strategies listed above.

- Brainstorm and list everything that you know about the main topic or concept.
- Preview the text and turn the subheadings and bolded type into questions. Write these questions in the left margin of your Cornell Notes. As you read, record your answers to these questions.
- Identify and define new vocabulary, theorems, and formulas.
- Record the details of examples in your notes.
- When several examples are given, identify why each example is provided. What is unique about that example? Write a short explanation of why you think the example was included.
- Describe what prior principles are used in the example(s).
- Write a short summary outlining what you learned from the example(s).
- Explain why the illustrations, graphs, tables, and graphics are included.
- Write the questions and answers in your notes that you predict will be included on a quiz or test.
- Create visuals (graphs, illustrations, graphic organizers, charts, etc.) to clarify the meaning of the text.
- Write a summary/reflection describing what you have learned and how the examples, visuals, your background knowledge, and reading strategies contributed to your understanding of the text. What questions do you still have?

4.2: Highlighting and Annotating a Math Text

Topic

- Constructing meaning from a mathematics text

Objectives

Students will:

- Practice highlighting strategies
- Demonstrate understanding of the use and value of text annotation

Timeline

- One 50-minute class period to highlight and annotate a math text, and utilize content-specific questions as a reading comprehension strategy

WICR Strategies

- Collaboration
  - — Work in collaborative groups to complete individual and group assessments
- Reading to Learn
  - — Practice highlighting and text annotation skills

NCTM Standards

Communication

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- analyze and evaluate the mathematical thinking and strategies of others; and
- use the language of mathematics to express mathematical ideas precisely.

Rationale

Readers who simply highlight an academic text without questioning the material engage in a passive and often counter-productive activity. "Active readers" combine highlighting strategies *with* annotation practices. Using active highlighting and annotation strategies in conjunction with overall reading strategies like "PQ5R" will help students stay focused and involved with the text and greatly increase recall.

Highlighting and **Annotating** a text will:

- Help students locate important information when studying
- Identify main points and concepts to make them more recognizable
- Help organize the concepts presented

- Draw attention to new vocabulary and concepts
- Help to paraphrase and condense key ideas
- Monitor and improve comprehension
- Help students make connections to background knowledge
- Help identify clarification questions to ask in the next class or study group
- Significantly increase recall

Vertical Alignment

- Highlighting and annotation can be introduced at any level. Mastery and sophistication will develop as students progress to higher levels.

Materials/Preparation

- *Student Handout 4.2a:* "Guide for Highlighting and Annotating a Mathematics Text"
- Mathematics text
- Sticky notes
- Highlighters in multiple colors
- Pen/pencil and paper

Instructions

- Assign a selection from the students' textbook or other mathematics text.
- Distribute and review *Student Handout 4.2a:* "Guide for Highlighting and Annotating a Mathematics Text."
- Provide students with time to work individually and in collaborative groups to practice the skills outlined.

Higher-Level Questions

Level One

- How many words were highlighted in each "chunk" of text?

Level Two

- How do you decide what to highlight?

Formative Assessment

- Review the students pre-reading questions.
- Assess the students' use of margin notes.
- Assess the amount of text that is highlighted.
- Ask students to respond to questions that they posed in the margin or as pre-reading questions.

Guide for Highlighting and Annotating a Mathematics Text

I. Never begin reading without a question in mind.

A. Establish a purpose for reading because it will help you to identify what parts of the text are the most important.

B. Form an initial list of questions from questions at the end of the chapter, or turn section headings into questions.

1. Some generic questions may include:

a. How does this relate to what I already know?

b. Have I seen or used this concept before?

c. Can I relate this to a "real world" example?

d. Where could I use this idea?

II. Read an entire paragraph or "chunk" of text before beginning the annotation and highlighting process.

A. While reading, make notations in the margin where you think you have found the answer to your initial question(s). These margin notations will help you identify the most important part of the section you are reading.

B. When you do begin writing margin questions, annotating the text and highlighting, you will be making an informed judgment.

III. After reading a "chunk" of the text, write a question that relates to the text in the margin, on a sticky note or in a notebook.

A. Put yourself in the shoes of the teacher or tutor. What question would you ask from this section of text?

B. Write questions that help clarify processes. How does the author get from one step to the next in an example? What steps have been implied?

IV. After writing the question in the margin, selectively highlight and annotate the portion of the text that answers the question you have written. (Never highlight more than twenty to thirty percent of the text. Some authors recommend as little as ten to fifteen percent.) *Highlighting a large percentage of the text is generally indicative of* ***not*** *mastering the material.*

V. Review the relationship between the question and the highlighted answer and annotations.

VI. Use a different color highlighter or code to mark vocabulary or concepts that are unclear.

VII. Review the section just annotated and highlighted:

A. Cover the text

B. Ask the questions written in the margin

C. Recite the hidden answers in *Sotto voce.* It will be easy to check for understanding; *the answers and annotations are highlighted.*

1. Quizzing yourself with margin questions, in conjunction with highlighting and annotating, will help you avoid tricking yourself into thinking that you actually remember and understand a highlighted text. Rather than "reading over" the highlighted text, you will be demonstrating to yourself the mastery of the concepts, vocabulary, and processes.

VIII. After mastering the previous section, you are ready to begin the process of annotating and highlighting the next section.

IX. After finishing the text:

A. Review the sections and prioritize the questions that you have written in the margin.

B. Create tasks for yourself. For example: After reading an example and working it out for yourself, think of other examples that fit the pattern, think of how this relates to other concepts in the chapter or in previous chapters, and think about how this idea could be applied in a practical way.

X. Finally, write a short summary/reflection in the margin at the end of each sub-unit.

4.3: Reading Comprehension—Looking for Clues

Topic

- Reading comprehension activity for grades 6-12

Objectives

Students will:

- Improve their ability to pick out context clues and further develop their critical reading skills in mathematics
- Strengthen their text-processing strategies

Timeline

- One 50–60-minute class period for students to solve problems in a language other than English and create original problems in their primary language

WICR Strategies

- Inquiry
  — Explore math problems in a language other than English
- Collaboration
  — Work collaboratively to construct meaning from a text written in a language other than English
  — Work in small groups to construct and present rigorous math problems based on a model
- Reading to Learn
  — Read and identify contextual clues in a language other than English
  — Apply reading strategies in the native language

NCTM Standards

Communication

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- analyze and evaluate the mathematical thinking and strategies of others; and
- use the language of mathematics to express mathematical ideas precisely.

Connections

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- recognize and use connections among mathematical ideas;
- understand how mathematical ideas interconnect and build on one another to produce a coherent whole; and
- recognize and apply mathematics in contexts outside of mathematics.

Representation

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- create and use representations to organize, record, and communicate mathematical ideas;
- select, apply, and translate among mathematical representations to solve problems; and
- use representations to model and interpret physical, social, and mathematical phenomena.

Rationale

Mathematics is a language of its own. Students struggling with reading comprehension need to acquire a variety of skills to identify key information when faced with challenging mathematical problems. In this exercise, students will test their ability to make sense of a problem by utilizing context clues. This activity will help students recognize their ability to identify and use context clues even when problems are written in a language other than English. For this activity, students will demonstrate their ability to understand context clues by solving a mathematical problem written in Spanish. Given the bilingual aspect of the activity, Spanish-speaking students may naturally assume a leadership role.

Vertical Alignment

- The reading strategy of identifying "Context Clues" can be introduced at any level. Mastery and sophistication will develop as students progress to higher levels.

Materials/Preparation

- *Student Handout 4.3a:* "Context Clues"
- Active Learning Methodologies (see the *Introduction*)

Instructions

- Divide students into small groups.
- Distribute *Student Handout 4.3a:* "Context Clues."
- Provide students with time to read and to identify the context clues needed to solve Problem #1.
- Ask students to record their interpretations and deduce an answer to the question.
- Ask students to identify context clues and speculate about the answer to Problem #2.
- Ask students to identify context clues and propose an answer to Problem #3.
- Provide groups an opportunity to review Problem #4 (the English version) and brainstorm about the differences and similarities in the three versions of the problem.
- Ask students to list and present the context clues they found in each version of the problem.
- Ask students to create problems in their primary language that includes context clues that they noticed and are based on a math concept they have recently studied. *Note:* Students who speak a character-based language can write their problems with the characters if they use context clues like numbers, English language variables like x and y, or include an illustration that gives a clue to the content of the problem. Group students who cannot write in their primary language with other students who can write in the primary language.

- Use one of your favorite Active Learning Methodologies for students to share their problems with the class.

Higher-Level Questions

Level Two

- Compare and contrast the different forms of the problem presented.

Level Three

- Create your own problem in a different language and be prepared to have a peer solve it.

Formative Assessment

- Assess student responses to the problems presented.
- Review students' written reflections of their thought processes when trying to interpret the problem.
- Assess context clues that students identified.

Context Clues: Problem #1

Felix lavó su carro, un ‘67 Chevy Impala, para que estuviera listo para llevarlo a una exhibición de carros este miércoles. El carro no puede correr más que *x* millas por hora, porque tiene un systema de hidráulica. ¿Si tiene que caminar *y* millas, y no puede manejar más de *x* millas por hora, cuánto tiempo tardaría en llegar a su destinación?

Context Clues: Problem #2

Felix lavó su carro, un '67 Chevy Impala, para que estuviera listo para llevarlo a una exhibición de carros este miércoles. El carro no puede correr más que x millas por hora, porque tiene un systema de hidráulica. ¿Si tiene que caminar y millas, y no puede manejar más de x millas por hora, cuánto tiempo tardaría en llegar a su destinación?

a. $\frac{x}{y}$ horas

b. $x + y$ horas

c. $\frac{y}{x}$ horas

d. $\frac{xy}{10}$ horas

e. la respuesta no está presente

AVID® Proven Achievement. Lifelong Advantage.

Context Clues: Problem #3

Felix lavó su carro, un ‘67 Chevy Impala, para que estuviera listo para llevarlo a una exhibición de carros este miércoles. El carro no puede correr más que *20* millas por hora, porque tiene un systema de hidráulica. ¿Si tiene que caminar *140* millas, y no puede manejar más de *20* millas por hora, cuánto tiempo tardaría en llegar a su destinación?

a. 1/7 horas

b. 160 horas

c. 7 horas

d. 280 horas

e. la respuesta no está presente

AVID®
Proven Achievement.
Lifelong Advantage.

Context Clues: Problem #4

Felix just cleaned his 67’ Chevy Impala to have it ready for the car show on Tuesday. Due to the hydraulic system, he can only drive the car 20 miles per hour. If Felix needs to travel 140 miles and he cannot go more than 20 miles per hour, how long will it take him to get to his destination?

a. 1/7 hours

b. 160 hours

c. 7 hours

d. 280 hours

e. None of the above

4.4: The Whole Picture

Topic

- The four representations of linear functions

Objectives

Students will:

- Identify "real-world" mathematical situations as linear
- Write other representations for the given representation of a linear function
- Identify attributes of linear functions in each of the four representations of function

Timeline

- One 50–60-minute class period to explore the four representations of function

WICR Strategies

- Writing to Learn
  — Write equations, tables, graphs, and written descriptions of functions
- Collaboration
  — Work in collaborative groups to write the four representations of each function
- Reading to Learn
  — Read and interpret graphs, equations, tables, and written descriptions of linear mathematical situations

NCTM Standards

Focal Point Grade 6

Algebra: Students write mathematical expressions and equations that correspond to given situations

Focal Point Grade 8

Students use linear functions, linear equations, and systems of linear equations to represent, analyze, and solve a variety of problems

Algebra

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- represent and analyze mathematical situations and structures using algebraic symbols; and
- use mathematical models to represent and understand quantitative relationships.

Representation

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- create and use representations to organize, record, and communicate mathematical ideas;
- select, apply and translate among mathematical representations to solve problems; and
- use representations to model and interpret physical, social and mathematical phenomena.

Rationale

The importance of multiple representations in mathematics is widely accepted. A student's ability to make connections between the various representations of linear functions is fundamental in acquiring deep understanding of the concept of function. Recognizing attributes of linear functions such as slope and *y*-intercept in each of the four representations develops students' ability to model mathematical situations with linear functions.

Vertical Alignment

- There are many examples of topics in mathematics that can be explored through multiple representations. One such example from elementary grades is the fraction, decimal, and percentage representations of the same value. For functions, exploring the four representations can be explored well beyond the study of linear functions. Quadratic, exponential, logarithmic, and trigonometric functions are all well-explored utilizing the four representations of function.

Materials/Preparation

- Red and green highlighters (or other colors)
- Calculator (optional)
- *Student Handout 4.4a:* "The Whole Picture"
- Instead of two-sided copies, teachers may want to copy *Student Handout 4.4a* onto two separate sheets.

Instructions

- Divide students into collaborative groups of two, three, or four.
- Distribute and review *Student Handout 4.4a:* "The Whole Picture."
- Ask students to take the four situations given and write the corresponding table, graph, situation, title, and equation for each of the representations.
- Students will need red and green highlighters (or other colors) to show where the slope and y-intercept can be found in each representation.
- Ensure students understand that each of the four given representations is for a different function and not related to each other.
- If time permits, ask each group to choose one of the functions to present to the class.

Higher-Level Questions

Level Two

- What characteristic of each function makes the function linear?
- Can you find the x-intercept of each function in a representation other than the graph?

Level Three

- Predict how a change in the rate of descent for the balloon situation would impact the other three representations.

Formative Assessment

- Monitor each group to determine which of the four representations is most difficult for the students to write.

The Whole Picture

Instructions

For each of the four representations shown below:

- Determine the other three representations.
- Give the mathematical situation a title.
- Complete the "Four Representations of Function" table on the following page.
- Using a green highlighter, indicate where you find the slope in each of the four representations.
- Using a red highlighter, indicate where you find the *y*-intercept in each of the four representations.
- You should have four final products, one for each of the representation shown below.

Representation 1:

$C(m) = \$60 + \$15h$

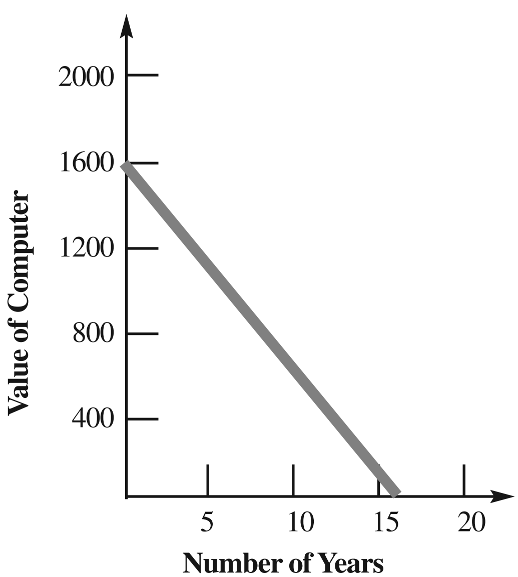
Representation 2:

Denise and Don took a hot air balloon trip for their anniversary. On the way down, the hot air balloon descended from a height of 1000 feet to the ground, at a constant rate of 2 feet per second.

Representation 3:

Number of Text Messages	0	10	20	30	40
Total Cost	\$25	\$26	\$27	\$28	\$29

Representation 4:



Four Representations of Function

SITUATION

GRAPH

EQUATION

TABLE OF VALUES

4.5: Advanced Sentence Frames—Logical Connectors

Topic

- Acquisition and use of the logical connectors used in mathematical academic language

Objectives

Students will:

- Practice the use of logical connectors in making oral arguments related to mathematics
- Create declarative statements relating two or more mathematical propositions using appropriate logical connectors

Timeline

- One 50–60-minute class period for students to practice oral arguments by using logical connectors

WICR Strategies

- Collaboration
  — Work in small groups and/or whole group to complete the activity.
- Reading to Learn
  — Practice reading and speaking the academic language of mathematics

NCTM Standards

Communication

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- analyze and evaluate the mathematical thinking and strategies of others; and
- use the language of mathematics to express mathematical ideas precisely.

Connections

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- recognize and use connections among mathematical ideas;
- understand how mathematical ideas interconnect and build on one another to produce a coherent whole; and
- recognize and apply mathematics in contexts outside of mathematics.

Rationale

In *The Write Path I: Mathematics,* the reader is introduced to the idea of using "Sentence Frames" as a strategy for students to practice the use of target mathematical academic language terms and phrases. In *The Write Path II: Mathematics*, we take this activity to the next level by focusing on the logical connectors used in the formal register of mathematics. Logical connectors are words or phrases that connect mathematical statements to make declarative mathematical sentences. "If, then, and, or, because, therefore, thus, either, neither, and since" are all examples of logical connectors. For many students, facility in the use of logical connectors is a major stumbling block not only in the acquisition of academic language, but also in making connections between independent mathematical truths. In the "Sentence Frames" activity in *Write Path I*, we ask students to fill in the blanks with numbers, expressions, equations, etc., that make the sentence frame mathematically accurate. In *"Advanced Sentence Frames—Logical Connectors,"* students are asked to fill in the blanks with mathematical statements that can be joined using the target logical connector.

Appropriate use of logical connectors poses a serious language obstacle for most students as they learn the formal language of mathematics. However (another example of a common logical connector), this obstacle gives us an excellent opportunity to not only focus on vocabulary instruction, but to give our students the opportunity to make connections between mathematical ideas. Explicit instruction in the use of logical connectors will give students greater ability to justify their mathematical thinking, write about mathematics with precision, and communicate effectively in the formal register of mathematical academic language.

Vertical Alignment

- Logical connectors are used in mathematics instruction from a very early age and continue to be an important component of mathematical academic language throughout a student's K–12 math experience. As students progress from grade 6 to AP Calculus, they should have a variety of opportunities to increase their fluency with the logical connectors they encounter.

Materials/Preparation

- *Student Handout 4.5a:* "Advanced Sentence Frames—Logical Connectors"
- Active Learning Methodologies (see the *Introduction*)

Instructions

- Arrange the class into collaborative groups of two to four students. Distribute one *Student Handout 4.5a:* "Advanced Sentence Frames—Logical Connectors" to each group.
- Ask student volunteers to read the first example given in the student handout.
- Lead your class through a discussion about the sentence frame.
- Once you are satisfied that your students have a basic understanding of the sentence frame and the mathematics involved, ask your class to read the sentence frame again chorally.
- Repeat for each of the other four sentence frames given in *Student Handout 4.5a.*
- Ask each group to create a sentence for each of the five sentence frames given in the handout.
- Emphasize to your students that the example given is just an example. The sentence they create for each sentence frame does not have to relate to the mathematics of the example.
- Use one your favorite Active Learning Methodologies to share-out your students' sentences.

Higher-Level Questions

Level Two and Three

- Ask your students to turn the sentences they wrote in the activity into Level Two or Level Three questions.

Formative Assessment

- Monitor your students' use of logical connectors in their oral and written descriptions of the mathematics they explore in your class. Are your students using logical connectors as they discuss mathematics?

Advanced Sentence Frames—Logical Connectors

Use each sentence frame and the example given to create a sentence of your own.

1. If ______________________________, then _________________________________.

   Example: If **both coordinates of an ordered pair are negative,** then **the point is located in the third quadrant.**

2. A _______________ is _________________ because ___________________________.

   Example: A **trapezoid** is **a quadrilateral** because **it has four sides.**

3. Every ___________________ is either ___________________ or __________________.

   Example: Every **fraction** is either **proper** or **improper**.

4. Since ______________________, it must ____________________________________.

   Example: Since **the discriminant of $f(x) = 0$ is positive,** it must **have two real solutions.**

5. ________________ and _______________. Therefore, ___________________________.

   Example: **A < B** and **B < C**. Therefore, **A < C.**

4.6: Academic Language—Looking at Multiple Meanings

Topic

- Distinguishing the meanings of academic terminology that have unique definitions in different subject areas

Objectives

Students will:

- Work together in collaborative groups to brainstorm definitions of high-use academic language words that have multiple meanings
- Write definitions, sentences, and examples of academic vocabulary words that have multiple meanings

Timeline

- 15–30 minutes to explore the ways words are used differently in different subject areas

WICR Strategies

- Writing to Learn
  — Write definitions and sentences for high-use academic vocabulary words
- Collaboration
  — Work in small groups to brainstorm words that have multiple meanings that are dependent on the context in which the word is used
- Reading to Learn
  — Practice reading and speaking the academic language of mathematics

NCTM Standards

Communication

Instructional programs from pre-kindergarten through grade 12 should enable all students to—

- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others; and
- use the language of mathematics to express mathematical ideas precisely.

Rationale

One of the most difficult aspects of learning the academic language of mathematics is distinguishing the definitions of words that have multiple meanings in various subject areas. Words such as function, area, linear, and expression all have definitions both in and out of the context of mathematics. A solution in mathematics is much different than a solution in Chemistry or Social Studies. As students labor to acquire the academic language of mathematics, they need opportunities to explore high-use vocabulary words that have multiple definitions. By engaging in activities like *"Academic Language—Looking at Multiple Meanings,"* students can explore the different ways vocabulary words are used in various subject areas.

Vertical Alignment

Exploring multiple-meaning words can occur at all levels of instruction. A sixth grade student can see that the word *root* when used in conjunction with *square* has a much different definition in mathematics than it does in science when discussing the *root* system of a plant. Geometry students should be aware that a *chord* in mathematics is not the same as a guitar *chord* or striking a *chord* with a story or poem. AP Calculus students must know that *increasing* does not necessarily mean enlarging in size or amount. Students of all ages need to be given the ability to distinguish between the definitions of multiple meaning words and to apply the math-specific meaning when doing mathematics.

Materials/Preparation

- *Student Handout 4.6a:* "Multiple Meanings"
- *Student Handout 4.6b:* "Multiple Meanings: 'Function' "
- Active Learning Methodologies (see the *Introduction*)
- *Optional:* Student math texts or sample state test items

Instructions

- Arrange the class into collaborative learning groups of two to four students.
- Distribute one blank *Student Handout 4.6a:* "Multiple Meanings" to each group and one example *Student Handout 4.6b:* "Multiple Meanings: 'Function' " to each student within the group.
- Facilitate a whole group discussion of the example handout. Ask if anyone can think of another definition for Function they have seen in another class that is not shown on the handout.

— **Option 1:** Ask each group to brainstorm a list of math words that they believe to have multiple meanings. Have each group choose one of their words and complete *Student Handout 4.6a.*

— **Option 2:** Ask each group to go through the current chapter in their math text and pull out the multiple-meaning words. Have each group choose one of their words and complete *Student Handout 4.6a.*

— **Option 3:** Ask each group to examine a set of sample state test items for multiple meaning words. Have each group choose one of their words and complete *Student Handout 4.6a.*

- Use one your favorite Active Learning Methodologies to share-out the student work.

Higher-Level Questions

Level Two and Three

- Ask students to examine the key words that identify a question as Level One, Two, or Three to determine if any of the key words have unique definitions outside of mathematics (see Activity 2.1: "The Evolution of a Great Question: Costa's Triples" in Unit Two: "Inquiry in Mathematics").
- As a quickwrite activity, ask your students to compare and contrast the multiple definitions they identify for a key word.

Formative Assessment

- As students brainstorm their list of multiple-meaning words, ask yourself if any of these words could be causing cognitive conflict for your students. Are any of your students confusing the math-specific meaning of a word with its meaning in conversational English?

Subject: ***Mathematics***

Definition:

Sentence:

Example or Illustration:

Subject:

Definition:

Sentence:

Example or Illustration:

Subject:

Definition:

Sentence:

Example or Illustration:

Subject:

Definition:

Sentence:

Example or Illustration:

"Function"

Subject: ***Mathematics***

Definition: A rule that assigns for each value of a first set of numbers a unique value in a second set of numbers.

Sentence: We can use the vertical line test to determine if a relation is a *function.*

Example or Illustration:

$$f(x) = 2x^2 - 5$$

Subject: ***English Language Arts***

Definition: The grammatical role of a linguistic form.

Sentence: The *function* of a preposition is to show location.

Example or Illustration:

The mouse ran *under* the chair.

Subject: ***Social Studies***

Definition: The contribution made by a sociocultural phenomenon to an ongoing social system.

Sentence: The main *function* of the U.S. Supreme Court is to interpret the Constitution of the United States.

Example or Illustration:

Branch	*Function*
Executive	Enforces Law
Legislative	Enacts Law
Judicial	Interprets Law

Subject: ***Conversational English***

Definition: Any ceremonious public or social gathering or occasion.

Sentence: I only wear my tuxedo for formal *functions.*

Example or Illustration:
Wedding
Quinceañera
Birthday Party
Bar Mitzvah

"AVID relies on the competence and compassion of people. It is not simply a program or a set of materials. It is a process, philosophy, and an attitude that provides a strong base for teaching all students well." "AVID provides support to young people to help insure their success in school." "An unexpected bonus is that about half of our AVID tutors who have come to us as non-education majors have now decided to become teachers. That is exciting."

—Dr. Debra Duvall, Superintendent
Arizona's Mesa Public Schools

Resources for Use During Write Path Training

The Write Path II: Mathematics training expands upon a wide variety of activities designed to illustrate the use of Writing, Inquiry, Collaboration, and Reading (WICR) in the mathematics classroom for you, the practitioner, experienced in *The Write Path I* training. The goal of the training is to stimulate your thinking, and to spark new and creative applications for use in your daily practice. We hope that you will personalize the activities by substituting the Active Learning Methodologies and mathematics content of your choice.

During training, use the following Training Resources to record your innovative ideas:

The *Action Lesson Plan.*

During training, you will be given an opportunity to develop an *Action Lesson Plan;* a blank template has been provided for this purpose.

The *Reflective Journal.*

Provided so that you can practice using them before introducing them to your students.

Cornell Notes: Seven Formats.

Provided so that you can practice the note-taking strategy on different forms during the training and in the process, find the one that will work best for you and your students.

The Write Path II: Mathematics training will be an active training. You will not only learn more about WICR lessons and Active Learning Methodologies, you will have the opportunity to actively engage them. To facilitate this active involvement, several training handouts are provided. The original handouts are embedded in the lesson materials of *The Write Path II: Mathematics'* text and will remain copy-ready.

Action Lesson Plan Template

Title ____

Topic

•

Objectives

Students will:

•

•

Timeline

•

WICR Strategies

- Writing to Learn
  —
  —
  —
- Inquiry
  —
  —
  —
- Collaboration
  —
  —
  —
- Reading to Learn
  —
  —
  —

NCTM Standards

•

•

•

•

Rationale

Vertical Alignment

-

Materials/Preparation

-
-
-
-

Instructions

Higher-Level Questions

Level Two

-
-

Level Three

-
-

Formative Assessment

-

Reflective Journal

Name: ______________________________ Date: ____________________

Name of the Course: ____

In class today we…

(Describe what topics were covered, what problems were worked on, what presentations were made by students and teachers, or how otherwise you used your time.)

I learned…

(Sum it up in a few sentences using standard English. Be specific, include examples as evidence of your understanding.)

One or two questions or comments I still have are…

(You may start this sentence with, "I don't understand how to…," or "I didn't understand the difference between…," or "I still don't know why…," or "When am I supposed to…," or "At last I understand…,")

Question #1:

Question #2:

My plan for getting help with my homework, should I need it, is to…

This Page is Being Left Blank Intentionally

Cornell Note-taking Checklist

Name____ Period_______________

Do your notes have the following characteristics?

1. Consistent Cornell physical format, notes dated and titled, readable — 3 pts
2. Use of abbreviations, key words/phrases, underlining, starring — 1 pt
3. Main ideas are easily seen; correct sequencing of information — 1 pt
4. Questions are completed on left hand side; Level 2 and 3 questions — 3 pts
5. An accurate, complete summary follows the notes — 2 pts

Characteristics **Date**					
1. Consistent Cornell physical format, notes dated and titled, readable					
2. Use of abbreviations, key words/phrases, underlining, starring					
3. Main ideas are easily seen; correct sequencing of information					
4. Questions are completed on left hand side; Level 2 and 3 questions					
5. An accurate, complete summary follows the notes					
Total Points					

Rubric

Consistent Cornell physical format, notes dated and titled, readable

3. Vertical line drawn 2.5 inches from the left margin. Heading is complete with name, date, subject. The notes are titled. Notes are adequate in length.
2. Minor problem with format
1. No date or no title; short
0. Fails to use Cornell note-taking format or date and title are missing or notes are inadequate in length

Use of abbreviations, key words/phrases, underlining, starring

1. Techniques used throughout
0. Too much verbiage

Main ideas are easily seen; correct sequencing of information

1. Information is complete and in correct order
0. Notes confusing

Questions are completed on left hand side; Level 2 and 3 questions

3. A substantive number of higher order thinking questions are noted in the left margin which are answered in the notes to the right
2. Level 1 questions are many; level 2 and 3 questions minimal
1. Level 1 questions only
0. No questions in the left hand margin

An accurate, complete summary follows the notes

2. Detailed summary covers the main topics of the notes
1. Summary is generic or incomplete
0. Summary missing

Class Notes/Learning Logs/Textbook Notes

Level 2: sort, infer, analyze, sequence, organize, solve, explain, compare, contrast, classify, isolate, characterize, make analogies.

Level 3: conclude, criticize, reorganize, justify, judge, estimate, predict, speculate, make a model, extrapolate, apply a principle, interpret, hypothesize, if/then

Name: ______________________________

Class: ______________________________

Period: ________ Date: ______________________

Topic

Study/Review Questions

Connections, Summary, Reflection, Analysis:

Class Notes/Learning Logs/Textbook Notes

Level 2: sort, infer, analyze, sequence, organize, solve, explain, compare, contrast, classify, isolate, characterize, make analogies.

Level 3: conclude, criticize, reorganize, justify, judge, estimate, predict, speculate, make a model, extrapolate, apply a principle, interpret, hypothesize, if/then

Name: ______________________________

Class: ______________________________

Period: ________ Date: ______________________

Topic

Study/Review Questions	
Connections, Summary, Reflection, Analysis:	

Class Notes/Learning Logs/Textbook Notes

Level 2: sort, infer, analyze, sequence, organize, solve, explain, compare, contrast, classify, isolate, characterize, make analogies.

Level 3: conclude, criticize, reorganize, justify, judge, estimate, predict, speculate, make a model, extrapolate, apply a principle, interpret, hypothesize, if/then

Name: ______________________________

Class: ______________________________

Period: ________ Date: ______________________

Topic

Study/Review Questions

Connections, Summary, Reflection, Analysis:

Power Math Notes

Name:______________________________ Date: ______________________

Title: ____

Tool Box:

Connections, Summary, Reflection, Analysis:

Study Questions:

Two-Column Homework

Name: ______________________

Topic: ______________________

Class: ______________________

Period: ________ Date: ______________________

Questions	**Show all your work and box your solution**

Connections, Summary, Reflection, Analysis:

Questions	Notes

Connections, Summary, Reflection, Analysis:

Two-Column Quiz

Topic: ____________________

Name: ____________________

Class: ____________________

Period: ________ Date: ____________________

Questions	**Show all your work and box your solution**

Connections, Summary, Reflection, Analysis:

Questions	Notes

Connections, Summary, Reflection, Analysis:

Five Ws for Writing Prompts

Who is writing?

What is the "Frame/Questions?"

Who is reading?

What is the format of the writing?

What are the reminders?

What is Math Like?

List the words you would use to describe Math to one of your friends.	List the feelings you have when doing math in or out of school.	List the things (nouns) that describe what math is like for you.

Write a complete paragraph responding to the following prompt:

For me, math is most like ...

This Page is Being Left Blank Intentionally

Sample Inquiry Cube

4

$4x - y = 8$

3

2

$2x + y = 7$

3

1

$2x - y = 0$

2

2

$x - 2y = -4$

3

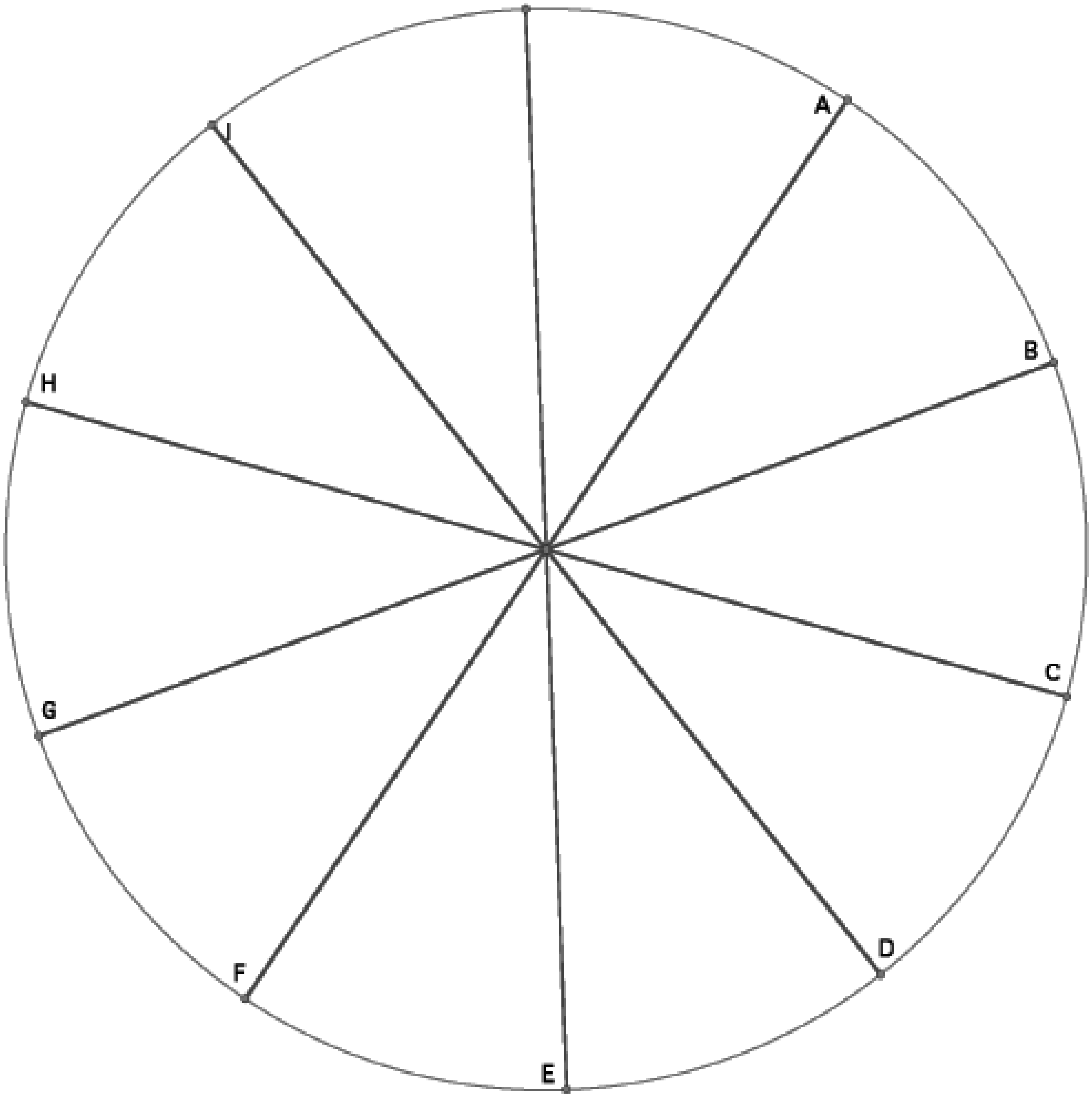
2

$x + 2y = 5$

1

This Page is Being Left Blank Intentionally

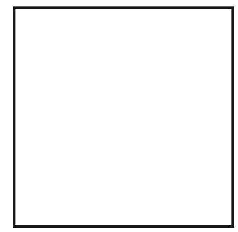
Volume of Cones Template



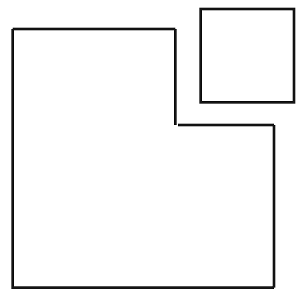
This Page is Being Left Blank Intentionally

The Difference of Two Squares

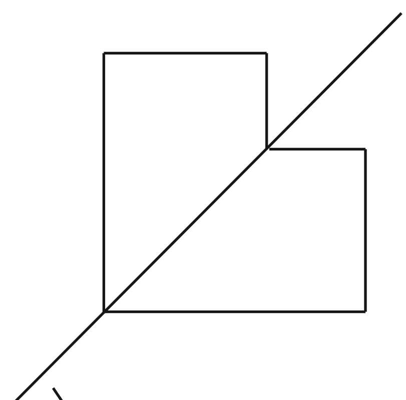
1. Each person in your group needs to have a square. No two squares can be the same size. The smallest square must have an area larger than one square unit.



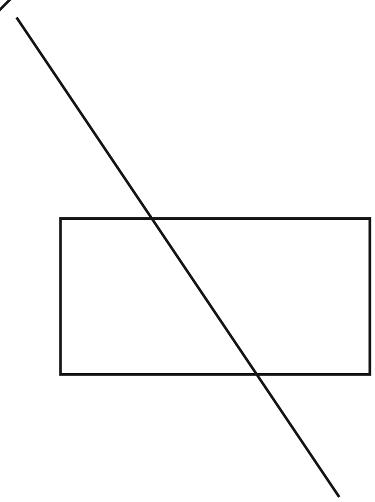
2. Each person needs to cut out a smaller square from the corner of his/her larger square. These smaller squares can be the same or different sizes. Put this square aside for a moment.



3. Imagine that the smaller square is still there. On what is left of the larger square, draw a diagonal line through the corners that would have bisected the smaller square. Cut along that line.



4. Rearrange the two irregular quadrilaterals you have just created into a rectangle. If this seems difficult, have someone in your group who is spatially talented assist you.



5. Fill in the chart below:

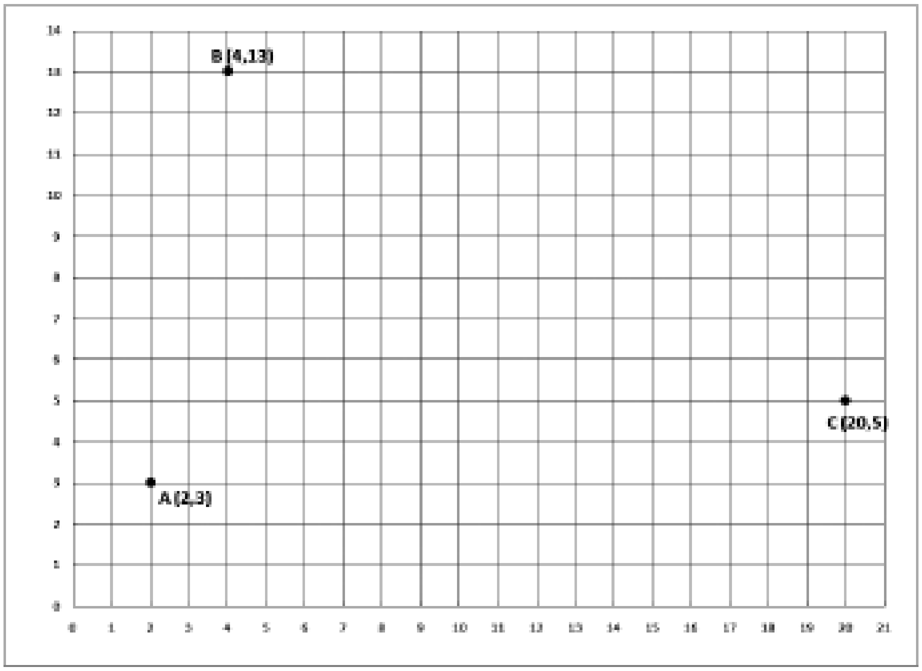
Group Member Names (Initials)	Large Square Side Length	Large Square Area	Small Square Side Length	Small Square Area	Difference of the Area of the Two Squares	Rectangle Length	Rectangle Width	Rectangle Area

6. Now summarize your findings. Use words, pictorial symbols, and/or algebraic notation to explain how this activity proves that your findings are true.

This Page is Being Left Blank Intentionally

Networks

Directions: Find the shortest distance necessary to connect the points A, B, and C by adding a junction point, D. Each member of your group should pick a different location for point D and find the sum of the distances DA, DB, and DC. Draw your shortest network on the coordinate plane below:



My point D (_____, _____)

DA = _________________ + DB _________________ + DC = _________________

Group Results:

Point								
Distance								

The Whole Picture

Instructions

For each of the four representations shown below:

- Determine the other three representations.
- Give the mathematical situation a title.
- Complete the "Four Representations of Function" table on the following page.
- Using a green highlighter, indicate where you find the slope in each of the four representations.
- Using a red highlighter, indicate where you find the y-intercept in each of the four representations.
- You should have four final products, one for each of the representation shown below.

Representation 1:

$C(m) = \$60 + \$15h$

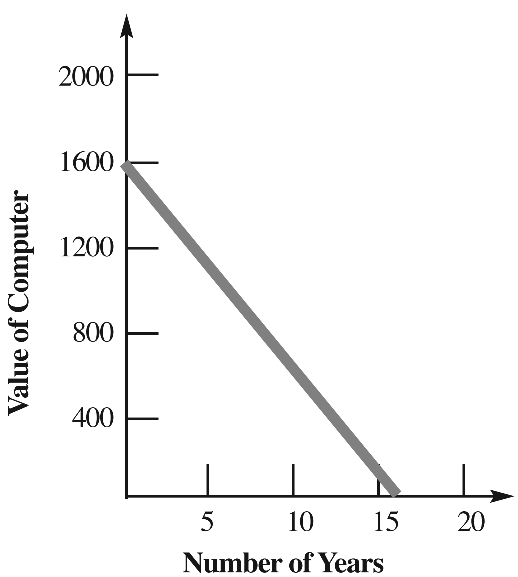
Representation 2:

Denise and Don took a hot air balloon trip for their anniversary. On the way down, the hot air balloon descended from a height of 1000 feet to the ground, at a constant rate of 2 feet per second.

Representation 3:

Number of Text Messages	0	10	20	30	40
Total Cost	$25	$26	$27	$28	$29

Representation 4:



Four Representations of Function

SITUATION	GRAPH

EQUATION	TABLE OF VALUES

Investigating Area under the Curve

Part 1:

1. Trace the outline of your hand onto grid paper.
2. Color the squares that lie fully inside the outline of your hand. (If any part of the square is outside the outline, don't color it.)
3. In another color, color the squares that lie on the outline of your hand. If any part of the square touches the outline, color the ENTIRE square.
4. (a) ________ Number of squares fully inside outline

   (b) ________ Number of squares partially inside outline

   (c) ________ Total number of squares colored
5. What does your answer to part (a) represent?

6. What does your answer to part (c) represent?

7. Write a ratio relating the number of squares to the number of inches.

8. Use this ratio to convert the number of squares you got for question 4, parts (a) and (c), to square inches.

(a) _________ $inch^2$

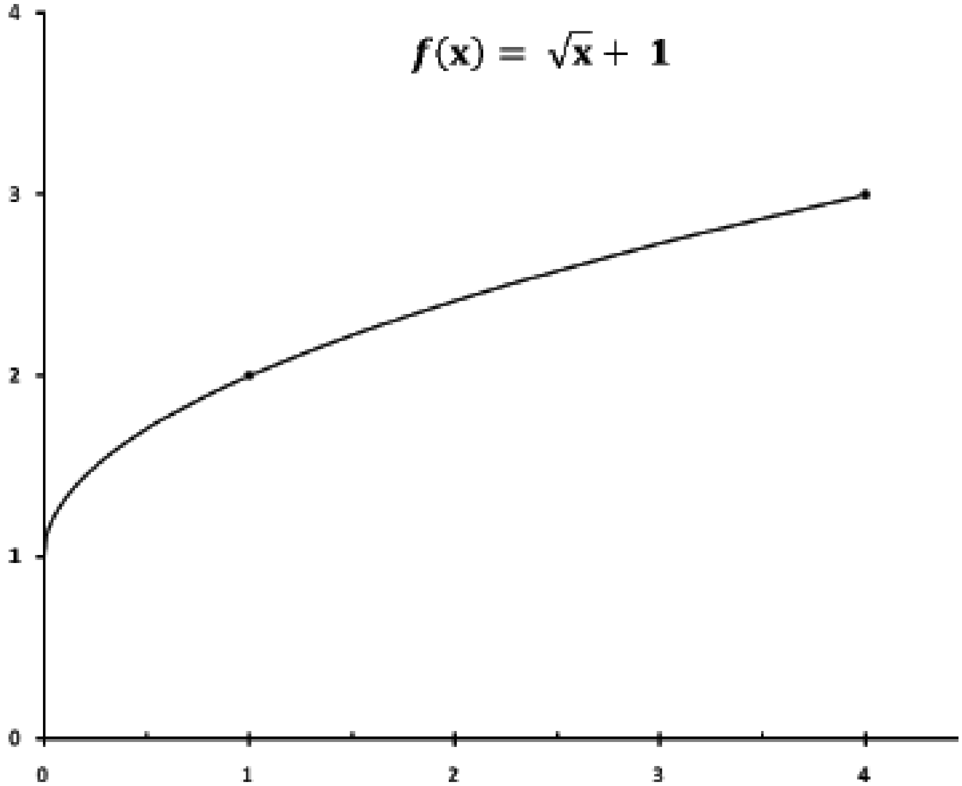
(c) _________ $inch^2$

9. Use your two answers from question 8 to estimate the area of your hand in square inches.

10. Compare your estimate to that of your classmates. Does your estimate make sense? Why?

Part 2:

1. Divide the following graph of $f(x) = \sqrt{x} + 1$ from 0 to 4 into four equal intervals.



2. Form four rectangles by drawing a horizontal line from the left of the interval to the right.
3. What is the width of each rectangle?
4. What is the height of the first rectangle?
   the second rectangle?
   the third rectangle?
   the fourth rectangle?
5. What is the area of the first rectangle?
   the second rectangle?
   the third rectangle?
   the fourth rectangle?
6. What is the total area of the four rectangles?

7. Do you think the answer to question 6 is an over estimate or under estimate to the area under the curve? Why?

Three-Column Proofs

Steps	Property or Reason	Verbal Description

This Page is Being Left Blank Intentionally